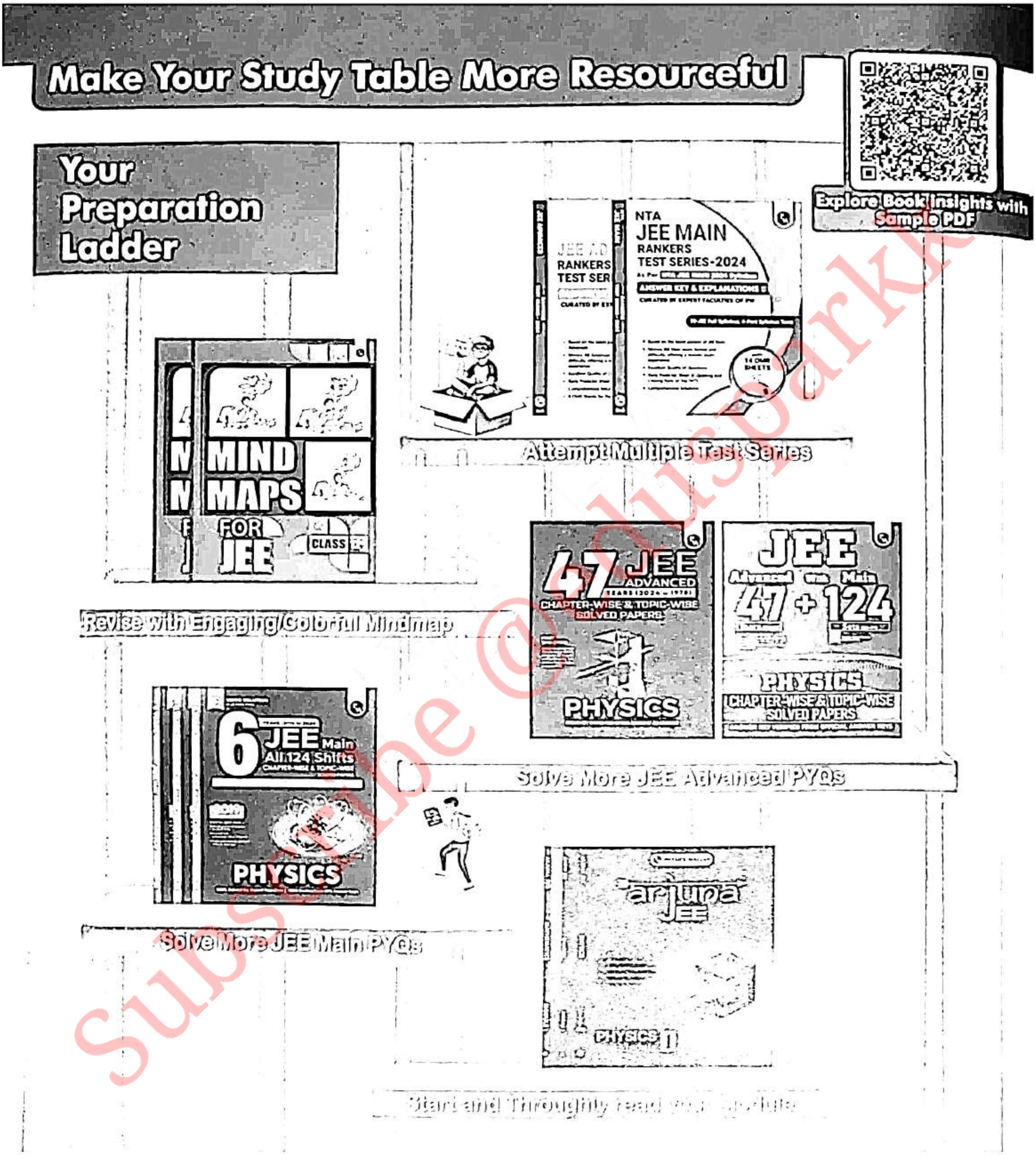

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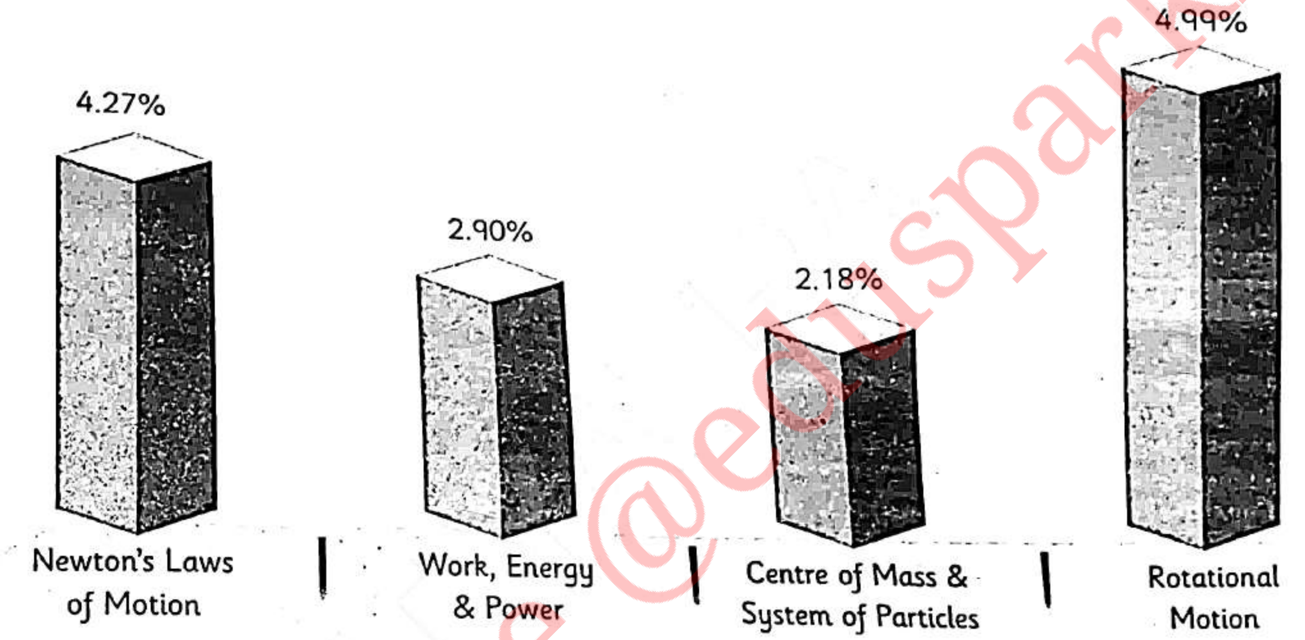

Chapter Wise Weightage (in %) Based on Past 6 Years JEE Main Papers

PHYSICS

Module 2

Practice Focus: In this module for the emotional well being we will practice Focus along with our regular academic learning

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PREFACE

A dedicated team at Physics Wallah (PW) works diligently to provide students with quality content for the **JEE** exam.

From the beginning, the whole content team comprising Associate faculties, DTP operators, Proofreaders and others are involved in shaping the material to their best knowledge and experience to produce powerful content for the students.

Faculties have adopted a new style of presenting the content in easy-to-understand language and have provided the team with their guidance and supervision throughout the creation of this Study Material.

Physics Wallah (PW) strongly believes in conceptual and fun-based learning. PW provides exam-focused content designed to offer clarity and value to students.

A plethora of **JEE Study Material** is available in the market but PW professionals are continuously Working to provide comprehensive study material for our **JEE** students.

This Study Material adopts a multi-faceted approach to master and understanding the concepts by having a rich diversity of questions asked in the examination and equip the students with the knowledge for the competitive exam.

The main objective of the study material is to provide a large number of quality problems with varying cognitive levels to facilitate the teaching-learning of concepts that are presented through the book.

It is widely used by aspirants because of its easy-to-understand language and clear illustrations with diagrams, mnemonics, flow charts and tables.

Students can benefit themselves by attempting the exercise given in this Study Material for the self-assessment and also mastering the basic techniques of problem-solving.

The questions are strictly designed in accordance with the exam relevant topics that helps to develop examination temperament within the students.

By mastering the Physics Wallah (PW) study material curated by the PW team, students can improve their chances of qualifying for the **JEE** exam and achieving a strong rank.

In each chapter, for better understanding, questions have been classified according to the latest syllabus of **JEE**

- The nature and diversity of the questions help students to ace the examination
- Quality questions to strengthen the concept of the topic at the zenith level

BOOK FEATURES

- Detailed theory with Train Your Brain and Concept Application Questions
- "Aarambh" Section for Solved Miscellaneous Examples
- School Level Problems for School Going Students
- "Prarambh" Section for Topicwise, "Prabal" Section for JEE Main Level and "Parikshit" Section for Advance Level MCQs covered in exercises
- Match the Column, Comprehension Based, Numerical Type and Multicorrect questions covered for complete practice
- JEE Past Year Questions
- PW Challengers Problems for Difficult Level Questions
- Elaborated Solutions Booklet you can get separately

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3P PROGRESSOR

PLANNER

CHAPTER NAMES	Start Date of Module	End Date of Module	Solve More PYQs	Revision Period	Re-revision Period (Before Test Series)
Newton's Laws of Motion					
Work, Energy & Power					
Centre of Mass & System of Particles					
Rotational Motion					

PROGRESS TRACKING

CHAPTER NAMES	Solved Examples	School Level	Topicwise	Main Level	Advanced Level	PYQs	Challengers
Newton's Laws of Motion	☐	☐	☐	☐	☐	☐	☐
Work, Energy & Power	☐	☐	☐	☐	☐	☐	☐
Centre of Mass & System of Particles	☐	☐	☐	☐	☐	☐	☐
Rotational Motion	☐	☐	☐	☐	☐	☐	☐

PREP POINTERS

Note down the topic you want to revise or section you want to re attempt while your preparation.

"The Power of Focus: Transforming Effort into Success"

"Focus is the compass that directs your efforts toward success. With clarity and determination, discover the art of channeling your energy and achieving your dreams."

As a JEE aspirant, focus is not just a skill—it's a habit that defines your preparation. With focus, even the most challenging topics can become manageable. Without it, even the simplest concepts might feel overwhelming.

> "Concentrate all your thoughts upon the work at hand. The sun's rays do not burn until brought to a focus."
> – Alexander Graham Bell

CLEAR GOALS DRIVE ACTIONS..
EFFECTIVE ACTIONS.

Without Focus

- Without a clear direction, efforts become scattered, leading to procrastination and distractions.
- This often results in frustration, confusion, and lost opportunities.

With Focus

- Clear goals drive effective actions, while regular, disciplined study keeps you on track.
- This leads to better retention, deeper understanding, and improved performance.

SCENARIOS

Without Focus		With Focus
Feeling demotivated, overwhelmed, and doubting ability.	After a Difficult Mock Test	Analyzing mistakes, learning from feedback, staying motivated for improvement.
Ignoring the feedback or taking it negatively.	Feedback from a Teacher	Taking the feedback constructively, using it as a guide for improvement.
Feeling drained, distracted, or disengaged after long hours.	Studying Long Hours	Maintaining energy and engagement with focused breaks and regular self-assessments.
Getting frustrated or giving up.	Facing a Tough Concept	Breaking the concept into smaller parts, staying persistent and asking for help when needed.
Feeling discouraged, jealous, or insecure.	Comparing Scores with Friends	Using comparisons as motivation to improve, not as a measure of self-worth.
Resenting the sacrifice and feeling left out.	Missing a Social Event for Study	Realizing the long-term benefit of prioritizing studies, staying positive about the decision.
Panicking or losing focus due to disruptions.	Unexpected Challenges During Preparation	Staying calm, adapting to changes, and sticking to the plan with flexibility.
Skipping sessions or procrastinating, feeling unorganized.	Daily Study Routine	Sticking to a structured plan, feeling in control, and making steady progress every day.

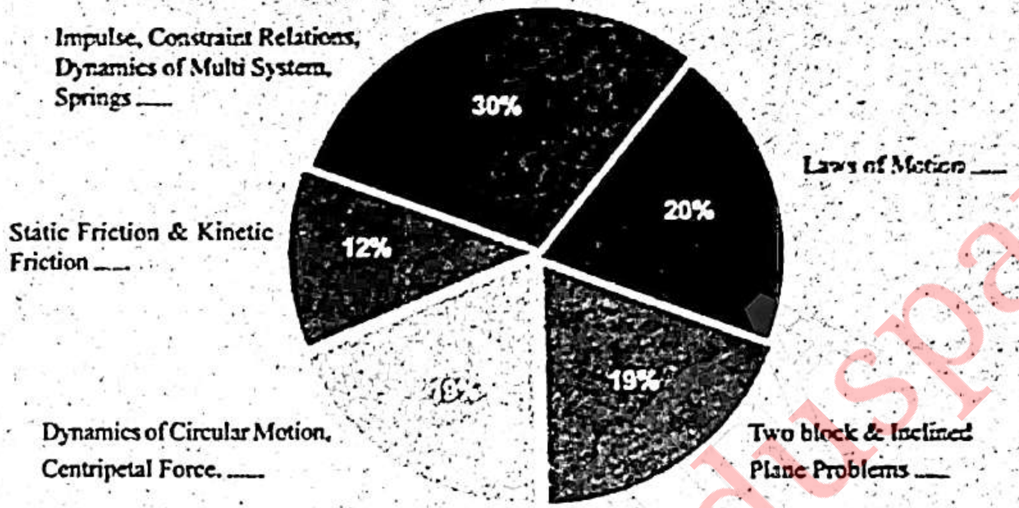
This demonstrates how adopting a mindset of Focus can help students navigate the stresses and challenges of exam preparation with more resilience and positivity. It highlights how a shift in perspective can transform obstacles into opportunities for growth and improvement.

CHAPTER

5

Newton's Laws of Motion

Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

FORCE

A force is a push or pull on a body which changes its state of motion. Body may speed up, retard or change its direction of velocity. Force is defined as an interaction between two bodies.

BASIC FORCES IN NATURE

(i) Gravitational force
(ii) Electromagnetic force
(iii) Strong nuclear force
(iv) Weak nuclear force

- Other well known forces like frictional force, elastic force, viscous force, spring force, intermolecular force are manifestations of electromagnetic forces.
- Electromagnetic force is about 10^{36} times stronger than gravitational force while weak nuclear forces are about 10^{25} times as strong as gravitational force.

  $F_N > F_E > F_W > F_G$

  $F_G : F_W : F_E : F_N :: 1 : 10^{25} : 10^{36} : 10^{38}$
- The gravitational and the electromagnetic forces are long range forces having infinite range while strong nuclear and weak nuclear forces are very short range forces.
- The strong nuclear force is the strongest force while the gravitational force is the weakest force of nature.
- Nuclear force is a non-central force and varies inversely with some higher power of distance.
- Beta decay can be explained only on the basis of weak nuclear forces.

NEWTON'S FIRST LAW OF MOTION OR LAW OF INERTIA

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external force to change that state.

Definition of force from Newton's first law of motion "Force is that push or pull which changes or tends to change the state of rest or of uniform motion in a straight line".

Inertia: Inertia is the property of the body due to which the body oppose the change of state of rest or of uniform motion by itself. Inertia of a body is measured by mass of the body.

Inertia ∝ mass: Heavier the body, greater is the force required to change its state and hence greater is inertia. The reverse is also true. i.e. lighter body has less inertia.

TYPES OF INERTIA

Inertia of rest: It is the inability of a body to change by itself, its state of rest.

- When we shake a branch of a mango tree, the mangoes fall down.
- When a bus or train starts suddenly the passengers sitting inside tends to fall backwards.
- The dust particles in a blanket fall off when it is beaten with a stick.

Inertia of motion: It is the inability of a body to change by itself its state of uniform motion.

- When a bus or train stop suddenly, a passenger sitting inside tends to fall forward.
- A person jumping out of a speeding train may fall forward.

Inertia of direction: It is the inability of a body to change by itself its direction of motion.

- When a car rounds a curve suddenly, the person sitting inside is thrown outwards.
- Rotating wheels of vehicle throw out mud. Mudguard over the wheels stop this mud.

NEWTON'S SECOND LAW OF MOTION

Rate of change of linear momentum of a body is directly proportional to the external force applied on it and the change in linear momentum takes place in the direction of force

$$\vec{F} \propto \frac{d\vec{p}}{dt} \quad or \quad \vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a} \text{ if } m = \text{constant}$$

$$\vec{F} = \vec{v}\frac{dm}{dt} \qquad \text{(as in case of conveyor belt)}$$

Special Points:

1. This law gives magnitude, unit and dimension of force.
2. There are two types of unit of force:
   (*a*) Absolute unit
   (*b*) Gravitational or Practical unit
   In M.K.S. : Newton (N)
   In M.K.S. : kg-wt or kg-f
   In C.G.S. : Dyne (dyne)
   In C.G.S. : gm-wt or gm-f
   1 kg-wt = 9.8 N
   1 gm-wt = 980 dyne

NEWTON'S THIRD LAW OF MOTION

It states that to every action, there is an equal (in magnitude) and opposite (in direction) reaction.

If a body A exerts force $\vec{F}$ on another body B, then B exerts a force $-\vec{F}$ on A. The two forces $\vec{F}$ and $-\vec{F}$ connected by Newton's third law are called action-reaction pair. Any one may be called 'action' and the other 'reaction'.

"Action and reaction acts on different bodies hence they never cancel each other".

Notes:

(i) Action-Reaction pair acts on two different bodies
(ii) Magnitude of force is same
(iii) Direction of forces are in opposite direction
(iv) For action-reaction pair there is no need of contact

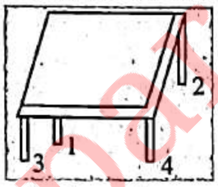
TYPES OF FORCES

Contact Forces

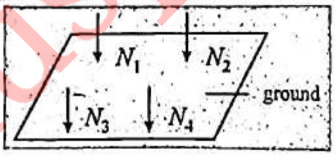
Whenever two bodies come in contact they exert forces on each other, that are called **contact forces.**

(*a*) **Normal Force (N):** It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force.

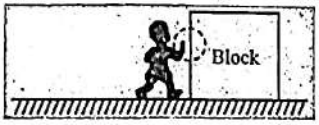
1. A table is placed on Earth as shown in figure.



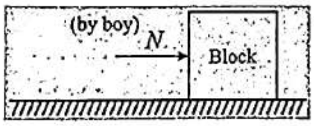
Here table presses the earth so normal force exerted by four legs of table on earth are as shown in figure.



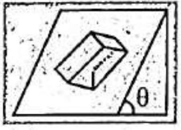
2. Now a boy pushes a block kept on a frictionless surface.



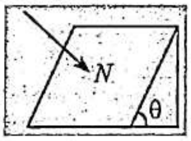
Here, force exerted by boy on block is electromagnetic interaction which arises due to similar charges appearing on finger and contact surface of block, it is normal force.



3. A block is kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block.



Normal force exerted by block on the surface of inclined plane is shown in figure.



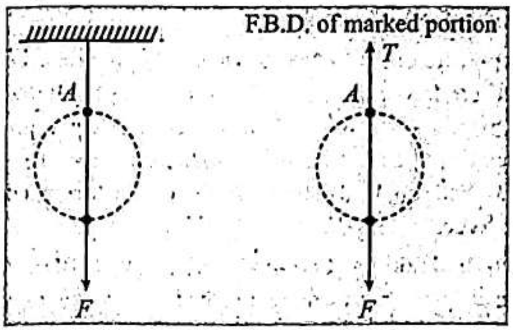
Force acts perpendicular to the surface

Notes:

1. Normal force acts in such a fashion that it tries to compress the body
2. Normal force is a self adjusting force, it comes in role when one surface presses the other.

(*b*) **Frictional Force (f):** It is the component of contact force parallel to the surface. It opposes the relative motion (or attempted motion) of the two surfaces in contact.

(*c*) **Tension:** Tension in a string is an electromagnetic force. It arises when a string is pulled. If a massless string is not pulled, tension in it is zero. A string suspended by rigid support is pulled by a force 'F' as shown in figure, for calculating the tension at point 'A' we draw F.B.D. of marked portion of the string; Here string is massless.



$\Rightarrow T = F$

String is considered to be made of a number of small segments which attracts each other due to electromagnetic nature. The attraction force between two segments is equal and opposite due to newton's third law.

Conclusion:

(i) Tension always acts along the string and in such a direction that it tries to reduce the length of string.

(ii) If the string is massless then the tension will be same along the string but if the string have some mass then the tension will continuously change along the string.

(iii) It is a self adjusting force.

(*d*) **Spring Force:** Every spring resists any attempt to change its length, the more you change its length the harder it resists. The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is spring constant or stiffness constant (units N/m).

Non-Contact Forces

These forces do not involve physical contact between two points and are also called **field forces** e.g. Gravitational force, electric force.

Weight: Weight of a body is the force with which earth attracts it. It is also defined as force of gravity or the gravitational force.

FREE BODY DIAGRAM

A free body diagram is a diagram showing the chosen body by itself, "free" of its surroundings, with vectors drawn to show the magnitudes and directions of all the forces applied to the body by the various other bodies that interact with it.

Be careful to include all the forces acting on the body, but the equally careful not to include any force that the body exerts on any other body. In particular, the two forces in an action-reaction pair must never appear in the same free-body diagram because they never act on the same body.

[Forces that one part of body exerts on other part never included, since these cancel out by Newton's third law]

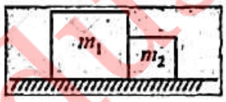
Notes:

- The forces exerted on bodies of a given system by bodies situated outside are called external forces.
- The forces of interaction between bodies composing a system are called internal forces.
- A single isolated force is physically impossible.
- Whenever one force acts on a body it gives rise to another force called reaction.
- Total internal force in an isolated system is always zero.

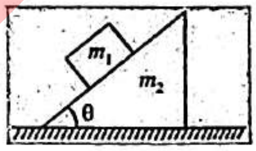
Train Your Brain

Example 1: Draw the *FBD* for the following individual parts of the systems

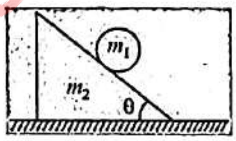
(*a*)



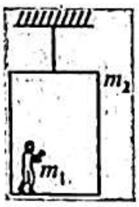
(*b*)



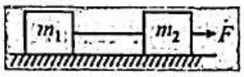
(*c*)



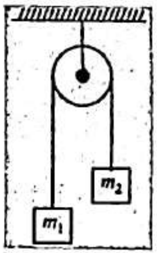
(*d*)



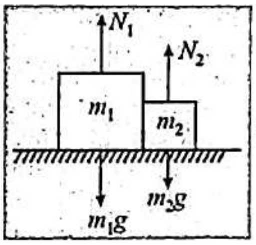
(*e*)



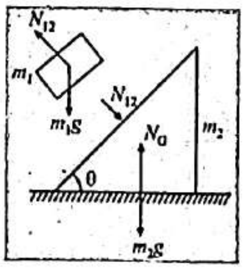
(*f*)



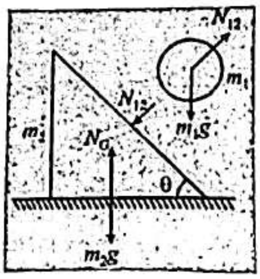
Sol. (*a*)



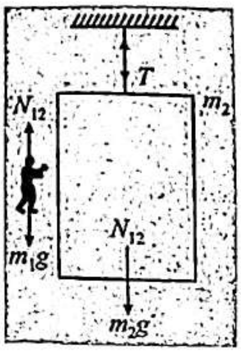
(*b*)



(c)



(d)



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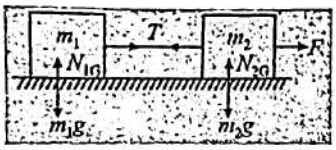
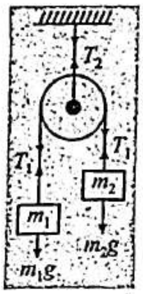
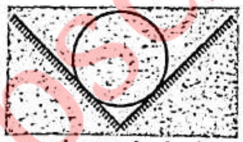


(f)

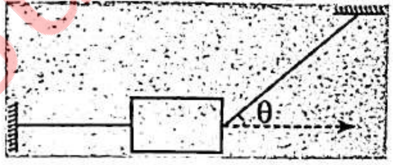


Concept Application

1. A cylinder of weight W is resting on a V-groove as shown in figure. Draw its free body diagram.



2. A block of mass m is attached with two strings as shown in figure. Draw the free body diagram of the block.



PROBLEM SOLVING STRATEGY

Newton's laws refer to a particle and relate the forces acting on the particle to its mass and to its acceleration. But before writing any equation from Newton's law, you should be careful about which particle you are considering. The laws are applicable to an extended body too which is nothing but collection of a large number of particles.

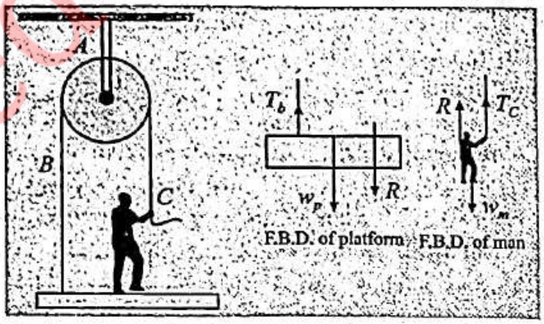
Follow the steps given below in writing the equations:

Step-1: Select the body: The first step is to decide the body on which the laws of motion are to be applied. The body may be a single particle, an extended body like a block, a combination of two blocks-one kept over another or connected by a string. The only condition is that **all the parts of the body or system must have the same acceleration.**

Step-2: Identify the forces: Once the system is decided, list down all the forces acting on the system due to all the objects in the environment such as inclined planes, strings, springs etc. However, any force applied by the system shouldn't be included in the list. You should also be clear about the nature and direction of these forces.

Step-3: Make a Free-body diagram (FBD): Make a separate diagram representing the body by a point and draw vectors representing the forces acting on the body with this point as the common origin.

This is called a free-body diagram of the body.



Look at the adjoining free-body diagrams for the platform and the man. Note that the force applied by the man on the rope hasn't been included in the FBD.

Once you get enough practice, you'd be able to identify and draw forces in the main diagram itself instead of making a separate one.

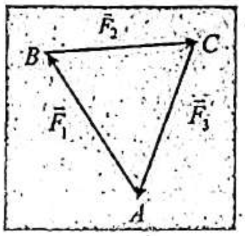
Step-4: Select axes and Write equations: When the body is in equilibrium then choose the axis in such a manner that maximum number of force lie along the axis.

If the body is moving in a straight line with some acceleration then first find out the direction of real acceleration and choose the axis. One is along the real acceleration direction and other perpendicular to it.

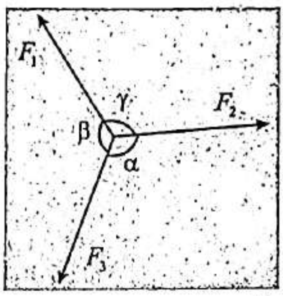
Write the equations according to the newton's second law ($F_{net} = ma$) in the corresponding axis.

EQUILIBRIUM OF CONCURRENT FORCES AND EQUILIBRIUM PROBLEMS

1. If all the forces working on a body are acting on the same point, then they are said to be concurrent.
2. A body, under the action of concurrent forces, is said to be in equilibrium, when there is no change in the state of rest or of uniform motion along a straight line.
3. The necessary condition for the equilibrium of a body under the action of concurrent forces is that the vector sum of all the forces acting on the body must be zero.
4. Mathematically for equilibrium $\sum \vec{F}_{\text{net}} = 0$ or $\sum F_x = 0$; $\sum F_y = 0; \sum F_z = 0$
5. Three concurrent forces will be in equilibrium, if they can be represented completely by three sides of a triangle taken in order.



6. **Lami's Theorem:** For concurrent forces $\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$



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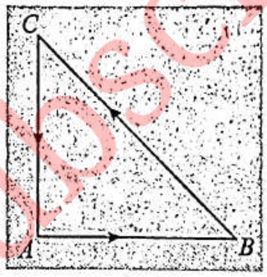
Example 2: Three forces starts acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle *ABC* (as shown). The particle will now move with velocity.



(*a*) $\vec{v}$ remaining unchanged
(*b*) Less than $\vec{v}$
(*c*) Greater than $\vec{v}$
(*d*) $\vec{v}$ in the direction of the largest force *BC*

Sol. (*a*) Given three forces are in equilibrium i.e. net force will be zero. It means the particle will move with same velocity.

Example 3: Two forces are such that the sum of their magnitudes is 18 N and their resultant is perpendicular to the smaller force and magnitude of resultant is 12 N. Then the magnitudes of the forces are

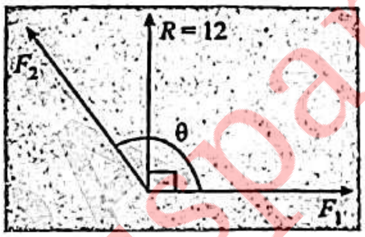
(*a*) 12 N, 6 N
(*b*) 13 N, 5 N
(*c*) 10 N, 8 N
(*d*) 16 N, 2 N

Sol. (*b*) Let two forces are F_1 and $F_2 \, (F_1 < F_2)$.

According to problem: $F_1 + F_2 = 18$ ... (*i*)

Angle between F_1 and resultant (R) is 90°

$$\therefore \ \tan 90 = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta} = \infty$$



$F_1 + F_2 \cos\theta = 0$

$\Rightarrow \cos\theta = -\dfrac{F_1}{F_2}$ ... (*ii*)

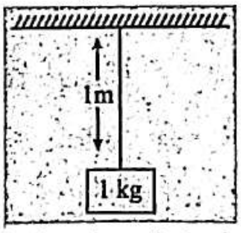
and $R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$144 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$ ... (*iii*)

By solving (*i*), (*ii*) and (*iii*)

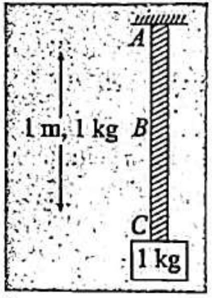
we get $F_1 = 5\text{N}$ and $F_2 = 13\text{N}$

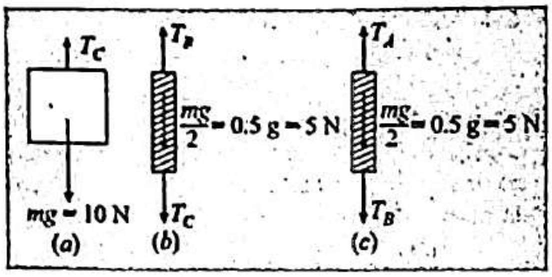
Example 4: A block of mass 1 kg is suspended by a string of mass 1 kg, length 1 m as shown in figure. ($g = 10 \text{ m/s}^2$) Calculate:



(*a*) The tension in string at its lowest point.
(*b*) The tension in string at its mid-point.
(*c*) Force exerted by support on string.

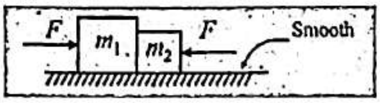
Sol.





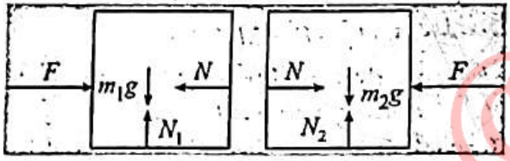
$$T_C - 10 = 0 \qquad \text{[Equilibrium of (a)]}$$
$$\Rightarrow T_C = 10\text{ N}$$
$$T_B - T_C - 5 = 0 \qquad \text{[Equilibrium of (b)]}$$
$$\Rightarrow T_B - 10 - 5 = 0$$
$$\Rightarrow T_B = 15\text{ N}$$
$$T_A - T_B - 5 = 0 \qquad \text{[Equilibrium of (c)]}$$
$$\Rightarrow T_A = 20\text{ N}$$

Example 5: Two blocks of masses m_1 and m_2 are placed on ground as shown in figure. Two forces of magnitude F act on m_1 and m_2 in opposite directions.



(i) Draw F.B.D. of masses m_1 and m_2.
(ii) Calculate the contact force between m_1 and m_2.
(iii) Calculate force exerted by ground surface on mass m_1 and m_2

Sol. (i)



It is obvious that both blocks will have same acceleration. If we take both block as one system then

$$F - F = (m_1 + m_2)\,a \left[\begin{array}{l}\text{Newton's second law in}\\ \text{horizontal direction}\end{array}\right]$$
$$\Rightarrow a = 0$$

Now take m_1 as a system

$$F - N = m_1\,a \left[\begin{array}{l}\text{Newton's second law in}\\ \text{horizontal direction}\end{array}\right]$$
$$\Rightarrow F - N = 0$$
$$\Rightarrow F = N$$
$$m_1g - N_1 = 0 \text{ [Equilibrium in vertical direction]}$$

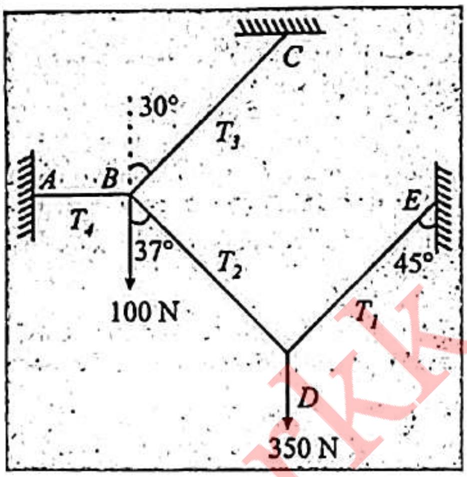
Now take m_2 as system

$$N - F = m_2\,a$$
$$\Rightarrow N - F = 0$$
$$N = F$$
$$m_2g - N_2 = 0 \text{ [Equilibrium in vertical direction]}$$
$$\Rightarrow N_2 = m_2g$$

(ii) $N = F$

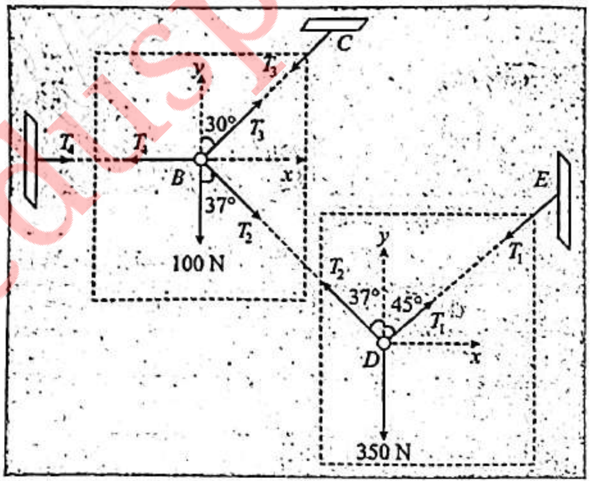
(iii) m_1g, m_2g.

Example 6: Two particles of masses 10 kg and 35 kg are connected with four strings at points B and D as shown in figure.



Determine the tensions in various segments of the string.

Sol. The free-body diagram of the whole system is shown in figure.



Analysing the equilibrium of point D:

$$\sum F_x = 0 \text{ or } T_1 \sin 45° - T_2 \sin 37° = 0 \qquad \ldots (i)$$

and $\sum F_y = 0$ or $T_1 \cos 45° + T_2 \cos 37° = 350 \ldots (ii)$

From (i), we have

$$T_2 = \frac{T_1 \sin 45°}{\sin 37°}$$

Now from (ii),

$$T_1 \cos 45° + \frac{T_1 \sin 45°}{\sin 37°} \times \cos 37° = 350$$

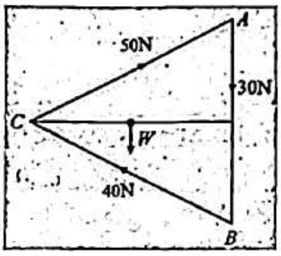
$$\text{or } \frac{T_1}{\sqrt{2}} + \frac{T_1}{\sqrt{2}} \times \frac{4}{3} = 350 \text{ or } \frac{T_1}{\sqrt{2}}\left[1 + \frac{4}{3}\right] = 350$$

$$\Rightarrow T_1 = 150\sqrt{2}\text{ N}$$

$$\text{and } T_2 = \frac{T_1 \sin 45°}{\sin 37°} = \frac{150\sqrt{2} \times \left(\frac{1}{\sqrt{2}}\right)}{3/5} = 250\text{ N}$$

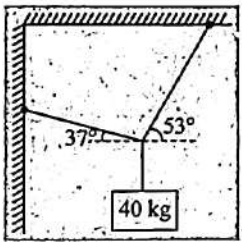
Concept Application

3. Forces of 30N, 40N, and 50N act along the sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ of an equilateral triangle ABC. The triangle is of mass 0.5 kg and kept in a vertical plane as shown in figure, with the side AB vertical. The net vertical force acting on the triangle will be ($g = 10$ m/s^2)

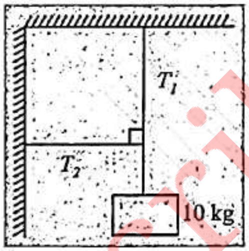


(a) 125 N upward (b) 5 N downwards
(c) 10 N upwards (d) 10 N downwards

4. The object weighs 40 kg and hangs at rest. Find the tensions (in N) in the three cords that hold it. [Take $g = 10$ m/s^2]



5. A block of mass $m = 10$ kg is suspended with the help of three strings as shown in Fig. Find the tensions T_1 and T_2 (in N). [Take $g = 10$ m/s^2]



MOTION OF A BLOCK ON A HORIZONTAL SMOOTH SURFACE

When a body is constraint to move in a straight line, then it is convenient to choose one axis along the direction of motion (say x-axis) and other perpendicular to it (say y-axis)

$\Sigma F_x = ma, \Sigma F_y = 0$

(a) When subjected to a horizontal pull.

(b) When subjected to a pull acting at an angle θ to the horizontal.

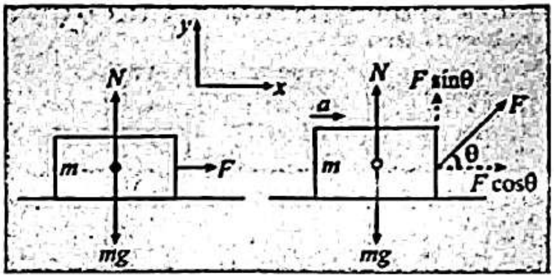


Figure-I

$N - mg = 0$ (y-axis)

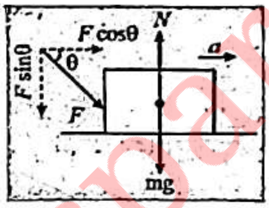
and $a = \dfrac{F}{m}$ (x-axis)

Figure-II

$N - mg + F\sin\theta = 0$ (y-axis)

$a = \dfrac{F\cos\theta}{m}$ (x-axis)

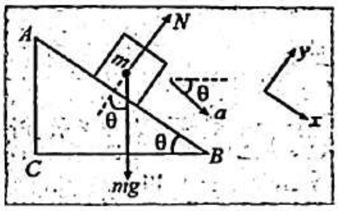
(c) When subjected to a push acting at an angle θ to the horizontal.



$N - mg - F\sin\theta = 0$ (for y-axis) and $ma = F\cos\theta$ (for x-axis)

$a = \dfrac{F\cos\theta}{m}$

MOTION OF A BODY ON A SMOOTH INCLINED PLANE

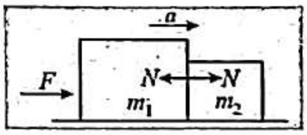


$N - mg\cos\theta = 0$ (for y-axis)

and $ma = mg\sin\theta$ (for x-axis), $a = g\sin\theta$

MOTION OF BODIES IN CONTACT

(a) **Two bodies are kept in contact and force is applied on the body of mass m_1**

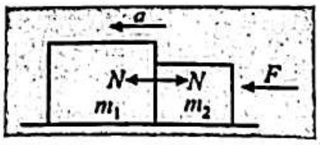


As both mass have same acceleration $a = \dfrac{F}{m_1 + m_2}$

for block 1 applying Newton's II law in horizontal direction.

$m_1 a = F - N \Rightarrow N = \dfrac{m_2 F}{m_1 + m_2}$

(*b*) **Two bodies are kept in contact and force is applied on the body of mass m_2**



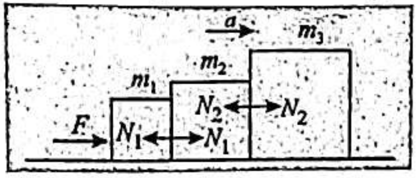
$$a = \frac{F}{m_1 + m_2}$$

for block I applying Newton's II law in horizontal direction.

$$m_1 a = N$$

$$\Rightarrow N = \frac{m_1 F}{m_1 + m_2}$$

(*c*) **Three bodies are kept in contact and force is applied on the body of mass m_1**



Since all three masses have same acceleration

$$a = \frac{F}{m_1 + m_2 + m_3} \quad \text{... (i)}$$

$m_1 a = F - N_1$ (for m_1) ... (*ii*)

$m_2 a = N_1 - N_2$ (for m_2) ... (*iii*)

$m_3 a = N_2$ (for m_3) ... (*iv*)

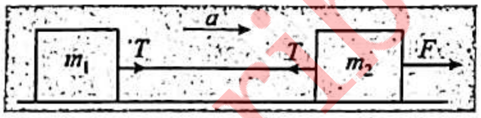
From equation (*i*), (*ii*), (*iii*) and (*iv*)

$$N_1 = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

$$N_2 = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

MOTION OF CONNECTED BODIES

(*a*) **Two bodies are connected by a string and placed on a smooth horizontal surface**



$m_1 a = T$ (for m_1) ... (*i*)

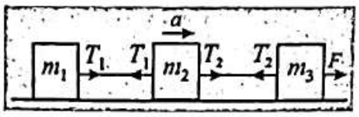
$m_2 a = F - T$ (for m_2) ... (*ii*)

From (*i*) and (*ii*)

$$a = \frac{F}{m_1 + m_2}$$

$$T = \frac{m_1 F}{m_1 + m_2}$$

(*b*) **When three bodies are connected through strings as shown in figure and placed on a smooth horizontal surface.**



$m_1 a = T_1$ (for m_1) ... (*i*)

$m_2 a = T_2 - T_1$ (for m_2) ... (*ii*)

$m_3 a = F - T_2$ (for m_3) ... (*iii*)

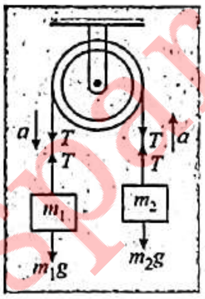
From (*i*), (*ii*) and (*iii*)

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

$$T_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)}$$

$$T_2 = \frac{(m_1 + m_2)F}{(m_1 + m_2 + m_3)}$$

(*c*) **When two bodies of mass m_1 and m_2 are attached at the ends of a string passing over a pulley** (neglecting the mass of pulley).



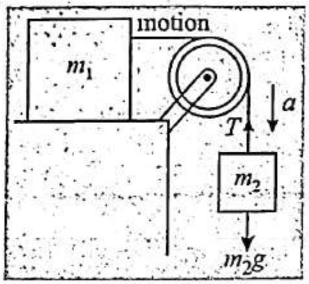
$m_1 a = m_1 g - T$ (for m_1) ... (*i*)

$m_2 a = T - m_2 g$ (for m_2) ... (*ii*)

From (*i*) and (*ii*)

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)},\ T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)g$$

(*d*) **When two bodies of masses m_1 and m_2 are attached at the ends of a string passing over a pulley in such a way that mass m_1 rests on a smooth horizontal table and mass m_2 is hanging vertically**



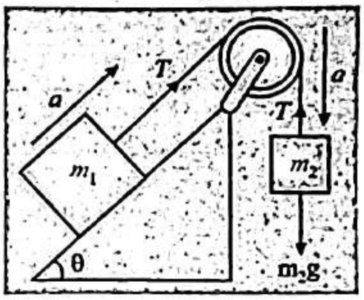
$m_1 a = T$ (for m_1) ... (*i*)

$m_2 a = m_2 g - T$ (for m_2) ... (*ii*)

From (*i*) and (*ii*)

$$a = \frac{m_2 g}{(m_1 + m_2)},\ T = \frac{m_1 m_2 g}{(m_1 + m_2)}$$

(*e*) **If in the above case, mass m_1 is placed on a smooth inclined plane making an angle θ with horizontal**



$m_1a = T - m_1g\sin\theta$ (for m_1) ... (*i*)

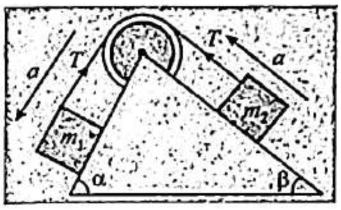
$m_2a = m_2g - T$ (for m_2) ... (*ii*)

From (*i*) and (*ii*)

$$a = \frac{(m_2 - m_1\sin\theta)g}{m_1 + m_2},\ T = \frac{m_1m_2g(1+\sin\theta)}{m_1+m_2}$$

If the system remains in equilibrium, then $m_1g\sin\theta = m_2g$

(*f*) **If case (*d*), masses m_1 and m_2 are placed on inclined planes making angles α and β with the horizontal respectively.**



$m_1a = m_1g\sin\alpha - T$ (for m_1) ... (*i*)

$m_2a = T - m_2g\sin\beta$ (for m_2) ... (*ii*)

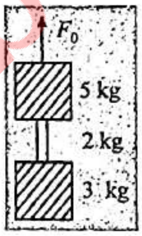
From (*i*) and (*ii*)

$\Rightarrow$ If $m_1\sin\alpha > m_2\sin\beta$ then $a = \dfrac{g(m_1\sin\alpha - m_2\sin\beta)}{(m_1+m_2)}$,

$$T = \frac{m_1m_2}{m_1+m_2}(\sin\alpha + \sin\beta)g$$

Train Your Brain

Example 7: A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward at an acceleration of 2 m/s² by an external force F_0. ($g = 10$ m/s²)

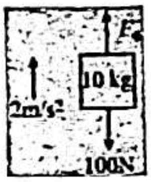


(*a*) What is the value of F_0?

(*b*) What is the net force on the rope?

(*c*) What is the tension at middle point of the rope?

Sol. For calculating the value of F_0, consider two blocks with the rope as a system.



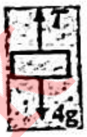
F.B.D. of whole system

(*a*) $F_0 - 100 = 10 \times 2$

$F = 120$ N ... (*i*)

(*b*) According to Newton's second law, net force on rope

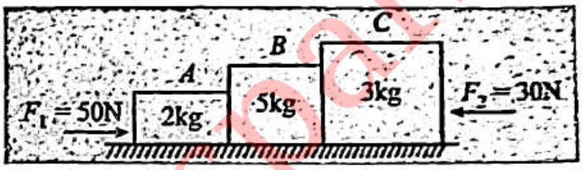
$F = ma = (2)(2) = 4$ N



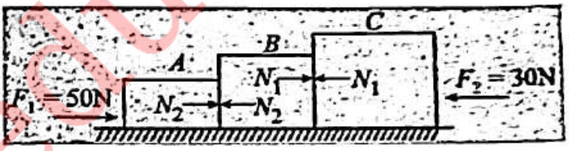
(*c*) For calculating tension at the middle point we draw F.B.D. of 3 kg block with half of the rope (mass 1 kg) as shown.

$T - 4g = 4 \times (2) = 48$N

Example 8: Find the contact force between the blocks and acceleration of the blocks as shown in figure.

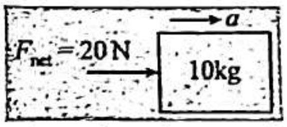


Sol. Considering all the three block as a system to find the common acceleration

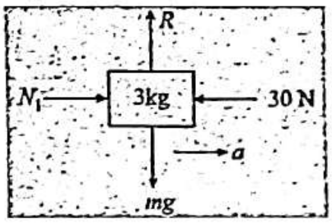


$F_{net} = 50 - 30 = 20$ N

$$a = \frac{20}{10} = 2\text{ m/s}^2$$

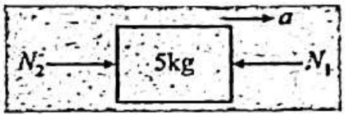


To find the contact force between *B* and *C* we draw F.B.D. of 3 kg block.



$\Sigma F_{net} = ma \Rightarrow N_1 - 30 = 3(2) \Rightarrow N_1 = 36$ N

To find contact force between *A* and *B* we draw F.B.D. of 5 kg block

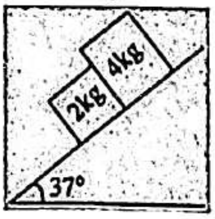


$\Rightarrow N_2 - N_1 = 5a$

$\Rightarrow N_2 = 5 \times 2 + 36$

$\Rightarrow N_2 = 46$ N

Example 9: Find out the contact force between the 2 kg and 4 kg block as shown in figure.



Sol. On an incline plane acceleration of the block is independent of mass. So both the blocks will move with the same acceleration ($g \sin 37°$) so the contact force between them is zero.

Concept Application

In Q.6 to Q.10, rigid blocks are kept in contact on a frictionless surface and acted upon by force (s). In each case, find

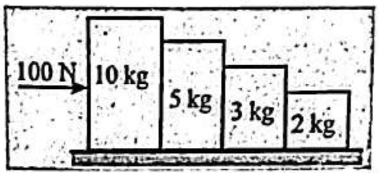
(*a*) Acceleration of each block (in m/s^2)

(*b*) Contact forces between the blocks (in N) and

(*c*) Net force on each block (in N).

Masses of blocks are indicated in diagram. Wherever required take $g = 10$ m/s^2.

6.



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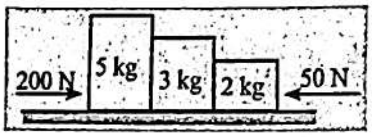

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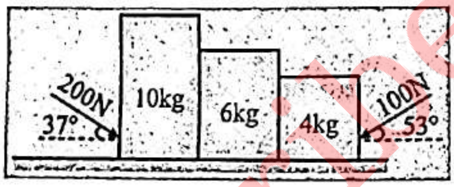
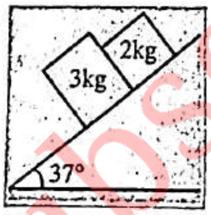


9.



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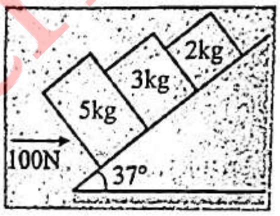
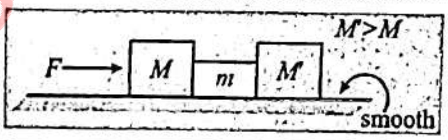


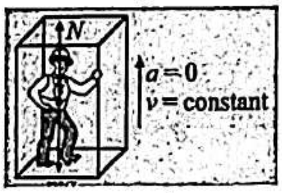
11. A constant force F is applied in horizontal direction as shown. Contact force between M and m is N and between m and M' is N' then find the relation between N' and N



APPARENT WEIGHT OF A BODY IN A LIFT

The weight recorded in the weighing machine is equal to the force with which your feet press the weighing machine. It is therefore equal to normal reaction.

- **The lift is at rest or moving with uniform velocity**



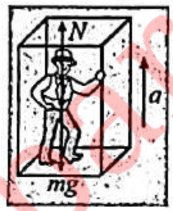
$a = 0$

$mg - N = 0$ or $N = mg$ or $W_{app.} = W_0$

$W_{app.} = N =$ reaction of supporting surface

and $W_0 = mg =$ true weight.

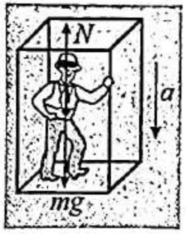
- **When the lift moves upwards with an acceleration a:**



$$N - mg = ma \text{ or } N = m(g + a) = mg\left(1 + \frac{a}{g}\right)$$

$$\therefore W_{app.} = W_0\left(1 + \frac{a}{g}\right)$$

- **When the lift moves downwards with an acceleration a:**

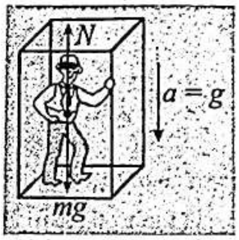


$$mg - N = ma \text{ or } N = m(g - a) = mg\left(1 - \frac{a}{g}\right)$$

$$\therefore W_{app.} = W_0\left(1 - \frac{a}{g}\right)$$

Here, if $a > g$, $W_{app.}$ will be negative. Negative apparent weight will mean that the body is pressed against the roof of the lift instead of floor.

- **When the lift falls freely, i.e., $a = g$:**



$N = m(g - g) = 0$ or $W_{app.} = 0$

CONSTRAINED MOTION

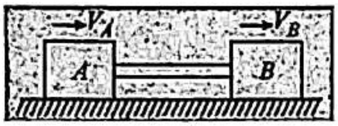
String Constraint

When two objects are connected through a string and if the string have the following properties:

(*a*) The length of the string remains constant i.e. inextensible string.

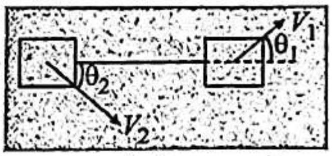
(*b*) String always remains tight, does not slacks.

(i) If the velocities of the block is along the string as shown, then their velocities must be same.



$V_A = V_B = V$

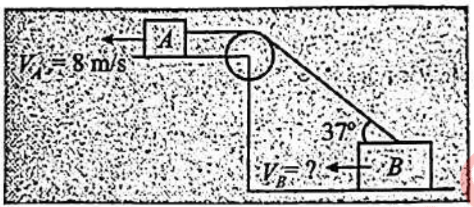
(ii) If the velocities of the blocks are not along the string as shown below, then components of velocities of the block along the string must be same as string is inextensible



$V_1\cos\theta_1 = V_2\cos\theta_2$

Train Your Brain

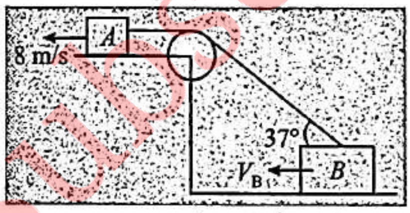
Example 10:



Sol. Block *A* is moving with velocity 8 ms^{-1}.

∴ velocity of every point on the string must be 8 m/s along the string.

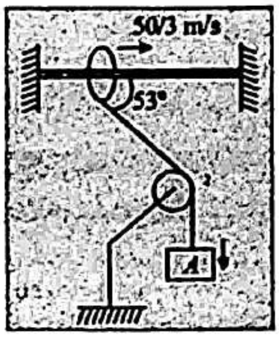
The velocity of *B* is v_B. Then the string will not break only when the component of v_B along string is 8 m/s.



$\Rightarrow v_B \cos 37° = 8$

$$\Rightarrow v_B = \frac{8}{\cos 37°} = 10 \text{ m/sec}$$

Example 11: What is the velocity of block *A* in the figure as shown below.



Sol. The component of velocity of ring along string

= velocity of *A*

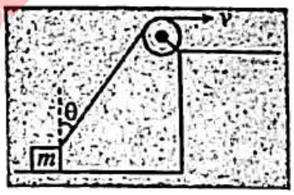
$= \frac{50}{3}\cos 53°$

$= v_A$

$\Rightarrow v_A = 10$ m/s

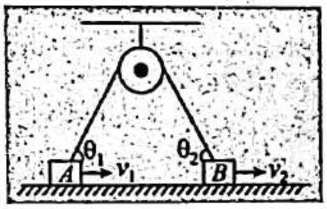
Concept Application

12. A block is dragged on a smooth plane with the help of a rope which moves with velocity *v*. The horizontal velocity of the block is



(*a*) v (*b*) $\frac{v}{\sin\theta}$ (*c*) $v\sin\theta$ (*d*) $\frac{v}{\cos\theta}$

13. In the figure shown, blocks *A* and *B* move with velocities v_1 and v_2 along horizontal direction. Find the ratio of $\frac{v_1}{v_2}$.



Virtual Work Method

For more complex constrain relation, virtual work method can be used.

An ideal (massless and inextensible) string can neither possess kinetic energy nor store potential energy. So it can never gain or supply energy. Total work done by the string on block attached to it in displacing them is always zero or equivalently total power input by it at any instant is zero.

Now, $P = \vec{F} \cdot \vec{v}$

$P_{\text{Total}} = \Sigma \vec{F}_i \cdot \vec{v}_i$

$= \Sigma \vec{T}_i \cdot \vec{V}_i = 0$

This gives us relation between velocities of bodies constrained to move when string is attached between them.

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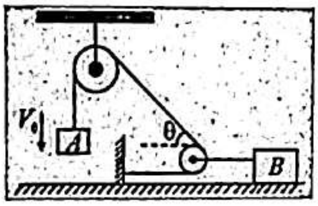
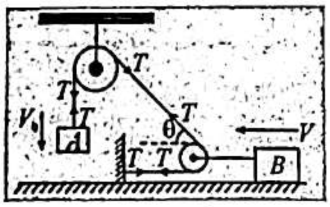
Example 12: In the figure, find the velocity of block B, if velocity of A is V_0 in downward direction?



Sol.



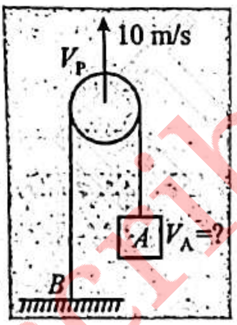
T
T
T
T
V_0
A
θ
T
T
V
B

Power input $= TV_0\cos180° + TV\cos\theta + TV\cos(0)$

$-TV_0 + TV\cos\theta + TV = 0$

$\Rightarrow V = \dfrac{V_0}{1+\cos\theta}$

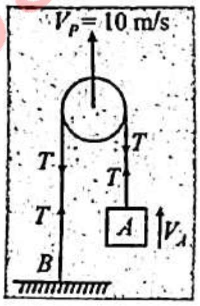
Example 13: Find $V_A = ?$



10 m/s
V_P
A
V_A=?
B

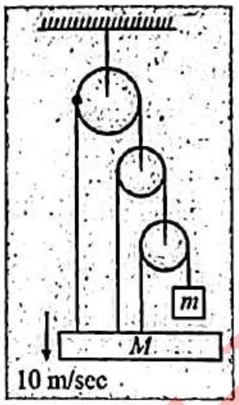
Sol. $P_{\text{total}} = P_{\text{pulley}} + P_{\text{block}} = -2T \times 10 + TV_A = 0$

$V_A = 20$ m/s.



V_P= 10 m/s
T
T
T
T
A
V_A
B

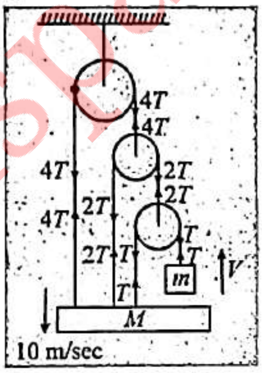
Example 14: Find the velocity of m?



Sol. $P_{\text{input}} = P_M + P_m$

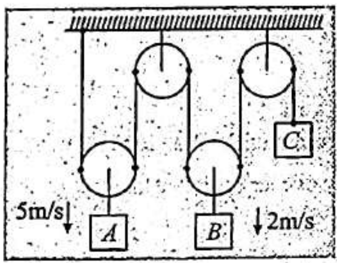
$(-7T \times 10) + TV = 0$

$\Rightarrow V = 70$ m/s



4T
4T
4T
2T
2T
4T
2T
2T
T
T
T
T
m
V
M
10 m/sec

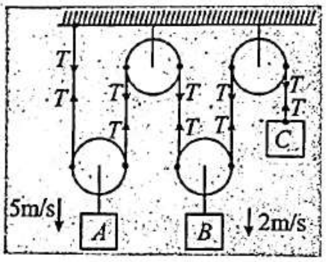
Example 15: Find velocity of block C?



Sol. $P_{\text{input}} = P_A + P_B + P_C$

$(-2T \times 5) - (2T \times 2) + TV_C = 0$

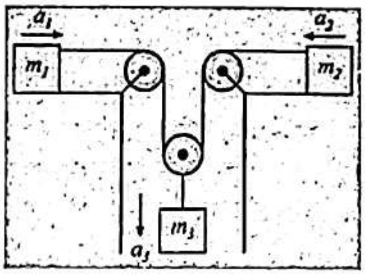
$\Rightarrow V_C = 10 + 4 = 14$ m/s.



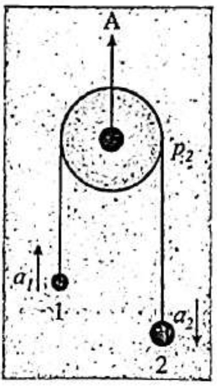
T
T
T
T
T
T
T
T
T
T
C
5m/s
A
B
2m/s

Concept Application

14. In the arrangement of three blocks as shown in fig, the string is inextensible. If the directions of accelerations are as shown in the figure, then determine the constraint relation among a_1, a_2 and a_3.



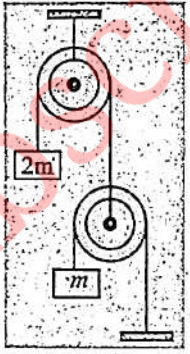
15. Two masses are connected by a string which passes over a pulley accelerating upward with an acceleration A as shown. If a_1 and a_2 be the accelerations of bodies 1 and 2 respectively then



(*a*) $A = a_1 - a_2$ (*b*) $A = a_1 + a_2$

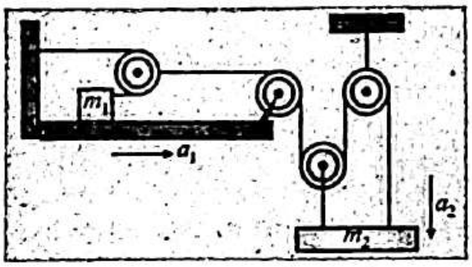
(*c*) $A = \frac{a_1 - a_2}{2}$ (*d*) $A = \frac{a_1 + a_2}{2}$

16. In the system shown in the figure, the friction and mass of rope is negligible then acceleration of the block of mass 2m is



(*a*) $\frac{g}{5}$ (*b*) $\frac{2g}{5}$ (*c*) 0 (*d*) $\frac{5g}{2}$

17. Two blocks are arranged as shown in the figure. The relation between acceleration a_1 and a_2 is



(*a*) $a_1 = a_2$ (*b*) $a_1 = 6a_2$

(*c*) $a_1 = 3a_2$ (*d*) $a_1 = 4a_2$

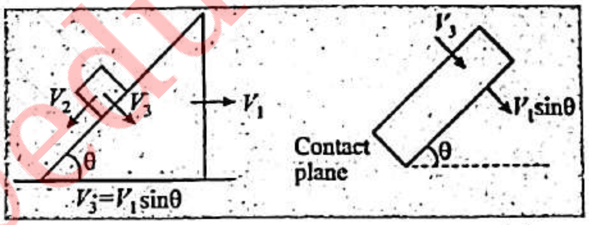
WEDGE CONSTRAINT

Conditions

(i) There is a regular contact between two objects.

(ii) Objects are rigid.

The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.



In other words,

Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations and they remain in contact.

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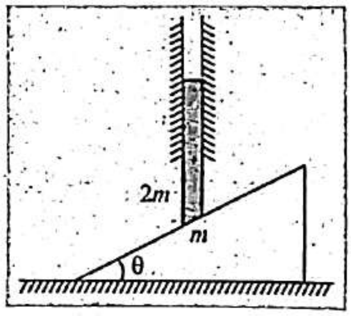
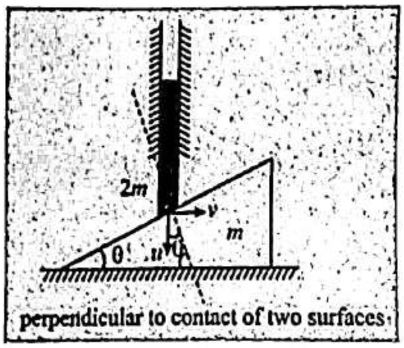
Example 16: A rod of mass 2*m* moves vertically downward on the surface of wedge of mass *m* as shown in figure. Find the relation between velocity of rod and that of the wedge at any instant.



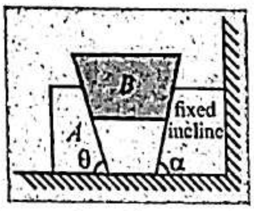
Sol. Using wedge constraint.

Component of velocity of rod along perpendicular to inclined surface is equal to velocity of wedge along that direction.

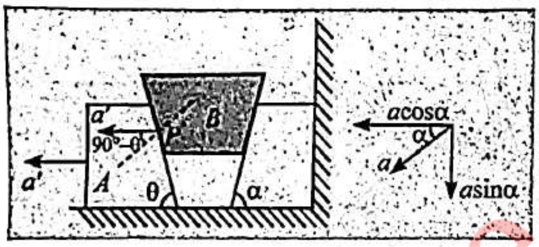


$$u\cos\theta = v\sin\theta \Rightarrow \frac{u}{v} = \tan\theta \Rightarrow u = v\tan\theta$$

Example 17: In the arrangement shown in figure, if the acceleration of B is a, then find the acceleration of A.



Sol. Let a' be the acceleration of block A using wedge constraint at point P



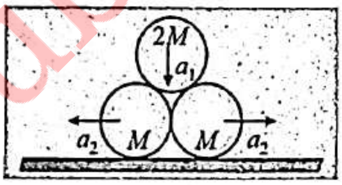
$$a\sin\alpha\cos\theta + a\cos\alpha\sin\theta = a'\cos(90° - \theta)$$

$$a\sin(\alpha+\theta) = a'\sin\theta$$

$$a' = \frac{a\sin(\theta+\alpha)}{\sin\theta}$$

Concept Application

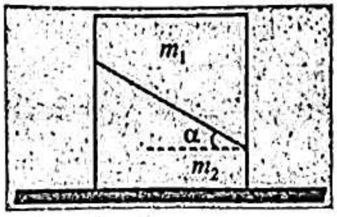
18. The relation between acceleration a_1 and a_2, if the radius of each sphere is equal to R.



(a) $a_2 = a_1\sqrt{3}$ (b) $a_1 = a_2\sqrt{3}$

(c) $a_1 = 2a_2$ (d) $a_2 = 2a_1$

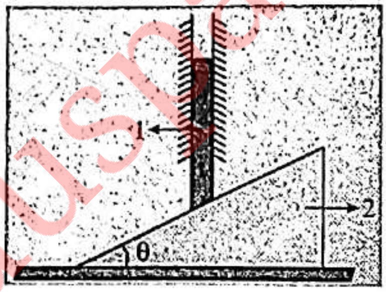
19. A block is kept on a smooth horizontal floor. It is cut along an inclined plane making an angle α with the horizontal in two parts of masses m_1 and m_2. Neglecting the friction between the surfaces, the ratio of magnitude of horizontal acceleration of m_1 and m_2 is



(a) $\frac{m_2}{m_1}\sin\alpha$ (b) $\frac{m_1}{m_2}\sin\alpha$

(c) $\frac{m_2}{m_1}$ (d) $\frac{m_1}{m_2}$

20. Acceleration of (1) w.r.t. ground is "a" downwards. Acceleration of (2) w.r.t. (1) will be:



(a) $a\sec\theta$ (b) $a\operatorname{cosec}\theta$

(c) $a\tan\theta$ (d) $a\cot\theta$

SPRING FORCE

Many spring follow Hooke's law for small extension and compression.

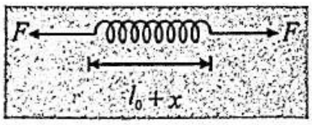
That is, the extension or compression – the increase or decrease in length from the relaxed length – is proportional to the force applied to the ends of the spring.

Hooke's Law for an ideal spring $F = k\Delta L$

In equation F is the magnitude of the force exerted on each end of the spring and ΔL is the modulus of change in length of the spring from its relaxed length. The constant k is called the spring constant for a particular spring. The *SI* units of a spring constant are N/m.

When we say an ideal spring, we mean a spring that obeys Hooke's law and is also massless. Since we have assumed spring to be massless we know forces acting on both ends have to be equal and opposite, to have net force on spring to be zero.

Note: If we look at *FBD* of the spring we will note that force on spring must act from both ends.



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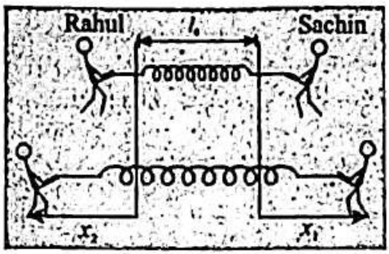
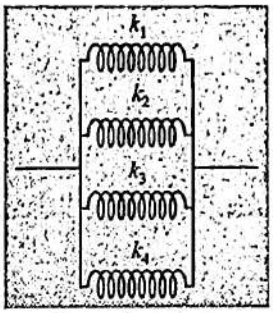
Lets say Rahul and Sachin are pulling a spring from two ends as shown. Rahul moves x_2 and Sachin moves x_1.



The force acting on Rahul and Sachin is $k(x_1 + x_2)$, Not kx_2 on Rahul and kx_1 on Sachin. Force due to spring is kx where x is defined as $|l - l_0|$, where l is present length and l_0 is natural length.

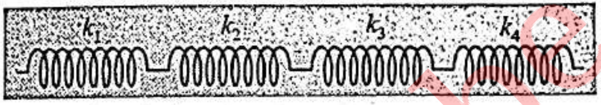
EQUIVALENT SPRING CONSTANT

(*a*) When two springs are connected in parallel then we can replace them by single spring of spring constant k_e where $k_e = k_1 + k_2$.



For more than two spring $k = k_1 + k_2 + k_3 +$ ....

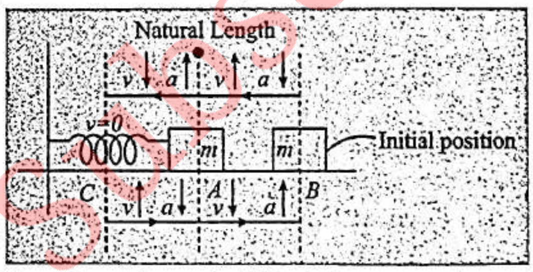
(*b*) When two springs are connected in series then we can replace them by single spring of spring constant k_e where $1/k_e = 1/k_1 + 1/k_2$. As spring constants are not equal so extensions will not be equal, but total extension y can be written as sum of two extensions $y = y_1 + y_2 + y_3 +$ .... for more than two springs.



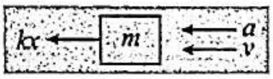
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \text{..........}$$

ANALYSIS OF MOTION OF BLOCK

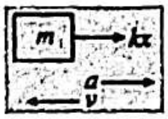
Suppose block is released from rest from position B in the figure shown



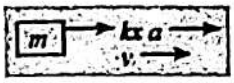
(i) From B to A speed of block increase and acceleration decreases. (due to decrease in spring force kx)



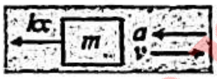
(ii) Due to inertia block crosses natural length at A. From A to C speed of the block decreases and acceleration increases. (due to increase in spring force kx)



(iii) At C the block stops momentarily at this instant and since the spring is compressed spring force is towards right and the block starts to move towards right. From C to A speed of block increases and acceleration decreases. (due to decrease in spring force kx)

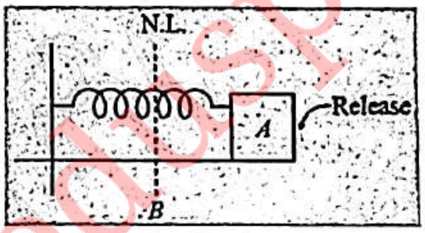


(iv) Again block crosses point A due to inertia then from A to B speed decreases and acceleration increases.

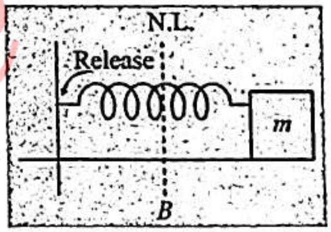


In this way block does SHM (to be explained later) if no resistive force is acting on the block.

Notes:

1. 

When the block A is released then it take some finite time to reach at B. i.e., spring force doesn't change instantaneously

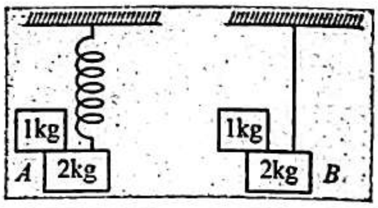
2. 

When point A of the spring is released in the above situation then the spring force changes instantaneously and becomes zero because one end of the spring is free.

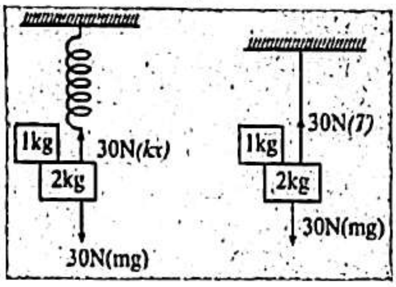
3. In string tension may change instantaneously.

Train Your Brain

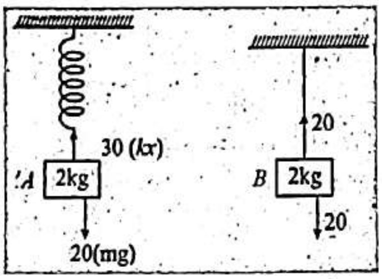
Example 18: Find out the acceleration of 2 kg block in the figures shown at the instant 1 kg block falls from 2 kg block. (at t = 0)



Sol. F.B.D.s before fall of 1kg block

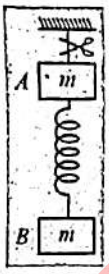


After the fall of the 1 kg block tension in string will change instantaneously but spring force (kx) doesn't change instantaneously. F.B.D.s just after the fall of 1 kg block



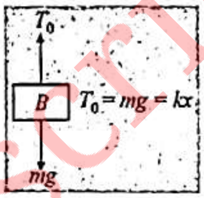
$$a_A = \frac{30-20}{2} = 5\text{m/s}^2\text{(upward)}, a_B = 0 \text{ m/s}^2$$

Example 19: Two blocks '*A*' and '*B*' of same mass '*m*' attached with a light spring are suspended by a string as shown in figure. Find the acceleration of block '*A*' and '*B*' just after the string is cut.



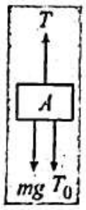
Sol. When block *A* and *B* are in equilibrium position

F.B.D of '*B*'



$T_0 = mg = kx$ ... (*i*)

F.B.D of '*A*'



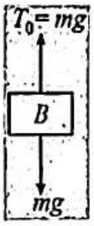
$T = mg + T_0$ ... (*ii*)

$T = 2\,mg$

when string is cut, tension *T* becomes zero. But spring does not change its shape instantaneously. So spring force continues to act on mass *B*.

Again draw F.B.D. of block *A* and *B* as shown in figure

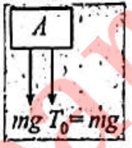
F.B.D of '*B*'



$T_0 - mg = m.a_B$

$\Rightarrow a_B = 0$

F.B.D. of '*A*'



$mg + T_0 = m.a_A$

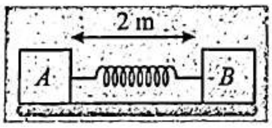
$\Rightarrow 2mg = m.a_A$

$\Rightarrow a_A = 2g$ (downwards)

Concept Application

21. Force constant of a spring is 100 N/m. If a 10 kg block attached with the spring is at rest, then find extension in the spring (in m) ($g = 10$ m/s^2)

22. Two blocks are connected by a spring of natural length 2 m. The force (in N) constant of spring is 200 N/m. Find spring force in following situations.



(*a*) If block '*A*' and '*B*' both are displaced by 0.5 m in same direction.

(*b*) If block '*A*' and '*B*' both are displaced by 0.5 m in opposite direction.

PSEUDO FORCE

Motion in Accelerated Frames

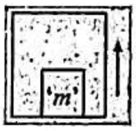
Till now we have restricted ourselves to applying Netwon's laws of motion, to describe observation that are made in an inertial frame of reference. In this section we learn how Newton's laws can be applied by an observer in a non-inertial reference frame. For example consider a block kept on smooth surface of a compartment of train.

If the train accelerates, the block accelerates toward the back of the train. We may conclude based on Newton's second law $F = ma$ that a force is acting on the block to cause it to accelerate, but the Newton's second law is not applicable in this non - inertial frame. So we can not relate observed acceleration with the force acting on the block.

If we still want to use Newton's second law we need to apply a pseudo force, acting in backward direction, i.e. opposite to the acceleration of non-inertial reference frame. This force explains the motion of block towards the back of car. The fictitious force is equal to $-ma$, where a is the acceleration of the non-inertial reference frame. Fictitious force appears to act on an object in the same way as a real force, but real forces are always interactions between two objects. On the other hand there is no second object for a fictitious force.

Train Your Brain

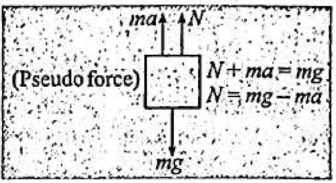
Example 20: A box is moving upward with retardation 'a' $< g$, find the direction and magnitude of "pseudo force" acting on block of mass 'm' placed inside the box. Also calculate normal force exerted by surface on block.



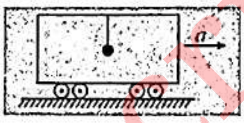
Sol. Pseudo force acts opposite to the direction of acceleration of reference frame.

Pseudo force $= ma$ in upward direction.

F.B.D of 'm' w.r.t box (non-inertial)



Example 21: Figure shows a pendulum suspended from the roof of a car that has a constant acceleration a relative to the ground. Find the deflection of the pendulum from the vertical as observed from the ground frame and from the frame attached with the car.



Sol. In inertial frame

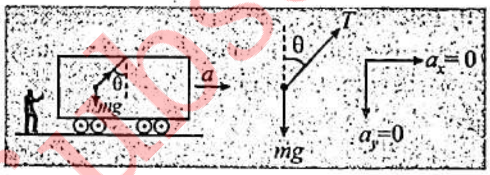


Figure represents free Body diagram of the bob w.r.t ground.

In an inertial frame the suspended bob has an acceleration a caused by the horizontal component of tension T.

$T \sin\theta = ma$ ... (*i*)

$T \cos\theta = mg$ ... (*ii*)

From equation (*i*) and (*ii*)

$$\tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

In a non-inertial frame

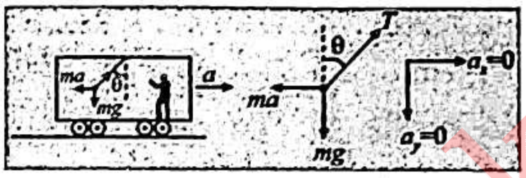


Figure represents free Body diagram of bob w.r.t car.

In the non-inertial frame of the car, the bob is in static equilibrium under the action of three forces, T, mg and ma (pseudo force)

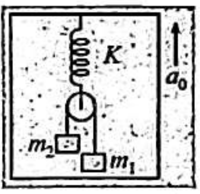
$T \sin\theta = ma$ ... (*iii*)

$T \cos\theta = mg$ ... (*iv*)

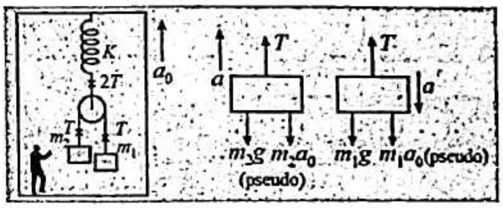
From equation (*iii*) and (*iv*)

$$\tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

Example 22: A pulley with two block system is attached to the ceiling of a lift moving upward with an acceleration a_0. Find the deformation in the spring.



Sol. Non-Inertial Frame



Let relative to the centre of pulley, m_1 accelerates downward with a and m_2 accelerates upwards with a.

Applying Newton's 2nd law.

$m_1g + m_1a_0 - T = m_1a$ ... (*i*)

$T - m_2g - m_2a_0 = m_2a$ ... (*ii*)

On adding (*i*) and (*ii*) we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)(g + a_0) \quad \text{... (iii)}$$

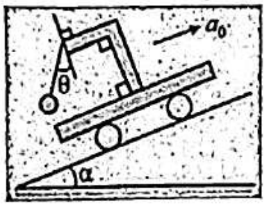
Substituting a in equation (*i*)

We get

$$T=\frac{2m_1m_2(g+a_0)}{m_1+m_2}$$

$$\therefore x=\frac{F}{k}=\frac{2T}{k}=\frac{4m_1m_2(g+a_0)}{(m_1+m_2)k}$$

Example 23: A pendulum of mass m hangs from a support fixed to a trolley. The direction of the string when the trolley rolls up a plane of inclination α with acceleration a_0 is (String and bob remain fixed with respect to trolley)



(a) $\theta=\tan^{-1}\alpha$ (b) $\theta=\tan^{-1}\left(\frac{a_0}{g}\right)$

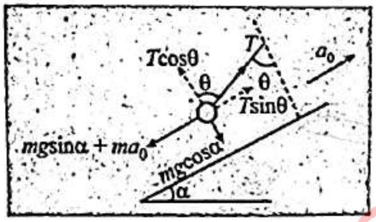
(c) $\theta=\tan^{-1}\left(\frac{g}{a_0}\right)$ (d) $\theta=\tan^{-1}\left(\frac{a_0+g\sin\alpha}{g\cos\alpha}\right)$

Sol. (*d*) Balancing forces in the frame of trolley (non-inertial frame)

$T\sin\theta=m(g\sin\alpha+a_0)$

$T\cos\theta=mg\cos\alpha$

$$\Rightarrow \tan\theta=\left(\frac{g\sin\alpha+a_0}{g\cos\alpha}\right)$$

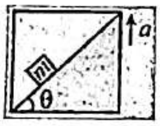


$$\theta=\tan^{-1}\left(\frac{g\sin\alpha+a_0}{g\cos\alpha}\right)$$

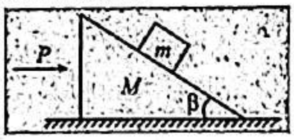
Concept Application

23. An object of mass 2 kg moving with constant velocity $10\hat{i}$ m/s is seen in a frame moving with constant velocity $10\hat{i}$ m/s. What will be the value of 'pseudo force' acting on object in this frame.

24. In the adjoining figure, a wedge is fixed to an elevator moving upwards with an acceleration 'a'. A block of mass 'm' is placed over the wedge. Find the acceleration of the block with respect to wedge. Neglect friction.



25. Two wooden blocks are moving on a smooth horizontal surface such that the mass m remains stationary with respect to block of mass M as shown in the figure. The magnitude of force P is



(a) $(M+m)g\tan\beta$ (b) $g\tan\beta$

(c) $mg\cos\beta$ (d) $(M+m)g\operatorname{cosec}\beta$

FRICTION

When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a force of friction. Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

Friction force is of two types

1. Static friction 'f_s'
2. Kinetic friction 'f_k'

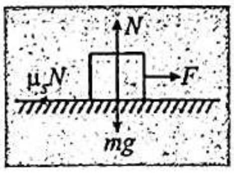
1. **Static Friction (f_s):** The static friction between two contact surfaces is given by $f_s\le\mu_s N$, where N is the normal force between the contact surfaces and μ_s is a constant which depends on the nature of the surfaces and is called 'coefficient of limiting friction'.

   Static friction acts between two objects when there is no relative motion between them. Static friction is a self adjusting force and its values satisfy the condition

   $f_s\le\mu_s N$

   μ_s = Co-efficient of limiting friction

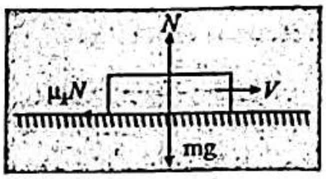
   Static friction takes its peak value ($f_{s(\max)}=\mu_s N$) when one surface is 'about to slide' on the other. Static friction in this case is called limiting friction.



2. **Kinetic Friction (f_k):** It acts on the two contact surfaces only when there is relative slipping or relative motion between two contact surfaces.

   $f_k=\mu_k N$

   The relative motion of a contact surface with respect to each other is opposed by a force given by $f_k=\mu_k N$, where N is the normal force between the contact surfaces and μ_k is a constant called **'coefficient of kinetic friction'**, which depends, largely, on the nature of the contact surfaces.

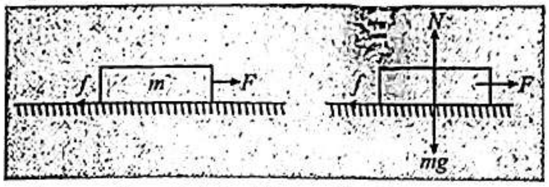


Notes:

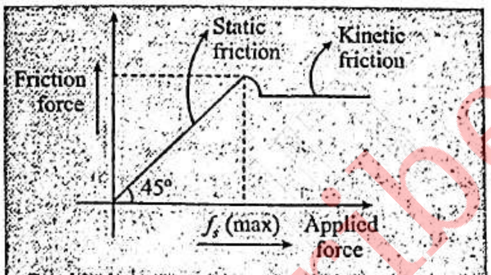
1. Value of μ_k is always less than μ_s ($\mu_k < \mu_s$) from experimental observation.
2. If only coefficient of friction (μ) is given in a problem then use $\mu_s = \mu_k = \mu$
3. Value of μ_s and μ_k is independent of surface area. It depends only on surface properties of contact surface.
4. μ_k is independent of relative speed.
5. μ_s and μ_k are properties of a given pair of surfaces i.e. for wood to wood combination μ_1, then for wood to iron μ_2 and so on.
6. Friction force acts along the surface.

Graph of Frictional Force Versus Applied Force:

Consider a block of mass m kept on a rough surface and a force F is applied on the block which gradually increases from zero. Let μ_s and μ_k be coefficient of static and kinetic friction respectively.



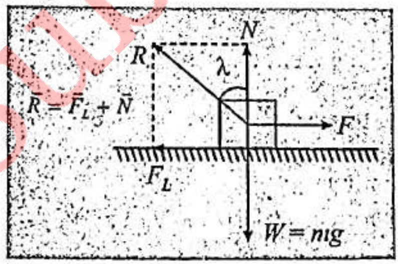
The maximum value of static friction is $\mu_s N$. As long as $F < \mu_s N$, block will not move. When F exceed $\mu_s N$, block will slip on the surface and kinetic friction $f_k = \mu_k N$ will act on the block which has a fixed value. (See graph below)



Graphical representation of variation of friction force with the force applied on a body

Angle of Friction

The angle of friction is the angle which the resultant of limiting friction F_L and normal reaction N makes with the normal reaction. It is represented by λ.



Thus from the figure.

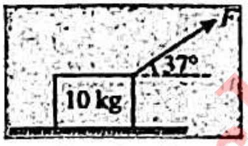
$$\tan\lambda = \frac{F_L}{N}$$

or $\tan\lambda = \mu_s$ $\qquad (\because F_L = \mu_s N)$

For smooth surfaces, $\lambda = 0$ (zero)

Train Your Brain

Example 24: A force F is applied on the block as shown in the figure. If the coefficient of static friction $\mu_s = 0.3$ and coefficient of kinetic friction is $\mu_k = 0.2$.

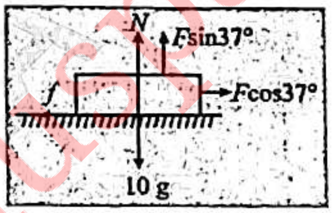


Find frictional force and acceleration of the block if

(i) $F = 10$ N

(ii) $F = 50$ N

Sol. F.B.D. of the block



(i) $F = 10$ N

Balancing forces normal to the surface

$N + 10\sin37° - 10g = 0 \Rightarrow N = 94$ N

$f_{s,\,max} = \mu_s N = 0.3 \times 94 = 28.2$ N

$$F\cos37° = 10 \times \frac{4}{5} = 8 \text{ N}$$

Since, $F\cos37° < \mu_s N$, block will not move and static friction $f_s = F\cos37° = 8$N.

(ii) $F = 50$ N

$N + 50\sin37° - 10\,g = 0 \Rightarrow N = 70$N

$f_{s,\,max} = \mu_s N = 0.3 \times 70 = 21$N

$$F\cos37° = 50 \times \frac{4}{5} = 40 \text{ N}$$

Since $F\cos37° > \mu_s N$, friction will not be able to prevent slipping.

Block will slide and kinetic friction will act on the block

$f_k = 0.2$ N $= 0.2 \times 70 = 14$ N

To find acceleration, we apply Newton's law along horizontal direction

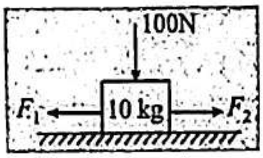
$F\cos37° - f_k = ma$

$50\cos37° - 14 = 10a$

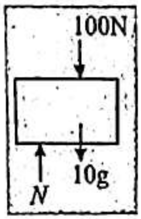
$$\Rightarrow a = \frac{40-14}{10} = 2.6 \text{ m/s}^2$$

Example 25: Find frictional force acting on the block in the following cases given that $\mu = 0.1$ [Take $g = 10\ m/s^2$]

Case-I: $F_1 = 0$	**Case-II:** $F_1 = 20\ N$
$F_2 = 5N$	$F_2 = 10\ N$
$F_2 = 15N$	$F_2 = 20\ N$
$F_2 = 25N$	$F_2 = 42\ N$
	$F_2 = 50\ N$



Sol. F.B.D. of the block



$N = 10g + 100 = 200\ N$

$f_{s,max} = \mu_s N = 200 \times 0.1 = 20\ N$

If $|F_2 - F_1| < \mu_s N$ block will not move and self adjusting static friction will act on its block and will be equal to $|F_2 - F_1|$.

If $|F_2 - F_1| > \mu_s N$

then, $f_k = \mu_k N$

Case-I: $F_1 = 0$

$F_2 = 5N$	$fr = 5\ N$ (towards left)
$F_2 = 15N$	$fr = 15\ N$ (towards left)
$F_2 = 25N$	$fr = 20\ N$ (towards left)

Case-II: $F_1 = 20\ N$

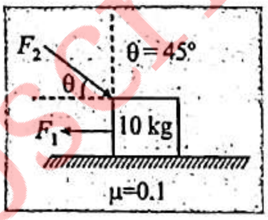
$F_2 = 10\ N$	$fr = 10\ N$ (towards right)
$F_2 = 20\ N$	$fr = 0$
$F_2 = 42\ N$	$fr = 20\ N$ (towards left)
$F_2 = 50\ N$	$fr = 20\ N$ (towards left)

Example 26: Find friction force acting on block in the following cases.

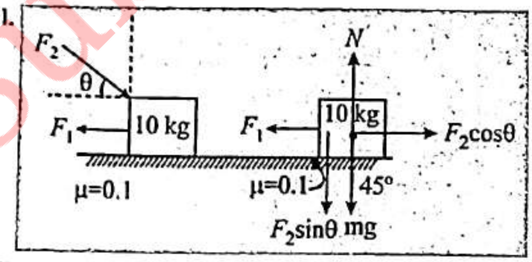
Case-I: $F_1 = 0, F_2 = 5\sqrt{2}$

Case-II: $F_1 = 0, F_2 = 50\sqrt{2}$

Case-III: $F_1 = 5N, F_2 = 5\sqrt{2}$

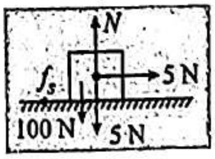


Sol.



Both of the above are same cases.

Case-I:



Along vertical:

$N - 100 - 5 = 0$

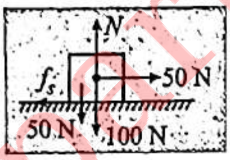
$N = 105\ N$

$f_{s,max} = 0.1 \times 105 = 10.5\ N$

$f_s = 5N$

Since, applied force along horizontal is less than $f_{s,max}$, block will not move and f_s = applied force = 5 N

Case-II:



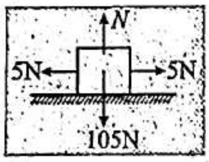
$N - 100 - 50 = 0$

$N = 150\ N$

$f_{s,max} = 0.1 \times 150 = 15\ N$

so, block will skid and $f_s = 15\ N$

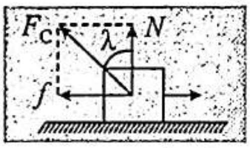
Case-III:



Since force along horizontal balance, there is no tendency to slip $f_s = 0$.

Example 27: A body of mass 400 g slides on a rough horizontal surface. If the frictional force is 3.0 N, find (*a*) the angle made by the contact force on the body with the vertical and (*b*) the magnitude of the contact force. Take $g = 10\ m/s^2$.

Sol. Let the contact force on the block by the surface be F_c which makes an angle λ with the vertical (shown in figure)



The component of F_c perpendicular to the contact surface is the normal force N and the component of F parallel to the surface is the friction f. As the surface is horizontal, N is vertically upward. For vertical equilibrium

$N = Mg = (0.400\ kg)\ (10\ m/s^2) = 4.0\ N$

The frictional force is $f = 3.0$ N

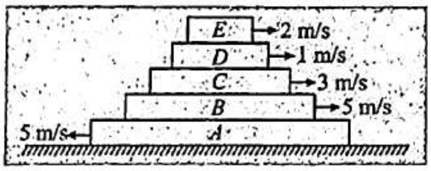
(i) $\tan\lambda = \frac{f}{N} = \frac{3}{4}$ or, $\lambda = \tan^{-1}(3/4) = 37°$

(ii) The magnitude of the contact force is

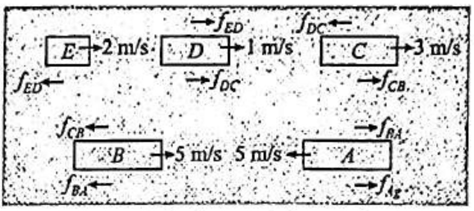
$$F = \sqrt{N^2 + f^2}$$

$$= \sqrt{(4.0\,\text{N})^2 + (3.0\,\text{N})^2} = 5.0\,\text{N}$$

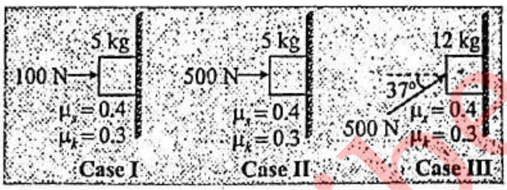
Example 28: In the given diagram find the direction of friction forces on each block and the ground (Assume all surfaces are rough and all velocities are with respect to ground).


E
2 m/s
D
1 m/s
C
3 m/s
B
5 m/s
5 m/s
A

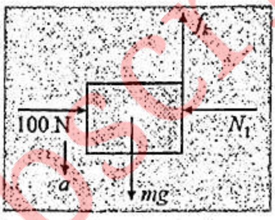
Sol. Since friction opposes relative motion, direction of friction will be as shown in below


f_{ED}
f_{DC}
E
2 m/s
D
1 m/s
C
3 m/s
f_{ED}
f_{DC}
f_{CB}
f_{CB}
f_{BA}
B
5 m/s
5 m/s
A
f_{BA}
f_{AG}

Example 29: Determine the magnitude of frictional force and acceleration of the block in each of the following cases: [Take g = 10 m/s²]


5 kg
100 N
$\mu_s = 0.4$
$\mu_k = 0.3$
Case I
5 kg
500 N
$\mu_s = 0.4$
$\mu_k = 0.3$
Case II
12 kg
37°
500 N
$\mu_s = 0.4$
$\mu_k = 0.3$
Case III

Sol. **Case-I:** $N_1 = 100$ N, $mg = 50$ N

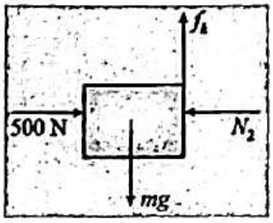

f_l
100 N
N_1
a
mg

$f_l = \mu_s N_1 = 0.4 \times 100 = 40$ N, $f_k = \mu_k N_1$

$= 0.3 \times 100 = 30$ N

Here mg (driving force) is greater than maximum friction $f_l = 40$ N. Hence the block will not be able to stay at rest. It will accelerate downwards. But when it starts slipping, then kinetic friction will come into play.

Case-II: $N_2 = 500$ N, $mg = 50$ N

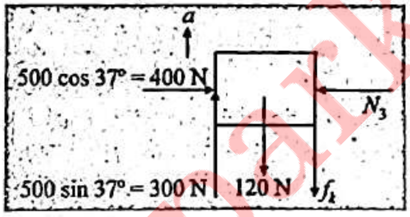

f_k
500 N
N_2
mg

$f_l = \mu_s N_2 = 200$ N, $f_k = \mu_k N_2 = 150$ N

Here f_l is greater than mg (driving force), Hence, block will not move. So in this case $a = 0, f = mg = 50$ N

Case-III:

$N_3 = 400$ N, $f_l = \mu_s N_3 = 160$ N, $f_k = \mu_k N_3 = 120$ N


a
500 cos 37° = 400 N
N_3
500 sin 37° = 300 N
120 N
f_k

Here driving force = 300 – 120 = 180 N in upward direction; hence, friction will act downwards. Driving force is more than f_l. So the block will accelerate upwards.

$$a = \frac{180 - f_k}{m} = \frac{180 - 120}{12} = 5\ \text{ms}^{-2}\ \text{(upwards)}$$

So in this case $f = f_k = 120$ N and $a = 5$ ms^{-2} (upwards)

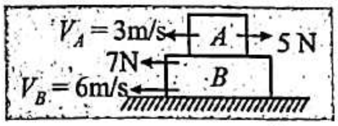
Now

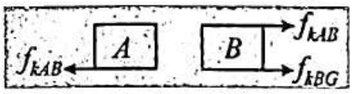
$$a = \frac{mg - f_k}{m} = \frac{50 - 30}{5} = 4\ \text{ms}^{-2}$$

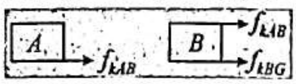
So in this case $f = f_k = 30$ N and $a = 4$ ms^{-2} (downwards)

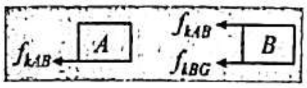
Concept Application

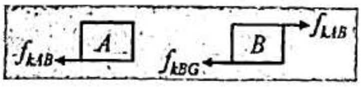
26. In the following figure, find the direction of friction on the blocks and ground .


$V_A = 3$m/s
A
5 N
7N
B
$V_B = 6$m/s

(a) 
f_{kAB}
A
B
f_{kAB}
f_{kBG}

(b) 
A
f_{kAB}
B
f_{kAB}
f_{kBG}

(c) 
f_{kAB}
A
f_{kAB}
f_{kBG}
B

(d) 
f_{kAB}
A
f_{kBG}
B
f_{kAB}

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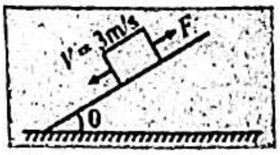
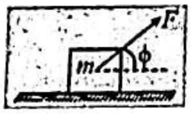
27. In the following figure, find the direction and nature of friction on the block.



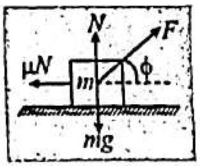
(*a*) Down the incline, kinetic friction.
(*b*) Up the incline, kinetic friction.
(*c*) Up the incline, static friction.
(*d*) Down the incline, static friction.

MINIMUM FORCE REQUIRED TO MOVE A BLOCK

A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body slide by applying the minimum possible force F.



Let the applied force F be at angle ϕ with the horizontal



N = Normal force

For vertical equilibrium,

$N + F\sin\phi = mg$ or, $N = (mg - F\sin\phi)$ ... (*i*)

For horizontal equilibrium i.e. when the block is just about to slide,

$F\cos\phi = \mu N$ ... (*ii*)

Substituting for N,

$F\cos\phi = \mu(mg - F\sin\phi)$ or $F = \mu mg/(\cos\phi + \mu\sin\phi)$

for minimum $F(\cos\phi + \mu\sin\phi)$ is maximum,

$\Rightarrow$ Let $x = \cos\phi + \mu\sin\phi$

$$\frac{dx}{d\phi} = -\sin\phi + \mu\cos\phi$$

for maximum of $x, \dfrac{dx}{d\phi} = 0$

$\tan\phi = \mu$ and at this value of ϕ

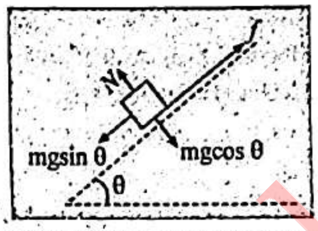
$$F_{\min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

MOTION ON A ROUGH INCLINED PLANE

Angle of Repose

Consider a rough inclined plane whose angle of inclination θ with ground can be changed. A block of mass m is resting on the plane. Coefficient of (static) friction between the block and plane is μ_s.

For a given angle θ, the FBD (Free body diagram) of the block is



where f is force of static friction on the block. For direction normal to the plane, we have $N = mg\cos\theta$

As θ increases, the force of gravity down the plane, mg sin θ, increases. Friction force resists the slide till it attains its maximum value.

$$f_{\max} = \mu_s N = \mu_s mg\cos\theta$$

which decreases with θ (because cos θ decreases as θ increases)

Hence, beyond a critical value $\theta = \theta_c$, the blocks starts to slide down the plane. The critical angle is the one when mg sin θ is just equal to $f_{\max}$, i.e. when

$mg\sin\theta_c = \mu_s\, mg\cos\theta_c$

or $\tan\theta_c = \mu_s$

where θ_c is called angle of repose

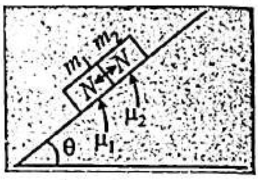
For $\theta < \theta_c$

$a = 0$ and $f_s = mg\sin\theta$

For $\theta > \theta_c$

$a = g\sin\theta - \mu_k g\cos\theta$ (as the block is moving)

TWO BLOCKS ON AN INCLINED PLANE

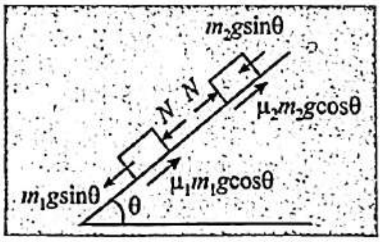


Consider two blocks having masses m_1 and m_2 placed on a rough inclined plane. μ_1 and μ_2 are the friction coefficient for m_1 and m_2 respectively. If N is the normal force between the contact surface of m_1 and m_2

Now two condition arises.

(i) If $\mu_1 > \mu_2$

Acceleration of m_2 will be greater than m_1 if they were not in contact. When in contact, block m_2 will press m_1 and both will move together with same acceleration.



$m_1 g\sin\theta + N - \mu_1 m_1 g\cos\theta = m_1 a$ ... (*i*)

$m_2 g\sin\theta - N - \mu_2 m_2 g\cos\theta = m_2 a$ ... (*ii*)

Adding (*i*) and (*ii*)

and solving we get

$$a = \frac{(m_1+m_2)g\sin\theta - (\mu_1 m_1 + \mu_2 m_2)g\cos\theta}{m_1+m_2}$$

we can also find N by putting value of a in equation (*i*).

(ii) If $\mu_1 < \mu_2$ then, blocks will move with different acceleration. $N = 0$ because, there is no contact between the blocks.

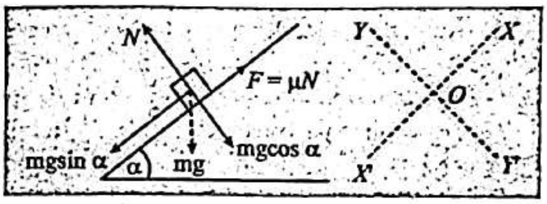
$a_1 = g\sin\theta - \mu_1 g\cos\theta$

$a_2 = g\sin\theta - \mu_2 g\cos\theta$

$\Rightarrow a_1 > a_2$

Train Your Brain

Example 30: A 20 kg box is gently placed on a rough inclined plane of inclination 30° with horizontal. The coefficient of sliding friction between the box and the plane is 0.4. Find the acceleration of the box down the incline.



Sol. In solving inclined plane problems, the X and Y directions along which the forces are to be considered, may be taken as shown. The components of weight of the box are

(i) $mg\sin\alpha$ acting down the plane and

(ii) $mg\cos\alpha$ acting perpendicular to the plane.

$N = mg\cos\alpha$

$mg\sin\alpha - \mu N = ma$

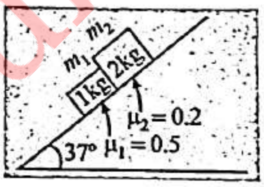
$\Rightarrow mg\sin\alpha - \mu\, mg\cos\alpha = ma$

$a = g\sin\alpha - \mu g\cos\alpha = g(\sin\alpha - \mu\cos\alpha)$

$$= 9.8\left(\frac{1}{2} - 0.4\times\frac{\sqrt{3}}{2}\right) = 4.9\times 0.3072 = 1.505\ \text{m/s}^2$$

The box accelerates down the plane at 1.505 m/s².

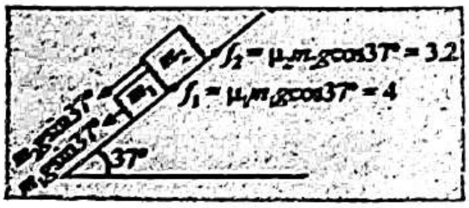
Example 31: Mass m_1 and m_2 are placed on a rough inclined plane as shown in figure. Find out the acceleration of the blocks and contact force in between these surface.



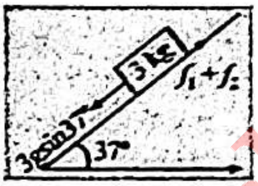
Sol. As we know if $\mu_1 > \mu_2$ both will travel together so

$a_1 = a_2 = a$

F.B.D

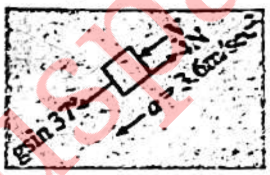


which is equivalent to



$$a = \frac{3g\sin 37^\circ - (f_1 + f_2)}{3} \Rightarrow a = \frac{18-7.2}{3} = 3.6\ \text{m/sec}^2$$

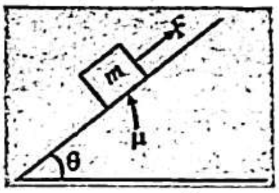
Now F.B.D of 1 kg block is



$g\sin 37^\circ + N - 4 = (1)\,a$

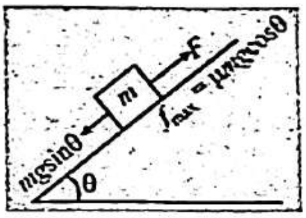
$N = 3.6 + 4 - 6 = 1.6$ Newton

Example 32:



Find out the range of force in the above situation for which m kg block does not move on the incline. ($\theta > \theta_c$)

Sol. F.B.D of the block



Block will slide upwards if

$F - mg\sin\theta > \mu mg\cos\theta$ (Friction acts down the incline)

$\Rightarrow F > \mu mg\cos\theta + mg\sin\theta$.

and block will slide downwards if

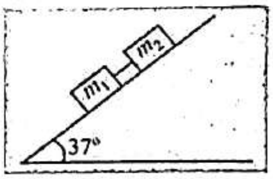
$mg\sin\theta - F > \mu mg\cos\theta$ (Friction acts up the incline)

$\Rightarrow F < mg\sin\theta - \mu mg\cos\theta$

Therefore, block will not move if

$mg\sin\theta - \mu mg\cos\theta < F < mg\sin\theta + \mu mg\cos\theta$

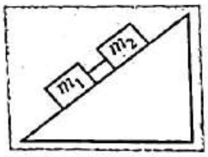
Example 33: Two blocks $m_1 = 4$ kg and $m_2 = 2$ kg, connected by a weightless rod on a plane having inclination of 37° as shown in figure. The coefficients of dynamic friction of m_1 and m_2 with the inclined plane are $\mu = 0.25$. Then the common acceleration of the two blocks and the tension in the rod are:



(*a*) 4 m/s², $T = 0$ (*b*) 2 m/s², $T = 5$ N

(*c*) 10 m/s², $T = 10$ N (*d*) 15 m/s², $T = 9$N

Sol. (*a*)

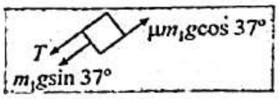


Apply Newton's law for system of m_1 and m_2

$$a = \frac{(m_1 + m_2)g\sin 37° - \mu[m_1 g\cos 37° + m_2 g\cos 37°]}{m_1 + m_2}$$

$= g[\sin 37° - \mu\cos 37°]$

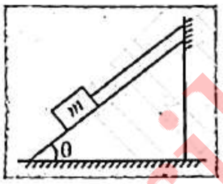
Now apply Newton's law for M_1



$m_1 g\sin 37° + T - \mu m_1 g\cos 37° = m_1 a = m_1 g[\sin 37° - \mu\cos 37°]$

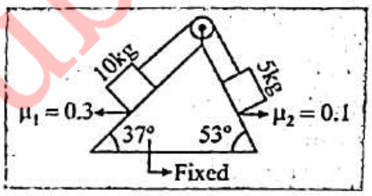
$\Rightarrow T = 0$ N and $a = 4\text{m/sec}^2$

Example 34: If $\mu = 0.9$ and $\theta = 45°$, then find tension in string



Sol. As $\theta < \theta_c$ block has no tendency to move hence tension in the string will be zero.

Example 35: Two blocks of masses 5 kg and 10 kg are attached with the help of light string and placed on a rough incline as shown in the figure. Coefficients of friction are as marked in the figure. The system is released from rest. Determine the acceleration of the two blocks. [Take $g = 10\text{ ms}^{-2}$]



Sol. Let 10 kg block is sliding down, then acceleration of both the blocks are given by.

$$a = \frac{(10\,g\sin 37° - \mu_1 \times 10\,g\cos 37° - 5\,g\sin 53° - \mu_2 \times 5\,g\cos 53°)}{15} = -\text{ve}$$

It means our assumed direction of motion is wrong and 5 kg block is going to slide down, if this would be the case, the direction of friction force will reverse and acceleration of blocks would be given by:

$$a_1 = \frac{(5g\sin 53° - \mu_2 \times 5g\cos 53° - \mu_1 \times 10g\cos 37° - 10\,g\sin 37°)}{15} = -\text{ve}$$

It means in this direction also there is no motion. So we can conclude that the system remains at rest and friction force is static in nature.

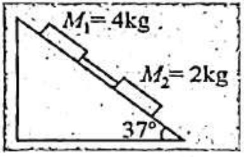
Example 36: Two blocks connected by a massless string slide down an inclined plane having angle of inclination 37°. The masses of the two blocks are $M_1 = 4$kg and $M_2 = 2$kg and the coefficients of friction 0.75 and 0.25 respectively [Take $g = 10\text{ m/s}^2$]

A. The common acceleration of the two masses is 1.3 ms^{-2}

B. The tension in the string is 14.7N

C. The common acceleration of the two masses is 2.94 ms^{-2}

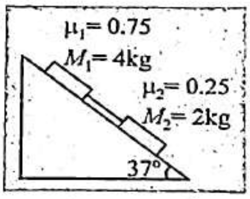
D. The tension in the string is 5.3N



(*a*) A, D (*b*) C, D

(*c*) B, D (*d*) B, C

Sol. (*a*)



Friction on M_1

$f_{1\max} = \mu_1 M_1 g\cos 37$

$= 0.75 \times 4 \times 10 \times \frac{4}{5}$

$= \frac{3 \times 4 \times 10 \times 4}{4 \times 5}$

$f_{1\max} = 24\text{N}$

Friction on M_2

$f_{2\max} = \mu_2 M_2 g\cos 37$

$= 0.25 \times 2 \times 10 \times \frac{4}{5} = \frac{1}{4} \times \frac{20 \times 4}{5}$

$f_{2\max} = 4\text{N}$

Downward force on M_1

$F_1 = M_1 g \sin 37$

$F_1 = 4 \times 10 \times \frac{3}{5}$

$F_1 = 24\text{ N}$

Downward force on M_2

$F_2 = M_2 g \sin 37$

$F_2 = 2 \times 10 \times \frac{3}{5}$

$F_2 = 12\text{ N}$

So resultant force on M_1 and M_2 is due to M_2

$F = F_2 - f_{2\max} = 12 - 4 = 8$

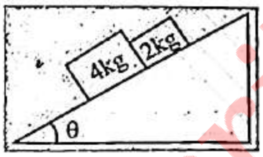
$8 = (M_1 + M_2)a$

$\therefore\ a = \frac{8}{6} = 1.3\text{ m/s}^2$

$T = M_1 a = 4 \times \frac{8}{6} = \frac{16}{3}\text{N}$

Concept Application

28. Figure shows two blocks in contact and sliding down the inclined surface of inclination 30°. Coefficient of friction between the block of mass 4 kg and the inclined plane is $\mu_1 = 0.3$ and that between the block of mass 2 kg and the inclined plane is $\mu_2 = 0.20$. Assume coefficients of static and kinetic friction to be equal. Find the acceleration of 2 kg block. Take $g = 10\text{ m/s}^2$.



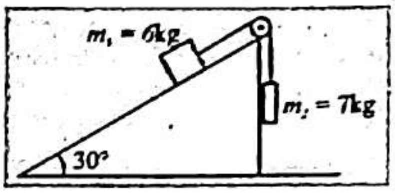
(*a*) $\frac{15-4\sqrt{3}}{3}\text{m/s}^2$

(*b*) $5-\sqrt{3}\text{ m/s}^2$

(*c*) $\sqrt{15}\text{ m/s}^2$

(*d*) 5 m/s^2

29. Two blocks of masses $m_1 = 6$ kg and $m_2 = 7$ kg are connected by a light string passing over a light frictionless pulley as shown in fig. The mass m_1 is at rest on the inclined plane and mass m_2 hangs vertically. If the angle of incline $\theta = 30°$. What is the magnitude and direction of the force of friction on the 6 kg block? (Take $g = 10\text{ ms}^{-2}$).



(*a*) 40 N up the plane

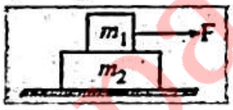
(*b*) 40 N down the plane

(*c*) 90 N up the plane

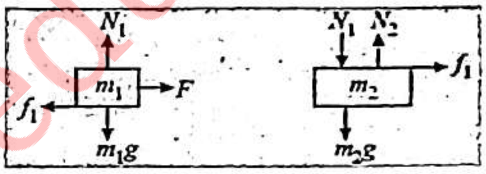
(*d*) 40 N down the plane

BLOCK OVER BLOCK PROBLEMS

Consider two block kept one over the other as shown in Figure where m_1 is kept on top of m_2.



Let the coefficient of friction between m_1 and m_2 is μ_1 and between m_2 and ground is $\mu_2 = 0$. To find acceleration of m_1 and m_2 we make their free-body diagrams and write Newton's second law



Along vertical

$N_1 - m_1 g = 0$ and $N_1 + m_2 g - N_2 = 0$

Solving the two gives.

$N_1 = m_1 g$ and $N_2 = (m_1 + m_2)g$

Along horizontal

$F - f_1 = m_1 a_1$ and $f_1 = m_2 a_2$

We know $f_1 < \mu_1 N_1$

There are two equations and three unknowns in equation along horizontal. This problem cannot be solved unless we make some assumptions.

Case-I: If we assume both blocks are moving together then friction between them need not be equal to maximum value. Thus, taking $a_1 = a_2$, we are down to two unknowns and we can solve the equations, but we must verify our answer by checking $f_1 \le \mu_1 N_1$. If this fails, then blocks slip an each other and we consider next possibility.

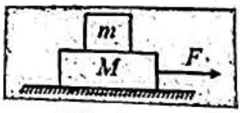
Case-II: If we assume blocks are moving relative to each other, then friction between them must have reached maximum value (Kinetic friction)

$$a_1 \ne a_2 \quad \text{but} \quad f_1 = \mu_1 N_1$$

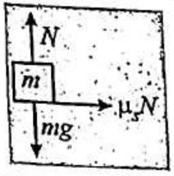
Again we get two unknowns and two equation. We can solve it.

When Force is Applied on Lower Block

Consider a block of mass m is placed on another block of mass M lying on a smooth horizontal surface. The coefficient of static friction between m and M is μ_s. The maximum force that should be applied to M so that the blocks remains at rest relative to each other then:



Draw the force diagrams of blocks at the moment when F is at its maximum value and m is about to slide relative to it.



Frictional force between m and $M = \mu_s N$

(N: normal reaction between the block)

Due to the friction. M will try to drag m towards right and hence frictional force will act on in towards right.

Let a = acceleration each block

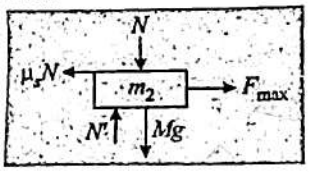
N' = normal reaction between M and the surface

From F.B.D. of m

$N = mg$

$\mu_s N = ma$

From F.B.D. of M.



$N + Mg = N'$

$F_{max} = \mu_s N = Ma$

combining these two equation, we get

$F_{max} = \mu_s(m + M)g$

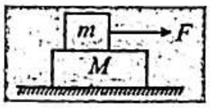
Hence $\mu_s(m + M)g$ is the critical value of force F.

If F is greater than this critical value, m begins to slip relative to M and their acceleration will be different.

If F is smaller than this critical value, m and M move together without any relative motion.

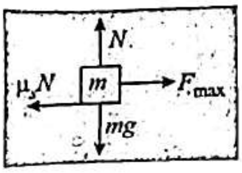
When Force is Applied on Upper Block

A block of mass m is placed on another block of mass M lying on a smooth horizontal surface. The coefficient of static friction between m and M is μ_s. The maximum force that can be applied to m so that blocks remains at rest relative to each other then:



Consider the situation when F is at its maximum value so that m is about to start slipping relative to M

Since, friction opposes relative motion. The mass m tries to drag M toward right due to friction.



From forces on m:

$F - \mu_s N = ma$ ... (i)

$N = mg$

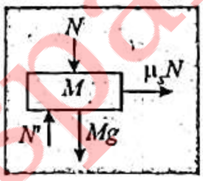
From F.B.D. of M:

$\mu_s N = Ma$

$\mu_s mg = Ma$ ... (ii)

or $a = \dfrac{\mu_s mg}{M}$

Putting the value of a in equation (i) we get,



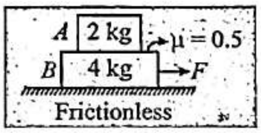
$$F_{max} = \frac{\mu_s(m + M)mg}{M}$$

If F is less than F_{max}, the blocks stick together without any relative motion.

If F is greater than F_{max} value, the blocks slide relative to each and their acceleration are different.

Train Your Brain

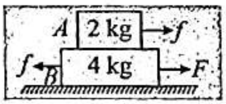
Example 37:



Find out the maximum value of F for which both the blocks will move together

Sol. In the given situation 2 kg block will move only due to friction force exerted by the 4 kg block

F.B.D.

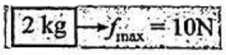


The maximum friction force exerted on the block B is

$f_{max} = \mu N$

$f_{max} = (0.5)(20) = 10$ N

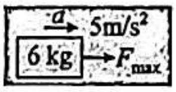
So the maximum acceleration of 2 kg block is



$$a_{2\max} = \frac{10}{2} = 5\ \text{m/s}^2$$

$a_{\max}$ is the maximum acceleration for which both the block will move together. i.e., for $a \leq 5\ \text{ms}^{-2}$ acceleration of both blocks will be same and we can take both the blocks as a system.

F.B.D



$F_{\max} = 6 \times 5 = 30$ N

For $0 < F < 30$

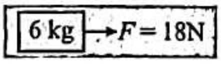
Both the block move together.

Example 38: In the above question find the acceleration of both the blocks when

(i) $F = 18$ N (ii) $F = 36$ N

Sol. (i) Since $F < 30$ both the blocks will move together

F.B.D

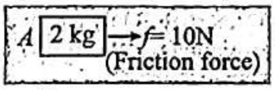


$$a = \frac{18}{6} = 3\ \text{m/s}^2$$

(ii) $F = 36$ N

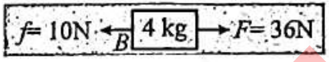
When $F > 30$ both the blocks will move separately so we treat each block independently

F.B.D of 2 kg block



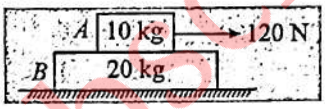
$a_A = 5\ \text{m/s}^2$.

F.B.D of 4 kg block



$$a_B = \frac{36-10}{4} = \frac{26}{4}\ \text{m/s}^2$$

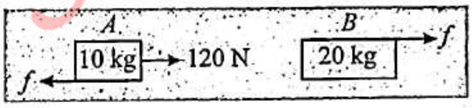
Example 39: Friction coefficient between the blocks is 0.5 and ground is smooth. If force of 120 N is applied on the upper block as shown in figure find acceleration of the blocks.



Sol. $f_{\max} = 0.5 \times 10g = 50$ N

Let's assume they have same acceleration. The value of common acceleration is $\frac{120}{30} = 4\ \text{m/s}^2$.

Making free-body diagram



Newton's second law given

$f = 20 \times 4 = 80$ N

Solving we get, $a = 4\ \text{m/s}^2$ and $f = 80$ N while limiting value of friction is 50 N. hence our assumption is wrong.

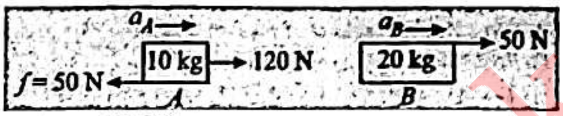
Let's say they are not moving together. Then $f_k = 50$ N.

By Newton's second law.

$120 - 50 = 10\, a_A$

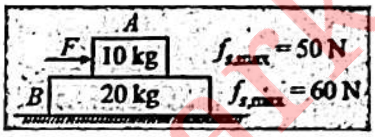
$50 = 20\, a_B$

$a_A = 7\ \text{m/s}^2$, $a_B = 2.5\ \text{m/s}^2$



This gives us the correct direction of friction.

Example 40:



Friction coefficient between block is 0.5 and between ground and 20 kg block is 0.2. Find acceleration of blocks if force $F = 80$ N is applied on 10 kg block as shown in figure

Sol. The values of limiting friction at contact surfaces ($f_{s,\max}$)

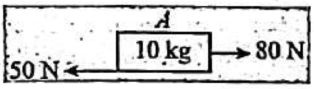
$f_{\max}$ between blocks $f_1 = 0.5 \times 10 \times 10 = 50$ N

$f_{\max}$ between 20 kg block and ground,

$f_2 = 0.2 \times 30 \times 10 = 60$ N

The only driving force that the block B can experience is the one applied by the lower surface of block A on block B.

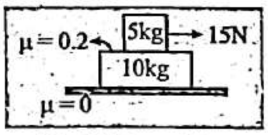
The maximum value of this force is 50 N. This is lower than the minimum force required to move block B with respect to ground $f_{\text{required}} = 0.2 \times 30 \times 10 = 60$ N. Hence only the block A will move.



$$\therefore a_A = \frac{80-50}{10} = 3\ \text{m/s}^2$$

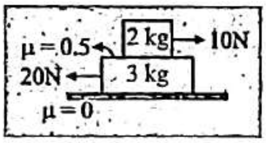
Concept Application

30. The frictional force between the blocks is:



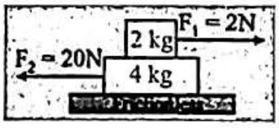
(*a*) 5 N (*b*) 10 N (*c*) 8 N (*d*) 6 N

31. The frictional force on block of mass 2 kg is:



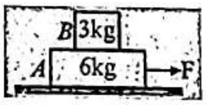
(*a*) 8 N (*b*) 10 N (*c*) 18 N (*d*) 6 N

32. In the arrangement shown in figure, coefficient of friction between the two blocks is $\mu = 1/2$. The force of friction acting between the two blocks is:



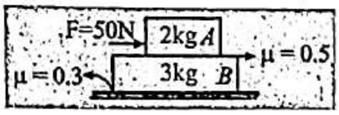
(*a*) 8 N (*b*) 10 N (*c*) 6 N (*d*) 4 N

33. Two blocks *A* and *B* of masses 6 kg and 3 kg rest on a frictionless horizontal surface as shown in the figure. If coefficient of friction between *A* and *B* is 0.4, the maximum horizontal force which can make them move without separation is



(*a*) 72 N (*b*) 40 N (*c*) 36 N (*d*) 20 N

34. From the figure shown, find out acceleration of 3kg block.



(*a*) 7 m/s^2 (*b*) 10 m/s^2

(*c*) 10/3 m/s^2 (*d*) None of these

DYNAMICS OF CIRCULAR MOTION

In circular motion or motion along any curved path Newton's law is applied in two perpendicular directions one along the tangent and other perpendicular to it. i.e., towards centre. The component of net force towards the centre is called centripetal force. The component of net force along the tangent is called tangential force.

tangential force $(F_t) = Ma_t = M\frac{dv}{dt} = M\alpha r$; where α is the angular acceleration

centripetal force $(F_c) = m\omega^2 r = \frac{mv^2}{r}$

Centripetal Force

Concepts: This is necessary resultant force towards the centre called the centripetal force.

$$F = \frac{mv^2}{r} = m\omega^2 r$$

(i) A body moving with constant speed in a circle is not in equilibrium.

(ii) It should be remembered that in the absence of the centripetal force the body will move in a straight line with constant speed.

(iii) It is not a new kind of force which acts on bodies. In fact, any force which is directed towards the centre may provide the necessary centripetal force.

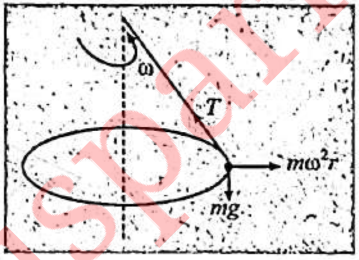
Centrifugal Force

When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. To an observer A who is not sharing the motion along the circular path, the body appears to fly off tangentially at the point of release. To another observer *B*, who is sharing the motion along the circular path (i.e., the observer *B* is also rotating with the body which is released, it appears to *B*, as if it has been thrown off along the radius away from the centre by some force. This inertial force is called centrifugal force.)

Its magnitude is equal to that of the centripetal force $= \frac{mv^2}{r}$.

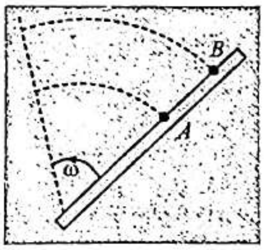
Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply Newton's law of motion in that frame)

FBD of ball w.r.t. non-inertial frame rotating with the ball.



Suppose we are working from a frame of reference that is rotating at a constant, angular velocity ω with respect to an inertial frame. If we analyse the dynamics of a particle of mass *m* at a distance *r* from the axis of rotation, we have to assume that a force $mr\omega^2$ act radially outward on the particle. Only then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called the centrifugal force.

Note: A rod move with ω angular velocity then we conclude following for point *A* and *B* in a rod.

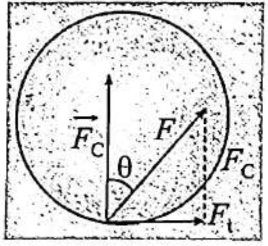


$\alpha_A = \alpha_B, s_B > s_A$

$\theta_A = \theta_B, v_B > v_A$

$\omega_A = \omega_B\ a_{tB} > a_{tA}$

Net Force on the Particle in Non-Uniform Circular Motion



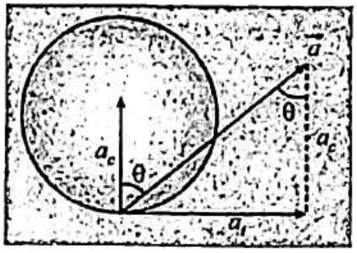
$$\vec{F} = \vec{F}_c + \vec{F}_t \Rightarrow F = \sqrt{F_c^2 + F_t^2}$$

If θ is the angle made by F with F_c

then $\tan\theta = \frac{F_t}{F_c} \Rightarrow \theta = \tan^{-1}\left[\frac{F_t}{F_c}\right]$

[angle between F_c and F_t is 90°]

Angle between F and F_t is $(90° - \theta)$



Net acceleration, $a = \sqrt{a_c^2 + a_t^2} = \frac{F_{net}}{m}$

The angle made by 'a' with a_c, $\tan\theta = \frac{a_t}{a_c} = \frac{F_t}{F_c}$

The direction of this resultant acceleration makes an angle θ with the radius.

where $\tan\theta = \dfrac{\frac{dv}{dt}}{\frac{v^2}{r}}$

Hint to Solve Numerical Problem

(i) Write down the required centripetal force

(ii) Draw the free body diagram of each component of system.

(iii) Resolve the forces acting on the rotating particle along radius and perpendicular to radius

(iv) Calculate net radial force acting towards centre of circular path.

(v) Make it equal to required centripetal force.

(vi) For remaining components see according to question.

Important Points in Uniform and Non-Uniform Circular Motion

- Since in both uniform and non-uniform circular motion F_c is perpendicular to velocity, so work done by centripetal force will be zero in both the cases.
- In uniform circular motion $F_t = 0$, as $a_t = 0$, so work done will be zero by tangential force.
- In non-uniform circular motion $F_t \neq 0$, so work done by tangential force is non-zero.
- Rate of work done by net force in non-uniform circular motion = rate of work done by tangential force

$$\Rightarrow P = \frac{dW}{dt} = \vec{F}_t.\vec{v} = \vec{F}_t.\frac{d\vec{x}}{dt}$$

- In a circle as tangent and radius are always normal to each other, so $\vec{a}_t \perp \vec{a}_r$.

Net acceleration in case of circular motion

$$a = \sqrt{a_r^2 + a_t^2}$$

Here it must be noted that a_t governs the magnitude of $\vec{v}$ while a_r its direction of motion, so that

⇒ If $a_r = 0$ and $a_t = 0$, $a \to 0$ motion is uniform translatory

⇒ If $a_r = 0$ but $a_t \neq 0$, $a \to a_t$ motion is accelerated translatory

⇒ If $a_r \neq 0$ but $a_t = 0$, $a \to a_r$ motion is uniform circular

⇒ If $a_r \neq 0$ and $a_t \neq 0$, $a \to \sqrt{a_r^2 + a_t^2}$ motion is non-uniform circular

The centripetal force can be expressed as:

$$\vec{F} = m\vec{a} = -m\omega^2\vec{r} = -m\omega^2 r\hat{r} = -\left(\frac{mv^2}{r}\right)\hat{r}$$

(*a*) For earth's motion around the sun, the centripetal force is provided by the gravitational force.

(*b*) For electron's motion around the nucleus in hydrogen atom, centripetal force is provided by the electrostatic force.

- If the body on a circular path comes to rest, i.e., $\vec{v} \to 0$, the body will move along the radius towards the centre and if radial acceleration a_r vanishes, the body will fly off along the tangent. So a tangential velocity and a radial acceleration are must for uniform circular motion.
- As $F = \frac{mv^2}{r} \neq 0$, so the body is not in equilibrium and linear momentum of the particle moving on the circle is not conserved. However, as the force is central, i.e., $\tau = 0$, so angular momentum is conserved.

Train Your Brain

Example 41: A body of mass m is attached with a string of length ℓ. If it is whirled in a horizontal circular path with velocity v. The tension in the string will be

(*a*) mv^2

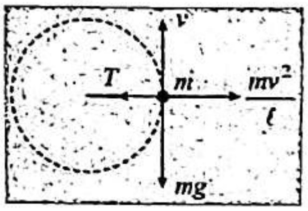
(*b*) $\frac{mv^2}{\ell}$

(*c*) $\frac{m\ell}{v^2}$

(*d*) $\frac{mv^2}{2\ell}$

Sol. (*d*) Required centripetal force,

$$F_c = \frac{mv^2}{\ell} = T$$



Example 42: A satellite of mass m is revolving around the earth of mass M in circular orbit of radius r. The orbital velocity of the satellite will be

(*a*) $\sqrt{\dfrac{GM}{r}}$ (*b*) $\sqrt{\dfrac{GM}{mr}}$

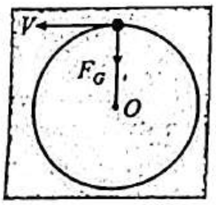
(*c*) $\sqrt{\dfrac{Gm}{Mr}}$ (*d*) $\dfrac{mv^2}{r}$

Sol. (*a*) The required centripetal force,

$$F_G=\frac{GMm}{r^2} \text{ (towards the centre)}$$

Net force towards the centre,

$$F_C=\frac{mv^2}{r}$$



(This force will provide required centripetal force)

Therefore $F_C=F_G$

$$\Rightarrow \frac{mv^2}{r}=\frac{GMm}{r^2}$$

$$\Rightarrow v=\sqrt{\frac{GM}{r}}$$

Hence correct answer is (*a*)

Notes:

(i) From above example we see that orbital velocity of a body is independent to its mass

(ii) If we are asked to find out time period of above body then time period can be calculated as

$$\Rightarrow T=\frac{2\pi}{\omega}=\frac{2\pi r}{v}$$

$$T=\sqrt{\frac{r^3}{GM}}=2\pi\frac{mv^2}{3\ell}$$

$T^2 \propto r^3$ this is **Kepler's law.**

Example 43: A small block of mass 100 g moves with uniform speed in a horizontal circular groove, with vertical side walls, of radius 25 cm. If the block takes 2.0 s to complete one round, find the normal contact force by the slide wall of the groove.

Sol. The speed of the block is

$$v=\frac{2\pi\times(25\text{ cm})}{2.0\text{ s}}=0.785\text{ m/s}$$

The acceleration of the block is

$$a=\frac{v^2}{r}=\frac{(0.785\text{m/s})^2}{0.25}=2.46\text{m/s}^2$$

Towards the center. The only force in this direction is the normal contact force due to the side walls. Thus from Newton's second law, this force is

$$N=ma=(0.100\text{kg})(2.46\text{m/s}^2)=0.246\text{N}$$

Example 44: A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.

Sol. Let ω be the angular speed of rotation of the bowl.

Two force are acting on the ball.

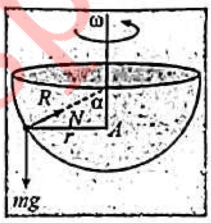
1. normal reaction N, 2. weight mg

The ball is rotating in a circle of radius r ($= R\sin\alpha$) with centre at A at an angular speed ω. Thus,

$$N\sin\alpha=m\omega^2 r=mR\omega^2\sin\alpha$$

$$N=mR\omega^2 \quad \text{... }(i)$$

$$\text{and } N\cos\alpha=mg \quad \text{... }(ii)$$



Dividing Eqs. (*i*) by (*ii*),

we get, $\dfrac{1}{\cos\alpha}=\dfrac{\omega^2 R}{g}$, $\therefore\ \omega=\sqrt{\dfrac{g}{R\cos\alpha}}$

Example 45: In above example, if friction is present between the surface of ball and bowl then find out the range of ω for which ball does not slip (μ is the friction coefficient)

Friction develop a range of ω for which the particle will be at rest.

Sol. When $\omega > \omega_0$

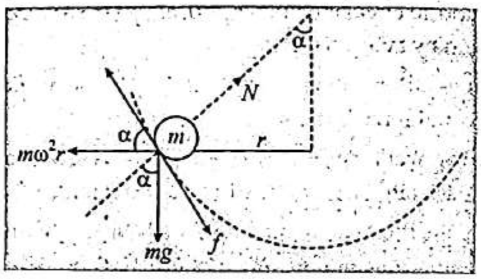
In this situation ball has a tendency to slip upwards so the friction force will act downwards. So F.B.D of ball

$$N=m\omega^2 r\sin\alpha+mg\cos\alpha$$

$$f+mg\sin\alpha=m\omega^2 r\cos\alpha \quad \text{... }(ii)$$

$$\therefore f_{\max}=\mu N=\mu(m\omega^2 r\sin\alpha+mg\cos\alpha)$$

$$r=R\sin\alpha$$



Substituting the values of f_{max} and r in eq. (ii) we get

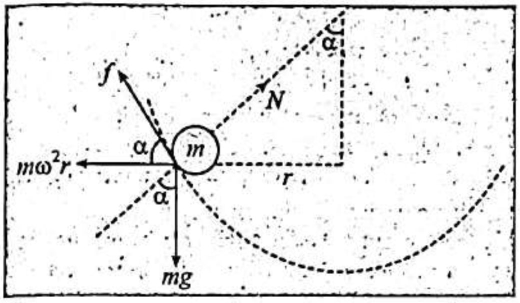
$$\Rightarrow \mu\left(m\omega^2 r\sin\alpha + mg\cos\alpha\right) \geq m\omega^2 r\cos\alpha - mg\sin\alpha$$

$$\therefore \mu\left(m\omega^2 R\sin^2\alpha + mg\cos\alpha\right) \geq m\omega^2 R\sin\alpha\cos\alpha$$
$$- mg\sin\alpha$$

$$\omega \leq \sqrt{\frac{\mu g\cos\alpha + g\sin\alpha}{R\sin\alpha\left(\cos\alpha - \mu\sin\alpha\right)}}$$

when $\omega < \omega_0$

In this situation ball has a tendency to slip downwards so the friction force will act upwards. So F.B.D of ball



$$\Rightarrow f + m\omega^2 r\cos\alpha = mg\sin\alpha$$

$$f = m\left(g\sin\alpha - \omega^2 r\cos\alpha\right) \quad \ldots (i)$$

$$N = mg\cos\alpha + m\omega^2 r\sin\alpha \quad \ldots (ii)$$

$$f_{max} = \mu N = \mu\left(mg\cos\alpha + m\omega^2 r\sin\alpha\right)$$

for equilibrium

$$\Rightarrow \mu\left(g\cos\alpha + \omega^2 r\sin\alpha\right) \geq \left(g\sin\alpha - \omega^2 r\cos\alpha\right)$$

$\Rightarrow$ Substituting $r = R\sin\alpha$ then

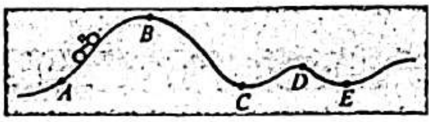
$$\omega \geq \sqrt{\frac{g\left(\sin\alpha - \mu\cos\alpha\right)}{R\sin\alpha\left(\mu\sin\alpha + \cos\alpha\right)}}$$

Concept Application

35. A body of mass 0.1 kg is moving on circular path of diameter 1.0 m at the rate of 10 revolutions per 31.4 seconds. The centripetal force acting on the body is
(a) 0.2 N (b) 0.4 N
(c) 2 N (d) 4 N

36. A body of mass 4 kg is moving in a horizontal circle of radius 1 m with an angular velocity of 2 rad/s. The required centripetal force, will be
(a) 16 N (b) 1.6 N
(c) 16 Dyne (d) 1.6 Dyne

MOTION OF A MOTORCYCLIST ON A CURVED PATH



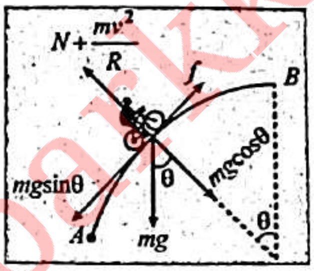
A cyclist having mass m move with constant speed v on a curved path as shown in figure.

We divide the motion of cyclist in four parts:

1. from A to B
2. from B to C
3. from C to D
4. from D to E

1 and 3 are same type of motion)

(A) Motion of Cyclist from A to B



$$N + \frac{mv^2}{R} = mg\cos\theta$$

$$\Rightarrow N = mg\cos\theta - \frac{mv^2}{R}$$

$f = mg\sin\theta$

(1) As cyclist move upward

$\because$ θ decreases and cos θ increases

$\therefore$ N increases

And

$\because$ θ decreases sin θ decreases

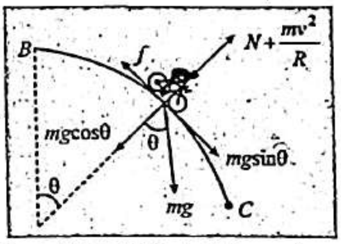
$\therefore$ friction force required to balance mg sin θ (As cyclist is moving with constant speed) also decreases

(B) Motion of Cyclist from B to C

$$N + \frac{mv^2}{R} = mg\cos\theta$$

$$\Rightarrow N = mg\cos\theta - \frac{mv^2}{R}$$

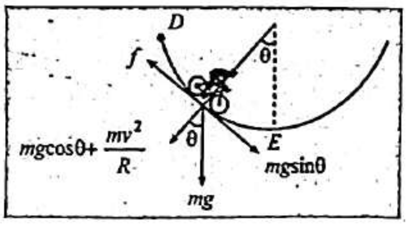
$f = mg\sin\theta$



Therefore from B to C Normal force decrease but friction force increase because θ increases.

(C) Motion of Cyclist from D to E

$$N = \frac{mv^2}{R} + mg\cos\theta, \quad f = mg\sin\theta$$



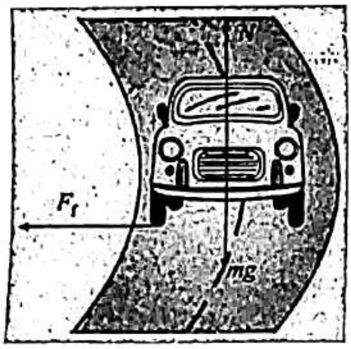
from D to E θ decreases

therefore mg cos θ increase

So N increase but f decreases

A CAR TAKING A TURN ON A LEVEL ROAD

When a car takes a turn on a level road, the portion of the turn can be approximated by an arc of a circle of radius r. If the car makes the turn at a constant speed v, then there must be some centripetal force acting on the car. This force is generated by the friction between the tyres and the road.



The maximum frictional force is $F_f = \mu_s N$, μ_s = the coefficient of static friction.

Then the maximum safe velocity v is given by

$$\frac{mv^2}{r} = \mu_s N = \mu_s mg \; (\because N = mg) \Rightarrow \boxed{v = \sqrt{\mu_s rg}} \text{ or } \mu_s = \frac{v^2}{rg}$$

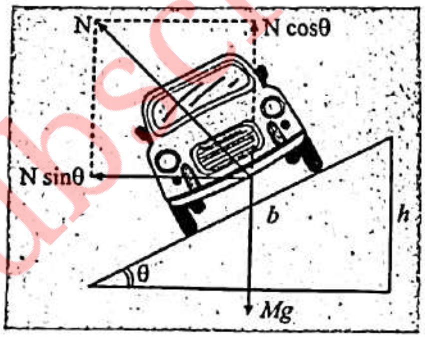
It is important to note that safe velocity is independent of the mass of the car.

Circular Turning by Banking of Tracks Only

Roads on large highways are generally banked so that a vehicle may make a safe and easier turn without depending on friction, road bend at the curved path is raised a little on the side away from the centre of the curved path.

Due to banking of the road, a component of the normal force points towards the centre of curvature of the road.

Necessary centripetal force required for circular motion is supplied by this component. The vertical component of the normal force is balanced by the weight of the vehicle.



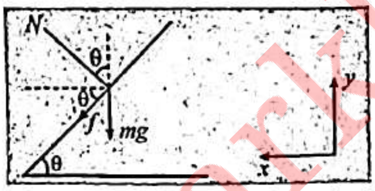
$$N\cos\theta = mg \text{ and } N\sin\theta = \frac{mv^2}{r}$$

$$\therefore \boxed{\tan\theta = \frac{v^2}{rg}} \Rightarrow v = \sqrt{rg\tan\theta}$$

For a road with angle of banking θ, the speed v at which minimum wear and tear of tyre takes place is $\boxed{v = \sqrt{rg\tan\theta}}$

Circular Turning by Banking of Tracks and Friction Both

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these force, the first force, i.e., weight (mg) is fixed both in magnitude and direction.



The direction of second force i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_L = \mu N$). So the magnitude of normal reaction N and directions plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the center of these m and r are also constant.

Therefore, magnitude of N and directions plus magnitude of friction mainly depends on the speed of the vehicle v. Thus, situation varies from problem to problem. Even though we can see that:

(i) Friction f will be outwards if the vehicle is at rest $v = 0$. Because in that case the component weight $mg\sin\theta$ is balanced by f.

(ii) Friction f will be inwards if

$v > \sqrt{rg\tan\theta}$

(iii) Friction f will be outwards if

$v < \sqrt{rg\tan\theta}$ and

(iv) Friction f will be zero if

$v = \sqrt{rg\tan\theta}$

(v) For maximum safe speed (figure (ii))

$$N\sin\theta + f\cos\theta = \frac{mv^2}{r} \quad \ldots (i)$$

$$N\cos\theta - f\sin\theta = mg \quad \ldots (ii)$$

As maximum value of friction

$f = \mu N$

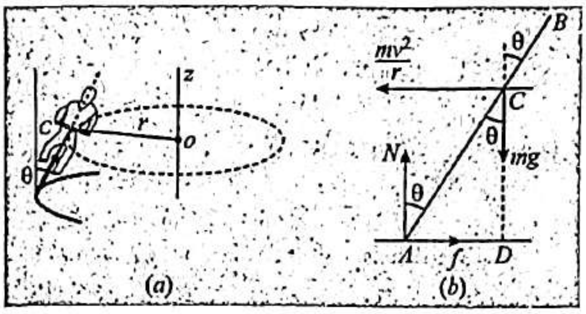
$$\therefore \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg} \therefore \boxed{v_{\max} = \sqrt{\frac{rg(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}}$$

$$\text{Similarly; } \boxed{v_{\min} = \sqrt{\frac{rg(\mu - \tan\theta)}{(1 + \mu\tan\theta)}}}$$

- The expression $\tan\theta = \frac{v^2}{rg}$ also gives the angle of banking for an aircraft, i,e., the angle through which it should tilt while negotiating a curve, to avoid deviation from the circular path.
- The expression $\tan\theta = \frac{v^2}{rg}$ also gives the angle at which a cyclist should lean inward, when rounding a corner: In this case, θ is the angle which the cyclist must make with the vertical to negotiate a safe turn.

TURNING OF A CYCLIST AROUND A CORNER ON THE ROAD

Suppose a cyclist is going at a speed v on a circular horizontal road of radius r which is not banked. Consider the cycle and the rider together as the system. The centre of mass C (figure shown) of the system is going in a circle with the centre at O and radius r.



Let us choose O as the origin, OC as the x-axis and vertically upward as the z-axis. This frame is rotating at an angular speed $\omega = \frac{v}{r}$ about the z-axis. In this frame the system is at rest. Since we are working from a rotating frame of reference, we will have to apply a centrifugal force on each $\omega^2 r = mv^2/r$, where m is the total mass of the system. This force will act through the centre of mass. Since the system is at rest in this frame, no other pseudo force is needed.

In the frame considered, the system is at rest. Thus, the total external force and the total external torque must be zero. Let us consider the torques of all the forces about the point A. The torques of N and f about A are zero because these forces pass through A. The torque of mg about A is $mg(AD)$ in the clockwise direction and that of $\frac{mv^2}{r}$ is $\frac{mv^2}{r}(CD)$ in the anticlockwise direction. For rotational equilibrium,

$$mg(AD) = \frac{mv^2}{r}(CD)$$

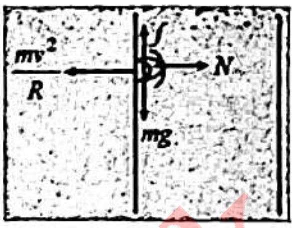
$$\text{or, } \frac{AD}{CD} = \frac{v^2}{rg}$$

$$\text{So, } \tan\theta = \frac{v^2}{rg}, \text{ i.e.,}$$

Angle θ should be: $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

Death Well

A motor cyclist is driving in a horizontal circle on the inner surface of vertical cylinder of radius R. Friction coefficient between tyres of motorcyclist and surface of cylinder is μ. Find out the minimum velocity for which the motorcyclist can do this. v is the speed of motor cyclist and m is his mass.



$$N = \frac{mv^2}{R}$$

$$f = mg$$

$$f_{\max} = \frac{\mu mv^2}{R}$$

Cyclist does not drop down when

$$f_{\max} \geq mg \Rightarrow \frac{\mu mv^2}{R} \geq mg$$

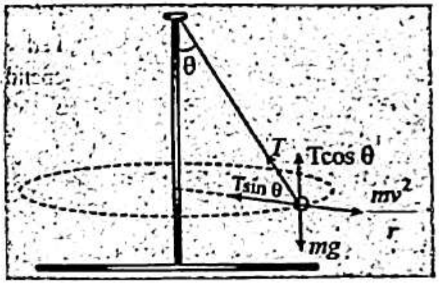
$$v \geq \sqrt{\frac{gR}{\mu}}$$

CONICAL PENDULUM

(i) If a small bob of mass m tied to a string is whirled in a horizontal circle, the string will not remain horizontal but the string becomes inclined to the vertical and sweeps a cone while the body moves on a horizontal circle with uniform speed. This arrangement is called **conical pendulum.**

(ii) In case of conical pendulum, the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force, i.e.,

$$T\cos\theta = mg \text{ and } T\sin\theta = \frac{mv^2}{r} \quad \text{or} \quad \tan\theta = \frac{v^2}{rg}$$



$$\text{Also } T = m\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}$$

$$\text{Hence } v = \sqrt{rg\tan\theta}$$

$$\text{i.e., } \omega = \sqrt{\frac{g\tan\theta}{r}}$$

Hence, time period

$$t = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

(iii) Time period t is independent of the mass of the body and depends on $L\cos\theta$ $(= h)$

i.e., distance between point of suspension and centre of circle.

(iv) If $\theta = 90°$, the pendulum becomes horizontal and it follows that $v = \infty$, $T = \infty$ and $t = 0$
It is practically impossible.

Train Your Brain

Example 46: A railway track is banked for a speed v, by making the height of the outer rail (h) higher than that of the inner rail. The distance between the rails is d. The radius of curvature of the track is r

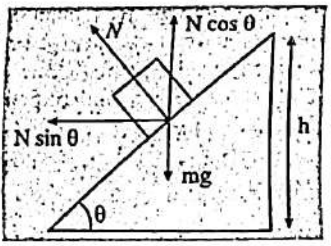
(*a*) $\frac{h}{d} = \frac{v^2}{rg}$

(*b*) $\tan\left(\sin^{-1}\frac{h}{d}\right) = \frac{v^2}{rg}$

(*c*) $\tan^{-1}\left(\frac{h}{d}\right) = \frac{v^2}{rg}$

(*d*) $\frac{h}{r} = \frac{v^2}{dg}$

Sol. (*b*)



$$\sin\theta = \frac{h}{d}, \quad N\cos\theta = mg \qquad \dots(i)$$

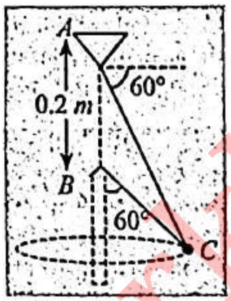
$$N\sin\theta = \frac{mv^2}{r} \qquad \dots(ii)$$

$$\therefore \frac{(2)}{(1)} \Rightarrow \tan\theta = \frac{v^2}{rg}$$

$$\Rightarrow \tan\left[\sin^{-1}\frac{h}{d}\right] = \frac{v^2}{rg}$$

Concept Application

37. Two wires AC and BC are each tied to a sphere at C. $AB = 0.2$ m the sphere is made to revolve in a horizontal circle at a constant speed v. Then when both wires are taut.



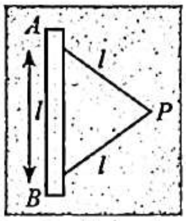
(*a*) $v_{\min} = 1$ m/s

(*b*) $v_{\min} = 2$ m/s

(*c*) $v_{\max} = \sqrt{3}$ m/s

(*d*) $v_{\max} = 4$ m/s

38. A particle P of mass m is attached to a vertical axis by two strings AP and BP of length l each. The separation $AB = l$. P rotates around the axis with an angular velocity ω. The tensions in the two strings are T_1 and T_2



(*a*) $T_1 = T_2$

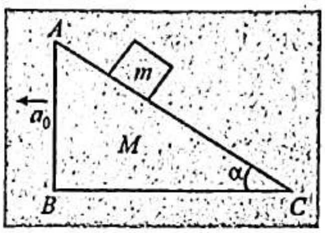
(*b*) $T_1 + T_2 = m\omega^2 l$

(*c*) $T_1 - T_2 = 2mg$

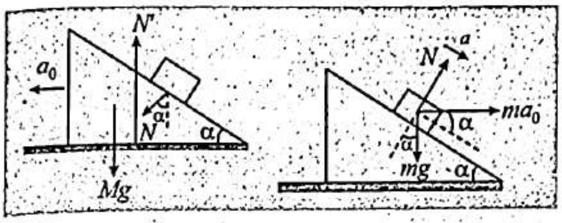
(*d*) BP will remain taut only if $\omega \geq \sqrt{\frac{2g}{\ell}}$

Aarambh (Solved Examples)

1. All the surfaces shown in figure are assumed to be frictionless. The block of mass m slides on the prism which in turn slides backward on the horizontal surface. Find the acceleration of the smaller block with respect to the prism.



Sol. Let the acceleration of the prism be a_0 in the backward direction. Consider the motion of the smaller block from the frame of the prism The forces on the block are (figure)



(i) N normal force

(ii) mg downward (gravity),

(iii) ma_0 forward (Pseudo Force)

The block slides down the plane. Components of the forces parallel to the incline give $ma_0 \cos\alpha + mg \sin\alpha = ma$

or; $a = a_0 \cos\alpha + g \sin\alpha$ ... (*i*)

Components of the forces perpendicular to the incline give

$N + ma_0 \sin\alpha = mg \cos\alpha$ ... (*ii*)

Now consider the motion of the prism from the ground frame. No pseudo force is needed as the frame used is inertial. The forces are (figure)

(i) Mg downward

(ii) N normal to the incline (by the block)

(iii) N' upward (by the horizontal surface)

Horizontal components give,

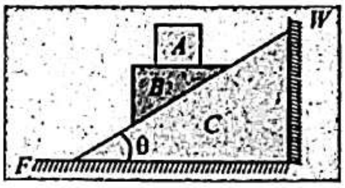
$N \sin\alpha = Ma_0$ or $N = Ma_0/\sin\alpha$ ... (*iii*)

Putting in (*ii*)

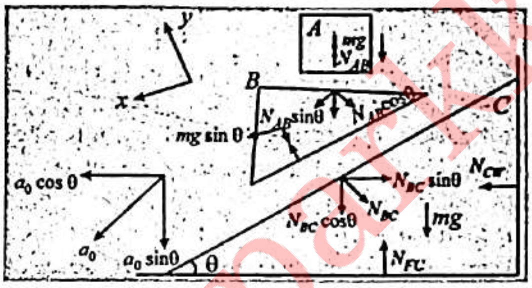
$$\frac{Ma_0}{\sin\alpha} + ma_0\sin\alpha = mg\cos\alpha \quad \text{or} \quad a_0 = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}$$

From (*i*), $a = \dfrac{mg\sin\alpha\cos^2\alpha}{M + m\sin^2\alpha} + g\sin\alpha = \dfrac{(M+m)g\sin\alpha}{M + m\sin^2\alpha}$

2. In the figure shown all the surfaces are smooth. The blocks A, B and C have the same mass m. F is floor and W is the wall. Find the magnitude of the contact forces at all the surfaces after the system is released from rest. The angle of inclination of the inclined plane with the horizontal is θ.



Sol.



$mg - N_{AB} = ma_A$

[Newton's II law for block A in vertical direction]

$mg\sin\theta + N_{AB}\sin\theta = ma_B$

[Newton's II law for block B in x direction]

$a_A = a_B \sin\theta$

[Constrained relation for contact surface between block A and B]

Solving above three equations we get

$$N_{AB} = \frac{mg\cos^2\theta}{1+\sin^2\theta}$$

$mg\cos\theta + N_{AB}\cos\theta - N_{BC} = 0$

[Equilibrium of block B in y direction]

$$\Rightarrow N_{BC} = mg\cos\theta + \frac{mg\cos^2\theta\cos\theta}{1+\sin^2\theta}$$

$$\Rightarrow N_{BC} = \frac{2mg\cos\theta}{1+\sin^2\theta}$$

$N_{BC}\sin\theta - N_{WC} = 0$

[Equilibrium of block C in horizontal direction]

$$N_{WC} = \frac{2mg\sin\theta\cos\theta}{1+\sin^2\theta}$$

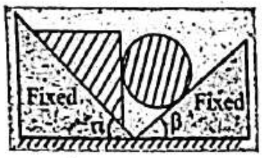
$N_{BC}\cos\theta + mg - N_{FC} = 0$

[Equilibrium of block C in vertical direction]

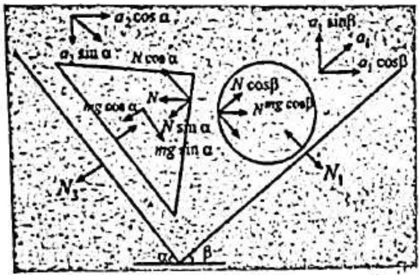
$$\Rightarrow N_{FC} = \frac{2mg\cos^2\theta}{1+\sin^2\theta} + mg$$

$$\Rightarrow N_{FC} = \frac{mg\,(2+\cos^2\theta)}{1+\sin^2\theta}$$

3. A cylinder and a wedge of same masses with a vertical face, touching each other, move along two smooth inclined planes forming the same angle α and β respectively with the horizontal. Determine the force of normal N exerted by the wedge on the cylinder, neglecting the friction between them.


Fixed
Fixed
α
β

Sol.


$a_2\cos\alpha$
$a_2\sin\alpha$
$N\cos\alpha$
$a_1\sin\beta$
a_1
$a_1\cos\beta$
$N\cos\beta$
N
$mg\cos\beta$
$mg\cos\alpha$
$N\sin\alpha$
$mg\sin\alpha$
N_1
N_3
α
β

It is obvious that acceleration of cylinder is parallel to the wedge 1 and acceleration of triangular block is parallel to the wedge 2.

$a_2 \cos\alpha = a_1 \cos\beta$ ... (*i*)

[constrained relation between the contact surface of block and cylinder]

$N\cos\beta - m_1 g\sin\beta = m_1 a_1$ ... (*ii*)

[Newton's II law for cylinder along the direction parallel to the wedge1]

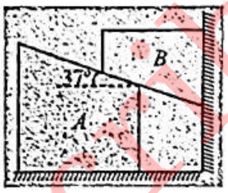
$m_2 g\sin\alpha - N\cos\alpha = m_2 a_2$ ... (*iii*)

[Newton's II law for block along the direction parallel to the wedge 2]

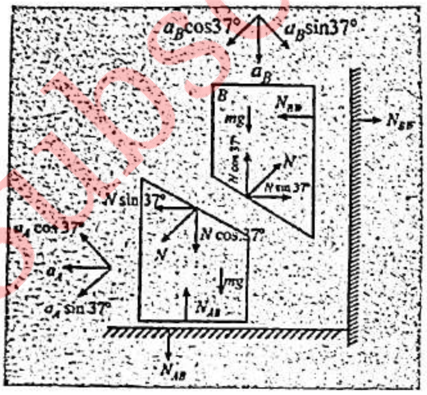
By solving equation (*i*), (*ii*) and (*iii*) we get

$$N = mg\left(\frac{\sin\alpha\cos\alpha + \sin\beta\cos\beta}{\cos^2\alpha + \cos^2\beta}\right)$$

4. The masses of blocks *A* and *B* are same and equal to *m*. Friction is absent everywhere. Find the magnitude of normal force with which block *B* presses on the wall and accelerations of the blocks *A* and *B*.



Sol.


$a_B\cos37°$
$a_B\sin37°$
a_B
B
mg
N_{BW}
N_{BW}
$N\cos37°$
N
$N\sin37°$
$N\sin37°$
$a_A\cos37°$
a_A
$a_A\sin37°$
N
$N\cos37°$
mg
N_{AB}
N_{AB}

$mg - N\cos 37 = ma_B$

[Newton's II law for block *B* in vertical direction]

$N\sin 37 = ma_A$

[Newton's II law for block *A* in horizontal direction]

$a_B\cos 37 = a_A\sin 37$

[constrained relation for contact surface between block *A* and *B*]

By solving above three equations we get

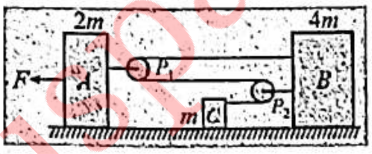
$a_A = \frac{12g}{25}$ $a_B = \frac{9g}{25}$ $N = \frac{4mg}{5}$

$N_{BW} = N\sin 37$

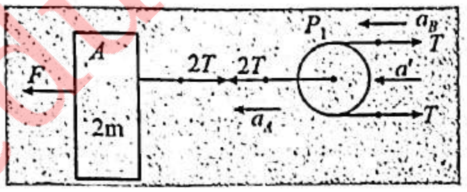
[Equilibrium of block *B* in horizontal direction]

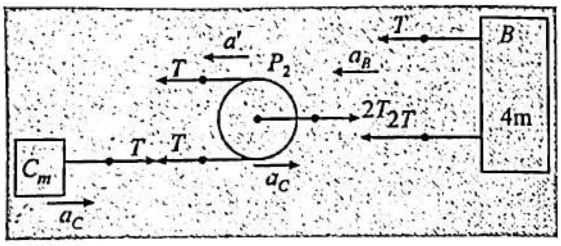
$\Rightarrow N_{BW} = \frac{12mg}{25}$

5. In the shown figure, find the acceleration of block *B*. All surfaces are frictionless.


2m
4m
F
A
P_1
P_2
B
m
C

Sol.


P_1
a_B
T
F
A
2T
2T
a'
2m
a_A
T


T
B
a'
P_2
a_B
T
2T
2T
4m
C_m
T
T
a_C
a_C

$-2TV_A + 3TV_B + TV_C = 0 \Rightarrow 3V_B + V_C = 2V_A$

From above equation

$3a_B + a_c = 2a_A$ ...(i) [Constraint relation]

$F - 2T = 2ma_A$ ...(ii) [Newton's II law for block *A*]

$3T = 4m\,a_B$ ....(iii) [Newton's II law for block *B*]

$T = ma_C$ ....(iv) [Newton's II law for block *C*]

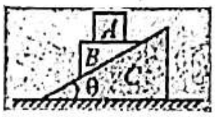
From (i), (ii), (iii) and (iv)

$$3\times\frac{3T}{4m} + \frac{T}{m} = 2\left(\frac{F-2T}{2m}\right)$$

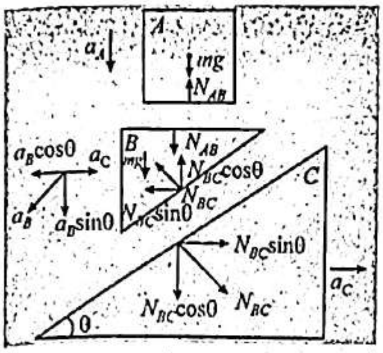
$$\frac{9T}{4} + T = F - 2T \Rightarrow T = \frac{4F}{21}$$

$$a_B = \frac{3T}{4m} \Rightarrow a_B = \frac{F}{7m}$$

6. In the figure shown all blocks are of equal mass 'm'. All surfaces are smooth. Find the acceleration of all the blocks.



Sol.


A
a_A
mg
N_{AB}
B
$a_B\cos\theta$
a_C
a_B
$a_B\sin\theta$
N_{AB}
$N_{BC}\cos\theta$
N_{BC}
$N_{BC}\sin\theta$
C
$N_{BC}\sin\theta$
$N_{BC}\cos\theta$
N_{BC}
a_C
θ

a_A and a_C are acceleration of block A and C w.r.t ground and a_B is the acceleration of block B w.r.t. block C

$N_{BC}\sin\theta = ma_C$

[Newton's II law for block C in horizontal direction in ground frame]

$N_{AB} + mg - N_{BC}\cos\theta = ma_B\sin\theta$

[Newton's II law for block B in vertical direction in ground frame]

$N_{BC}\sin\theta = m(a_B\cos\theta - a_c)$

[Newton's II law for block B in horizontal direction in ground frame]

$mg - N_{AB} = ma_A$

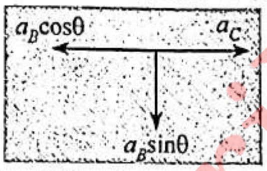
[Newton's II law for block A in vertical direction in ground frame]

$a_A = a_B\sin\theta$

[constrained relation for block A and B in vertical direction]

Now we have 5 equations and 5 unknowns.

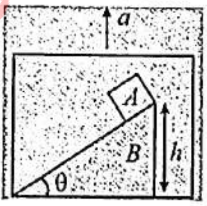
$$\Rightarrow a_A = \frac{4g\sin^2\theta}{1+3\sin^2\theta} \Rightarrow a_B = \frac{4g\sin\theta}{1+3\sin^2\theta} \Rightarrow a_C = \frac{2g\sin\theta\cos\theta}{1+3\sin^2\theta}$$


$a_B\cos\theta$
a_C
$a_B\sin\theta$

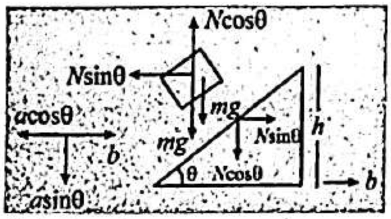
$\Rightarrow$ Acceleration of B w.r.t ground

$$= \sqrt{(a_B\cos\theta - a_C)^2 + (a_B\sin\theta)^2} = \frac{2g\sin\theta}{\sqrt{1+3\sin^2\theta}}$$

7. A lift is moving upwards with a constant acceleration $a = g$. A small block A of mass 'm' is kept on a wedge B of the same mass 'm'. The height of the vertical face of the wedge is 'h'. A is released from the top most point of the wedge. Find the time taken by A to reach the bottom of B. All surfaces are smooth and B is also free to move.



Sol.


$N\cos\theta$
$N\sin\theta$
$a\cos\theta$
mg
mg
$N\sin\theta$
h
b
b
θ
$N\cos\theta$
$a\sin\theta$

$N\sin\theta = mb$ ... (*i*)

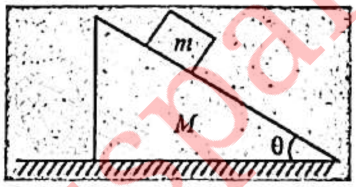
$N\sin\theta = m(a\cos\theta - b)$ ... (*ii*)

$2mg - N\cos\theta = ma\sin\theta$ ... (*iii*)

From (*i*), (*ii*) and (*iii*)

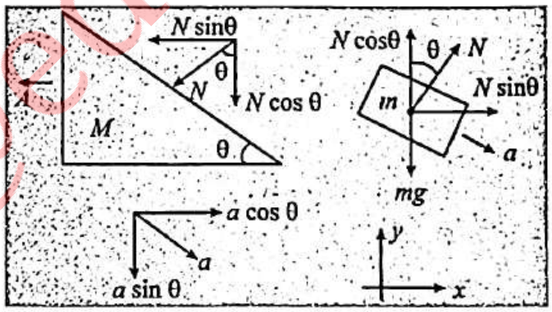
$$\Rightarrow a = \frac{4g\sin\theta}{1+\sin^2\theta} \Rightarrow h = \frac{1}{2}a\sin\theta\, t^2 \Rightarrow t = \sqrt{\frac{h(1+\sin^2\theta)}{2g\sin^2\theta}}.$$

8. A block of mass m is placed on the inclined surface of a wedge as shown. Calculate the acceleration of the wedge and the block when the block is released. Assume all surfaces are frictionless.



Sol. Let the acceleration of wedge be A and that of block be a (w.r.t. wedge). Then acceleration of m w.r.t. ground is

$(a\cos\theta - A)\hat{i} - a\sin\theta\ \hat{j}$


$N\sin\theta$
θ
N
$N\cos\theta$
A
M
θ
$N\cos\theta$
θ
N
m
$N\sin\theta$
a
mg
$a\cos\theta$
a
$a\sin\theta$
y
x

For M: $N\sin\theta = MA$ ... (*i*)

For m: $mg - N\cos\theta = ma\sin\theta$ ... (*ii*)

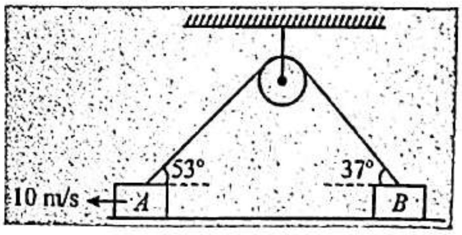
$N\sin\theta = m(a\cos\theta - A)$ ... (*iii*)

Solved (*i*), (*ii*) and (*iii*) to get

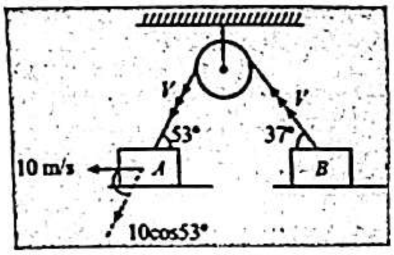
$$A = \frac{mg\sin\theta\cos\theta}{M+m\sin^2\theta} \text{ and } a = \frac{(M+m)g\sin\theta}{M+m\sin^2\theta}$$

Now acceleration of block can be found by putting the values of A and a in (i).

9. Find out the velocity of block B in a pulley block system as shown in figure.


53°
37°
10 m/s
A
B

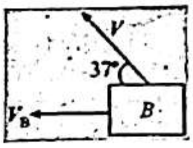
Sol. Let us assume the velocity of all the particle of string is v.


v
v
53°
37°
10 m/s
A
B
10cos53°

10 m/s is the velocity of block A then its component along string is v.

$\Rightarrow$ 10 cos 53° = v ... (*i*)

If v_B is the velocity of block B then it component



Along string is v then

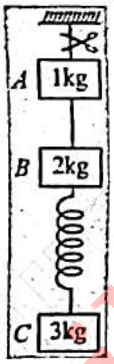
$v_B \cos 37° = v$ ... (*ii*)

From (*i*) and (*ii*)

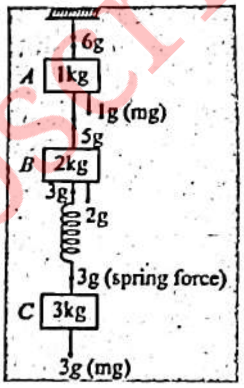
$v_B \cos 37° = 10 \cos 53°$

$$\Rightarrow v_B = \frac{10 \times 3/5}{4/5} = \frac{30}{4} = \frac{15}{2} m/sec$$

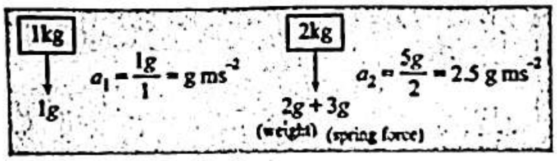
10. Find out the acceleration of 1 kg, 2 kg and 3 kg block and tension in the string between 1 kg and 2 kg block just after cutting the string as shown in figure.


A
1kg
B
2kg
C
3kg

Sol. F.B.D before cutting of string


6g
A
1kg
1g (mg)
5g
B
2kg
3g
2g
3g (spring force)
C
3kg
3g (mg)

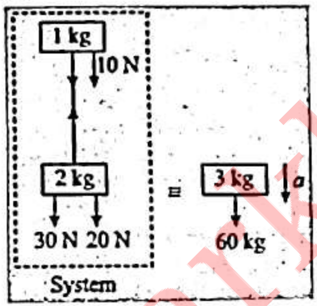
Let us assume the Tension in the string connecting blocks A and B becomes zero just after cutting the string then.


1kg
1g
2kg
2g + 3g
(weight) (spring force)

$a_1 = \frac{1g}{1} = g\ ms^{-2}$ $\quad$ $a_2 = \frac{5g}{2} = 2.5\ g\ ms^{-2}$

$\because a_2 > a_1$ i.e., $\therefore T \neq 0$

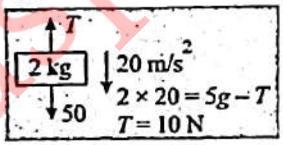
If $T \neq 0$ that means string is tight and both block A and B will have same acceleration. So we can take it as a system of 3 kg mass.


1 kg
10 N
2 kg
30 N 20 N
≡
3 kg
a
60 kg
System

Total force down ward = 10 + 30 + 20 = 60 N

Total mass = 3 kg $a = \frac{60}{3} = 20\ m/s^2$

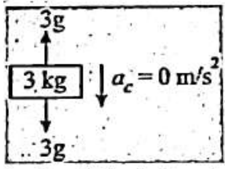
Now apply, $F_{net} = ma$ for block B.


T
2 kg
50
20 m/s²

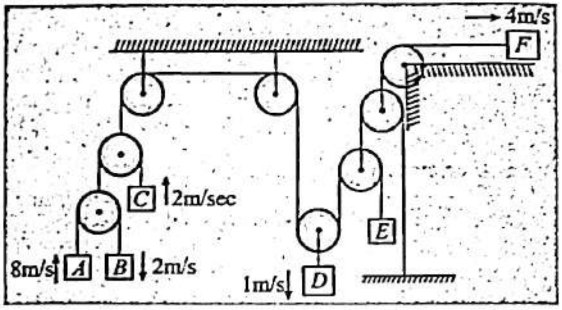
$2 \times 20 = 5g - T$

$T = 10$ N

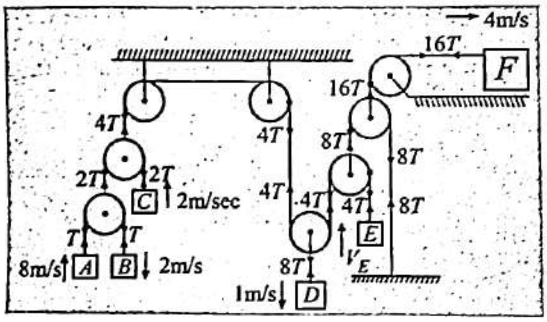
$\because$ The spring force does not change instantaneously the F.B.D of 'C'


3g
3 kg
$a_c = 0\ m/s^2$
3g

11. Find out the velocity of block E as shown in figure.


4m/s
F
C
2m/sec
E
8m/s
A
B
2m/s
1m/s
D

Sol.


4m/s
16T
F
16T
4T
4T
8T
8T
2T
2T
C
2m/sec
4T
4T
4T
8T
T
T
E
V_E
8m/s
A
B
2m/s
8T
1m/s
D

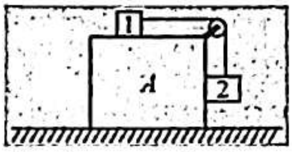
$P_{input} = P_A + P_B + P_C + P_D + P_E + P_F = 0$

$8T - 2T + (2T \times 2) - (8T \times 1) + 4TV_E - (16T \times 4) = 0$

$4V_E = -8 + 2 - 4 + 8 + 64 = 62$

$V_E = \frac{62}{4} = \frac{31}{2} \text{ ms}^{-1}$

12. What is the minimum acceleration with which bar A should be shifted horizontally to keep the bodies 1 and 2 stationary relative to the bar? The masses of the bodies are equal and the coefficient of friction between the bar and the bodies equal to μ. The masses of the pulley and the threads are negligible while the friction in the pulley is absent.



Sol. Let us place the observer on A.

Since we have non-inertial frame we have pseudo forces.

For body '1' we have,

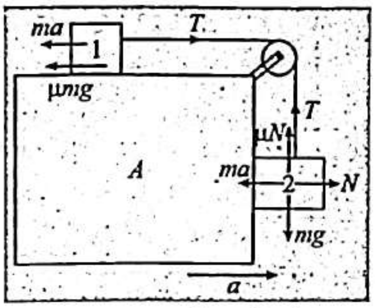
$T = ma + \mu mg$ ... (*i*)

For body '2' we have,

$N = ma$

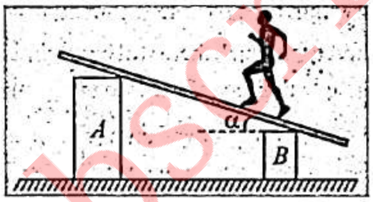
$mg - T - \mu ma = 0$

$\therefore mg = T + \mu ma$ ... (*ii*)



From (i) and (ii) $a_{\min} = g\left(\frac{1-\mu}{1+\mu}\right)$

13. A plank is held at an angle α to the horizontal (Fig.) on two fixed supports A and B. The plank can slide against the supports (without friction) because of its weight Mg. With what acceleration and in what direction, a man of mass m should move so that the plank does not move.



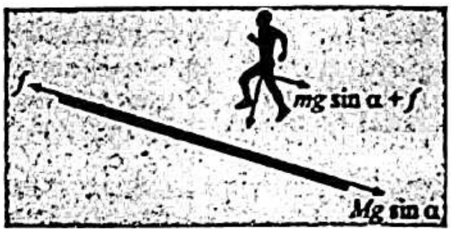
(*a*) $g \sin\alpha \left(1 + \frac{m}{M}\right)$ down the incline

(*b*) $g \sin\alpha \left(1 + \frac{M}{m}\right)$ down the incline

(*c*) $g \sin\alpha \left(1 + \frac{m}{M}\right)$ up the incline

(*d*) $g \sin\alpha \left(1 + \frac{M}{m}\right)$ up the incline

Sol. F.B.D. of man and plank are



For plank be at rest, applying Newton's second law to plank along the incline

$Mg \sin\alpha = f$ ... (*i*)

and applying Newton's second law to man along the incline.

$mg \sin\alpha + f = ma$ ... (*ii*)

$a = g \sin\alpha \left(1 + \frac{M}{m}\right)$ down the incline

Alternate Solution: If the friction force is taken up the incline on man, then application of Newton's second law to man and plank along incline yields.

$f + Mg \sin\alpha = 0$ ... (*i*)

$mg \sin\alpha - f = ma$ ... (*ii*)

Solving (*i*) and (*ii*)

$a = g \sin\alpha \left(1 + \frac{M}{m}\right)$ down the incline

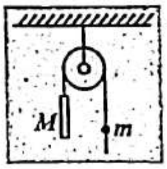
Alternate Solution: Application of Newton's seconds law to system of man + plank along the incline yields

$mg \sin\alpha + Mg \sin\alpha = ma$

$a = g \sin\alpha \left(1 + \frac{M}{m}\right)$ down the incline

Therefore, option (*b*) is the correct answer.

14. In the arrangement shown in figure the mass of the rod M exceeds the mass m of the ball. The ball has an opening permitting it to slide along the thread with some friction. The mass of the pulley, mass of the string and the friction in its axle are negligible. At the initial moment the ball was located opposite the lower end of the rod. When set free, both bodies began moving with constant accelerations. Find the friction force between the ball and the thread if t seconds after the beginning of motion the ball got opposite the upper end of the rod. The rod length equals ℓ.



Sol. **Step-I:** Draw force diagram separately: In fig B, P is a point on the string

From fig. A,

$mg - f = mw_1$

$\therefore w_1 = \frac{mg - f}{m}$ ... (*i*)

From figure B,

$T = f$ ... (*ii*)

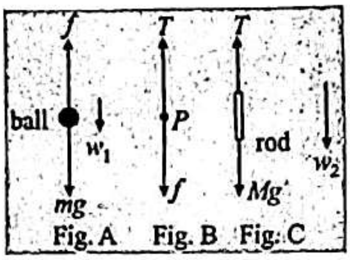


Fig. A Fig. B Fig. C

From fig. C

$Mg - T = Mw_2$

$Mg - f = Mw_2$

$\therefore w_2 = \dfrac{Mg - f}{M}$ ... (*iii*)

Step-II: Apply kinematic relation:

$s_{rel} = u_{rel}t + \frac{1}{2}w_{rel}t^2$ (Shown in fig. D)

Here $s_{rel} = \ell$, $u_{rel} = 0$

$w_{rel} = w_2 - w_1$

$\therefore \ell = \frac{1}{2}(w_2 - w_1)t^2$

$\therefore t = \sqrt{\dfrac{2\ell}{(w_2 - w_1)}}$

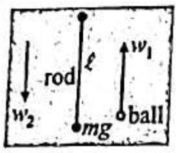
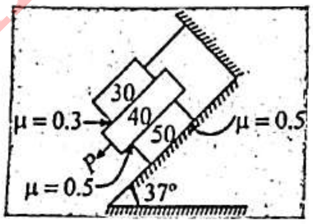


Fig. D

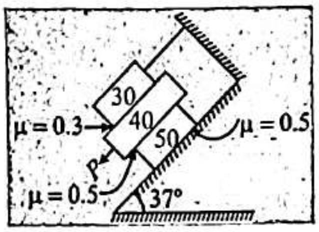
Putting the value of w_1 and w_2, w_1

$f = \dfrac{2\ell Mm}{(M-m)t^2}$

15. The three flat blocks as shown in the figure are positioned on the 37° incline and a force *P* parallel to the inclined plane is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The masses of three blocks in kg and coefficient of static friction for each of the three pairs of contact surfaces are shown in the figure. Determine the maximum value which force *P* may have before slipping take place anywhere. ($g = 10\text{ m/s}^2$)



Sol. $f_{AB\max} = 0.3 \times 30g\cos 37°$



$f_{AB\max} = 72\text{ N}$

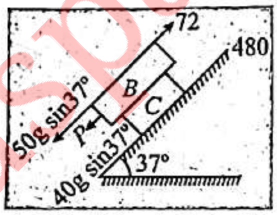
$f_{BC\max} = 0.5 \times 70g\cos 37° = 280\text{ N}$

$f_{C\max} = 0.5 \times 120g\cos 37° = 480\text{ N}$

when block '*B*' is pulled two cases are possible

1. *A* and *C* remains stationary only '*B*' tends to move downwards.
2. *B* and *C* both moves together and there is just slipping between *A* & *B* contact and *C* and wedge contact

Taking case (2) Let *B* and *C* moves together and there is just slipping between *A* & *B* and between wedge and *C*

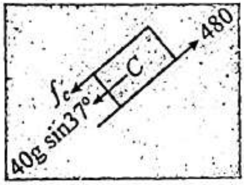


$\Rightarrow P + 40\text{ g}\sin 37° + 50\text{ g}\sin 37° - 72 - 480 = 0$

$\Rightarrow P = 12\text{ N}$

checking friction between *B* and *C*

$f_{BC} + 40\,g\sin 37° - 480 = 0$



$f_{BC} = 240$

which is within limit so case is correct.

so max. force for which there will be no slipping $P_{\max} = 12\,N$ at this force both *B* and *C* tends to move together.

16. A block of mass '*m*' moves on a horizontal circle against the wall of a cylindrical room of radius *R*. The floor of the room on which the block moves is smooth but the friction coefficient between the wall and the block is μ. The block is given an initial speed v_0. As a function of the instantaneous speed '*v*' write

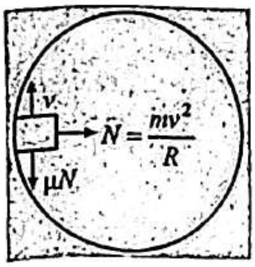
(i) the normal force by the wall on the block,

(ii) the frictional force by the wall and

(iii) the tangential acceleration of the block.

(iv) obtain the speed of the block after one revolution.

Sol.



(i) The normal reaction by wall on the block is $N = \frac{mv^2}{R}$

(ii) The friction force on the block by the wall is

$$f = \mu N = \frac{\mu m v^2}{R}$$

(iii) The tangential acceleration of the block $= \frac{f}{m} = \frac{\mu v^2}{R}$

(iv) $\frac{dv}{dt} = -\frac{\mu v^2}{R}$

or $v\frac{dv}{ds} = -\frac{\mu v^2}{R}$

$$\Rightarrow \int_{v_0}^{v} \frac{dv}{v} = -\int_{0}^{2\pi R} \frac{\mu}{R}\, ds$$

integrating we get

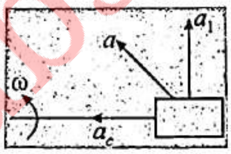
$$\ell n \frac{v}{v_0} = -\mu\, 2\pi \quad \text{or} \quad v = v_0\, e^{-2\mu\pi}$$

17. A block of mass m is kept on a horizontal ruler. The friction coefficient between the ruler and the block is μ. The ruler is fixed at one end and the block is at a distance L from the fixed end. The ruler is rotated about the fixed end in the horizontal plane through the fixed end. (i) What can be the maximum angular speed before which the block does not slip? (ii) If the angular speed of the ruler is uniformly increased from zero at a constant angular acceleration α, at what angular speed will the block slip?

Sol. (i) Ruler rotate with constant angular velocity for just slipping $m\omega^2 L = \mu mg$

$$\omega = \sqrt{\frac{\mu g}{L}}$$

(ii) Angular velocity increase with constant angular acceleration α



$a_c = \omega^2 L,\ a_t = \alpha L$

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(\omega^2 L)^2 + (\alpha L)^2} \qquad \dots (i)$$

for just slipping $\Rightarrow ma = \mu mg$

$$\Rightarrow a = \mu g \qquad \dots (ii)$$

form (*i*) and (*ii*)

$$\mu g = \sqrt{(\omega^2 L)^2 + (\alpha L)^2}$$

$$\omega = \left[\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right]^{1/4}$$

18. A smooth semicircular wire-track of radius R is fixed in a vertical plane shown in fig. One end of a massless spring of natural length $(3R/4)$ is attached to the lower point O of the wire track. A small ring of mass m, which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant $k = mg/R$. Consider the instant when the ring is released, and (i) draw free body diagram of the ring, (ii) Determine the tangential acceleration of the ring and the normal reaction.

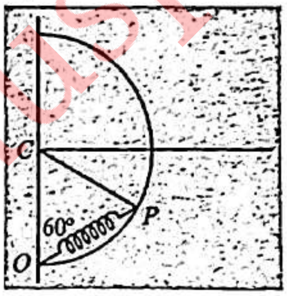
Sol. (i) $CP = CO =$ Radius of circle (R)

$\therefore \angle COP = \angle CPO = 60°$

$\therefore \angle OCP$ is also 60°

$\angle OCP = 60°$

Therefore, ΔOCP is an equilateral triangle.



Hence, $OP = R$

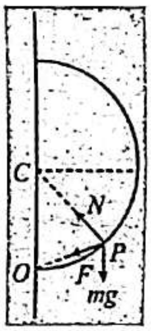
Natural length of spring is $3R/4$.

$\therefore$ Extension in the spring

$$x = R - \frac{3R}{4} = \frac{R}{4}$$

$$\Rightarrow \text{Spring force, } F = kx = \left(\frac{mg}{R}\right)\left(\frac{R}{4}\right) = \frac{mg}{4}$$

The free body diagram of the ring will be as shown.



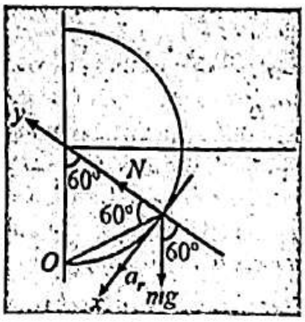
Here, $F = kx = \frac{mg}{4}$

and $N =$ Normal reaction

(ii) Tangential acceleration a_r = The ring will move towards the x-axis just after the release. So, net force along x-axis:

$$F_x = F\sin 60° + mg\sin 60° = \left(\frac{mg}{4}\right)\frac{\sqrt{3}}{2} + mg\left(\frac{\sqrt{3}}{2}\right)$$

$$F_x = \frac{5\sqrt{3}}{8}mg$$



Therefore, tangential acceleration of the ring.

$$a_t = a_x = \frac{F_x}{m} = \frac{5\sqrt{3}}{8}g$$

$$a_t = \frac{5\sqrt{3}}{8}g$$

Normal Reaction N: Net force along y-axis on the ring just after the release will be zero.

$F_y = 0$

$\therefore N + F\cos 60° = mg\cos 60°$

$$\therefore N = mg\cos 60° - F\cos 60° = \frac{mg}{2} - \frac{mg}{4}\left(\frac{1}{2}\right)$$

$$= \frac{mg}{2} - \frac{mg}{8}$$

$$N = \frac{3mg}{8}$$

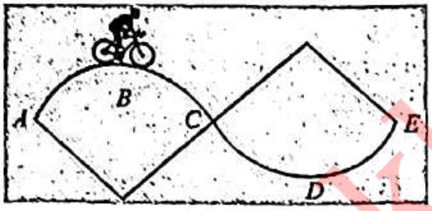
19. A track consists of two circular parts ABC and CDE of equal radius 100 m and joined smoothly as shown in fig. Each part subtends a right angle at its centre. A cycle weighing 100 kg together with the rider travels at a constant speed of 18 km/h on the track.

(*a*) Find the normal contact force by the road on the cycle when it is at B and D.

(*b*) Find the force of friction exerted by the track on the tyres when the cycle is at B, C and D.

(*c*) Find the normal force between the road and the cycle just, before and just after the cycle crosses C.

(*d*) What should be the minimum friction coefficient between the road and the tyre, which will ensure that the cyclist can move with constant speed? Take $g = 10\text{m/s}^2$.



Sol. Constant speed = 18 km/hr = 5m/sec.

$m = 100$ kg, $r = 100$ m

(*a*) At B, $mg - N_B = \frac{mv^2}{r} = \frac{100\times 5^2}{100} = 25$

$N_B = 975$ N **Ans.**

at D, $N_D - mg = \frac{mv^2}{r} \Rightarrow N_D = 1025$ N

(*b*) At B and D friction force acting is zero.

at $C \Rightarrow f = mg\sin 45 = 100 \times 10\frac{1}{\sqrt{2}}$

($\because v$ = constant)

= 707 N

(*c*) For BC part

$mg\cos 45 - N_{BC} = \frac{mv^2}{R} \Rightarrow N_{BC} = 682$ N

for CD part

$N_{CD} - mg\cos 45 = \frac{mv^2}{R} \Rightarrow N_{CD} = 732$ N

(*d*) $f \le \mu N \Rightarrow \mu \ge \frac{f}{N}$

position where f is maximum and N is minimum which is in part BC at C position.

$$\mu \ge \frac{mg\sin 45°}{mg\cos 45° - \frac{mv^2}{r}} \Rightarrow \mu \ge \frac{707}{682} = 1.037$$

School Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. Newton's first law is applicable

(*a*) In all reference frames

(*b*) Only in inertial reference frames

(*c*) Only in non-inertial reference frames

(*d*) None of these

2. From Newton's second law of motion, it can be inferred that

(*a*) No force is required to move a body uniformly along straight line

(*b*) Accelerated motion is always due to an external force

(*c*) Inertial mass of a body is equal to force required per unit acceleration in the body

(*d*) All of these

3. If a force of constant magnitude acts in direction perpendicular to the motion of a particle, then its

(*a*) Speed is uniform (*b*) Momentum is uniform

(*c*) Velocity is uniform (*d*) All of these

4. When an object is in equilibrium state, then

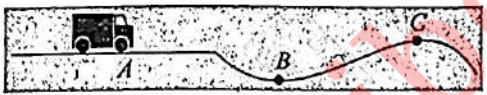
(*a*) It must be at rest

(*b*) No force is acting on it

(*c*) Its net acceleration must be zero

(*d*) All of these

5. A vehicle is moving on a track with constant speed as shown in figure. The apparent weight of the vehicle is



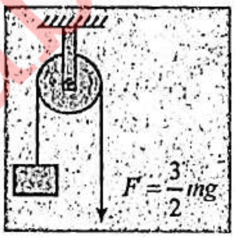
(*a*) Maximum at A

(*b*) Maximum at B

(*c*) Maximum at C

(*d*) Same at A, B and C

6. In the arrangement shown, the mass will ascend with an acceleration (Pulley and rope are massless)



(*a*) Zero (*b*) $\frac{g}{2}$

(*c*) g (*d*) 2g

7. A uniform rope of mass M and length L is fixed at its upper end vertically from a rigid support. Then the tension in the rope at the distance l from the rigid support is

(*a*) $Mg\frac{L}{L+l}$ (*b*) $\frac{Mg}{L}(L-l)$

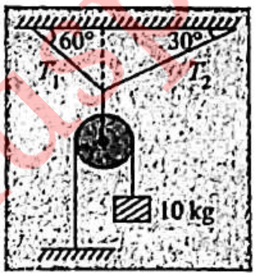
(*c*) Mg (*d*) $\frac{l}{L}Mg$

8. A man slides down a light rope whose breaking strength is η times the weight of man. The maximum acceleration of the man so that the rope just breaks is

(*a*) $g(1-\eta)$ (*b*) $g(1+\eta)$

(*c*) $g\eta$ (*d*) $\frac{g}{\eta}$

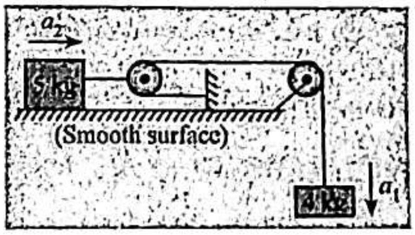
9. In the arrangement as shown, tension T_2 is ($g = 10 m/s^2$)



(*a*) 50 N (*b*) 100 N

(*c*) $50\sqrt{3}$ N (*d*) $100\sqrt{3}$ N

10. If pulleys shown in the diagram are smooth and massless and a_1 and a_2 are acceleration of blocks of mass 4 kg and 5 kg respectively, then



(*a*) $a_1 = a_2$ (*b*) $a_1 = 2a_2$

(*c*) $2a_1 = a_2$ (*d*) $a_1 = 4a_2$

11. **Assertion:** In case of free fall of a lift, the apparent weight of a man in it will be zero.

Reason: In free fall, acceleration of lift is equal to acceleration due to gravity.

(*a*) Both assertion and reason are true and the reason is the correct explanation of the assertion.

(*b*) Both assertion and reason are true but reason is not the correct explanation of the assertion.

(*c*) Assertion is true but reason is false.

(*d*) Assertion is false and reason is true.

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12. **Assertion:** According to Newton's third law of motion for every action, there is an equal and opposite reaction.

Reason: There is no time lag between action and reaction.

(*a*) Both assertion and reason are true and the reason is the correct explanation of the assertion.

(*b*) Both assertion and reason are true but reason is not the correct explanation of the assertion.

(*c*) Assertion is true but reason is false.

(*d*) Assertion is false and reason is true.

VERY SHORT ANSWER TYPE QUESTIONS

13. An impulsive force of 100 N acts on a body for 0.5s. What is the change in its linear momentum?

14. A boy jumps from the upper storey of a house with a load on his back. What is the force of the load on his back when he is in air?

15. The displacement of a body is directly proportional to time. Is there any external force acting on it?

16. You can push a brick gently on a smooth floor by applying force with your foot. Your foot gets hurt when you kick the brick. Why?

17. The two ends of a spring-balance are each pulled by a force of 10 kg wt. What will be the reading of the balance?

18. Calculate the net force acting on a body of mass 2kg moving with a uniform velocity of $2ms^{-1}$.

19. Why is it advisable to hold a gun tight to one's shoulder when it is being fired?

SHORT ANSWER TYPE QUESTIONS

20. A man stands in a lift going downward. He experiences loss of weight at the start but not when the lift is in uniform motion. Explain.

21. Define inertia of rest and give any two of its examples.

LONG ANSWER TYPE QUESTIONS

22. (i) What do you mean by equilibrium of concurrent forces?

(ii) A uniform rope of length *L*, resting on a frictionless horizontal surface is pulled at one end by a force *F*. Find the tension in the rope at a distance *x* from the end where the force is applied.

23. How does the weight of a person in an elevator will vary if

(i) It moves upward with acceleration *a*?

(ii) It moves downward with acceleration *a*?

(iii) It falls freely?

(iv) It is moving with uniform velocity?

(v) It is at rest?

CASE STUDY BASED QUESTIONS

24. According to Newtons 2nd law, the rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the force applied.

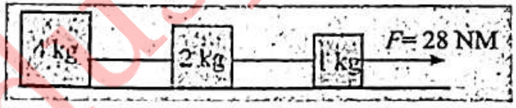
When an unbalanced force is applied on a body, the momentum of the body changes. The rate of change of momentum with respect to time is defined as the net external force acting on the body, i.e., $\vec{F}_{ext} = d\vec{p}/dt$, where linear momentum is the mass of the body multiplied by its instantaneous velocity.

$$\vec{F}_{ext} = \frac{dt}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

If m is constant, i.e., $\frac{dm}{dt} = 0 \Rightarrow \vec{F}_{ext} = m\vec{a}$

If the mass of the body is constant, the acceleration of the body is inversely proportional to its mass and directly proportional to the resultant force acting on it, i.e., $\sum \vec{F} = m\vec{a}$

In the arrangement shown in figure, the strings are light and inextensible. The surface over which blocks are placed is smooth. Find:



(i) The acceleration of 2kg block is,

(*a*) $5\ ms^{-2}$ (*b*) $4\ ms^{-2}$

(*c*) $6\ ms^{-2}$ (*d*) $8\ ms^{-2}$

(ii) The tension in string between 4kg and 2kg is

(*a*) 4 N (*b*) 8 N

(*c*) 12 N (*d*) 16 N

(iii) The tension in string between 1kg and 2kg is

(*a*) 16 N (*b*) 24 N

(*c*) 12 N (*d*) 32 N

(iv) According to the second law of motion the rate of change of momentum of a body is directly proportional to

(*a*) Velocity of a body

(*b*) Applied force

(*c*) Only the mass of the body

(*d*) None of the above.

(v) A body experiences no acceleration. What can be concluded about the net external force acting on it?

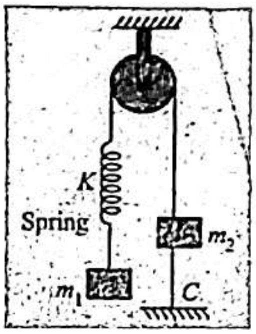
(*a*) It is acting in the opposite direction to motion.

(*b*) It is equal to the weight of the body.

(*c*) It is zero.

(*d*) It is infinite.

25. In the system shown in figure, $m_1 > m_2$. The system is held at rest by thread BC. Now thread is burnt. Answer the following:



(i) Before burning the thread, what is the tension in spring?

(a) m_2g (b) m_1g
(c) $m_2g - m_1g$ (d) $m_1g + m_2g$

(ii) Before burning the thread, what is the tension in thread BC?

(a) m_2g (b) $m_1g - m_2g$
(c) m_1g (d) $m_1g + m_2g$

(iii) Just after burning the thread, what is the tension in the spring?

(a) m_1g (b) m_2g
(c) Zero (d) Cannot say

(iv) Just after burning the thread, what is the acceleration of m_2?

(a) $\left(\frac{m_2 - m_1}{m_2}\right)g$ (b) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$
(c) Zero (d) $\left(\frac{m_1 - m_2}{m_2}\right)g$

(v) Just after burning the thread, what is the acceleration of m_1?

(a) $\left(\frac{m_2 - m_1}{m_2}\right)g$ (b) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$
(c) Zero (d) $\left(\frac{m_1 - m_2}{m_2}\right)g$

Prarambh (Topicwise)

LAWS OF MOTION

1. A rider on horse back falls when horse starts running all of a sudden because

(a) Rider is taken back
(b) Rider is suddenly afraid of falling
(c) Inertia of rest keeps the upper part of body at rest whereas lower part of the body moves forward with the horse
(d) None of the above

2. When a train stops suddenly, passengers in the running train feel an instant jerk in the forward direction because

(a) The back of seat suddenly pushes the passengers forward
(b) Inertia of rest stops the train and takes the body forward
(c) Upper part of the body continues to be in the state of motion whereas the lower part of the body in contact with seat remains at rest
(d) Nothing can be said due to insufficient data

3. A man getting down a running bus falls forward because

(a) Due to inertia of rest, road is left behind and man reaches forward
(b) Due to inertia of motion upper part of body continues to be in motion in forward direction while feet come to rest as soon as they touch the road
(c) He leans forward as a matter of habit
(d) Of the combined effect of all the three factors stated in (a), (b) and (c)

4. A force of 100 dynes acts on a mass of 5 gm for 10 sec. The velocity produced is

(a) 2 cm/sec (b) 20 cm/sec
(c) 200 cm/sec (d) 2000 cm/sec

5. A machine gun is mounted on a 2000 kg car on a horizontal frictionless surface. At some instant the gun fires bullets of mass 10 gm with a velocity of 500 m/sec with respect to the car. The number of bullets fired per second is ten. The average thrust on the system is

(a) 550 N (b) 50 N
(c) 250 N (d) 250 dyne

6. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15$ N/m. What will be its initial acceleration if it is released from a point 20 *cm* away from the origin

(a) 5 m/s^2 (b) 10 m/s^2
(c) 3 m/s^2 (d) 15 m/s^2

7. In a rocket of mass 1000 kg fuel is consumed at a rate of 40 kg/s. The velocity of the gases ejected from the rocket is 5×10^4 m/s. The thrust on the rocket is

(a) 2×10^3 N (b) 5×10^4 N
(c) 2×10^6 N (d) 2×10^9 N

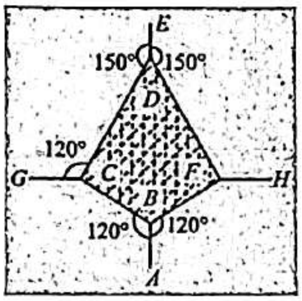
8. A force vector applied on a mass is represented as $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$ and accelerates with 1 m/s^2. What will be the mass of the body

(a) $10\sqrt{2}$ kg (b) $2\sqrt{10}$ kg
(c) 10 kg (d) 20 kg

9. A cricket ball of mass 250 g collides with a bat with velocity 10 m/s and returns with the same velocity within 0.01 second. The force acted on bat is

(*a*) 25 N (*b*) 50 N (*c*) 250 N (*d*) 500 N

10. The adjacent figure is the part of a horizontally stretched net. Section *AB* is stretched with a force of 10 N. The tensions in the sections *BC* and *BF* are



(*a*) 10 N, 11 N

(*b*) 10 N, 6 N

(*c*) 10 N, 10 N

(*d*) Can't calculate due to insufficient data

11. A ball of mass *m* moves with speed *v* and it strikes normally with a wall and reflected back normally, if its time of contact with wall is *t* then find force exerted by ball on wall

(*a*) $\frac{2mv}{t}$ (*b*) $\frac{mv}{t}$

(*c*) mvt (*d*) $\frac{mv}{2t}$

12. A particle moves in the *xy* - plane under the action of a force *F* such that the components of its linear momentum *p* at any time *t* are $p_x = 2\cos t$, $p_y = 2\sin t$. The angle between *F* and *p* at time *t* is

(*a*) 90° (*b*) 0° (*c*) 180° (*d*) 30°

13. If a force of 250 N act on body, the momentum acquired is 125 kg-m/s. What is the period for which force acts on the body

(*a*) 0.5 sec (*b*) 0.2 sec (*c*) 0.4 sec (*d*) 0.25 sec

14. An aircraft is moving with a velocity of 300 ms^{-1}. If all the forces acting on it are balanced, then

(*a*) It still moves with the same velocity

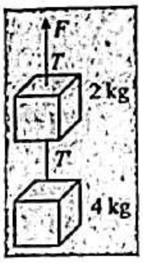
(*b*) It will be just floating at the same point in space

(*c*) It will fall down instantaneously

(*d*) It will lose its velocity gradually

APPLICATION OF FORCE/IMPULSE, STATICS AND DYNAMICS INVOLVING SINGLE SYSTEM

15. Two blocks are connected by a string as shown in the diagram. The upper block is hung by another string. A force *F* applied on the upper string produces an acceleration of 2 m/s^2 in the upward direction in both the blocks. If *T* and *T'* be the tensions in the two parts of the string, then



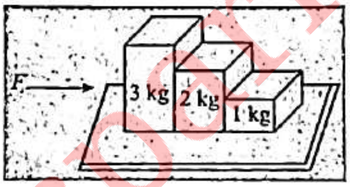
(*a*) $T = 70.8$ N and $T' = 47.2$ N

(*b*) $T = 58.8$ N and $T' = 47.2$ N

(*c*) $T = 70.8$ N and $T' = 58.8$ N

(*d*) $T = 70.8$ N and $T' = 0$

16. Consider the following statements about the blocks shown in the diagram that are being pushed by a constant force on a frictionless table



A. All blocks move with the same acceleration

B. The net force on each block is the same.

Which of these statements are/is correct.

(*a*) *A* only (*b*) *B* only

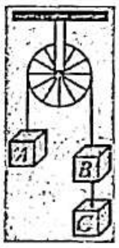
(*c*) Both *A* and *B* (*d*) Neither *A* nor *B*

17. A rope of length *L* is pulled by a constant force *F*. What is the tension in the rope at a distance *x* from the end where the force is applied

(*a*) $\frac{FL}{x}$ (*b*) $\frac{F(L-x)}{L}$

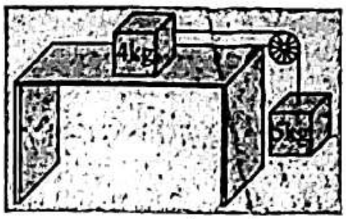
(*c*) $\frac{FL}{L-x}$ (*d*) $\frac{Fx}{L-x}$

18. Three equal weights *A*, *B* and *C* of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in the figure The tension in the string connecting weights *B* and *C* is



(*a*) Zero (*b*) 13 N (*c*) 3.3 N (*d*) 19.6 N

19. Two masses of 4 kg and 5 kg are connected by a string passing through a frictionless pulley and are kept on a frictionless table as shown in the figure. The acceleration of 5 kg mass is



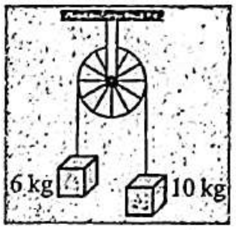
(*a*) 49 m/s² (*b*) 5.44 m/s² (*c*) 19.5 m/s² (*d*) 2.72 m/s²

20. Three solids of masses m_1, m_2 and m_3 are connected with weightless string in succession and are placed on a frictionless table. If the mass m_3 is dragged with a force T, the tension in the string between m_2 and m_3 is

(*a*) $\frac{m_2}{m_1+m_2+m_3}T$ (*b*) $\frac{m_3}{m_1+m_2+m_3}T$

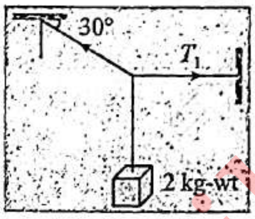
(*c*) $\frac{m_1+m_2}{m_1+m_2+m_3}T$ (*d*) $\frac{m_2+m_3}{m_1+m_2+m_3}T$

21. A light string passes over a frictionless pulley. To one of its ends a mass of 6 kg is attached. To its other end a mass of 10 kg is attached. The tension in the thread will be



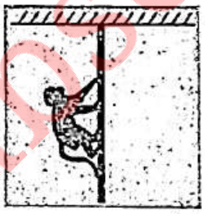
(*a*) 24.5 N (*b*) 2.45 N (*c*) 79 N (*d*) 73.5 N

22. A body of weight 2 kg is suspended as shown in the figure. The tension T_1 in the horizontal string (in kg wt) is



(*a*) $2/\sqrt{3}$ (*b*) $\sqrt{3}/2$ (*c*) $2\sqrt{3}$ (*d*) 2

23. A monkey of mass 40 kg climbs on a rope which can withstand a maximum tension of 600 N. In which of the following cases will the rope break the monkey



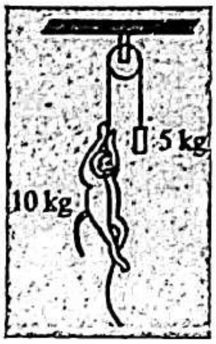
(*a*) Climbs up with an acceleration of 6 ms^{-2}

(*b*) Climbs down with an acceleration of 4 ms^{-2}

(*c*) Climbs up with an uniform speed of 5 ms^{-1}

(*d*) Falls down the rope nearly freely under gravity?

24. In the figure shown acceleration of monkey relative to the rope if it exerts a force of 80 N on string will be:



(*a*) 2 m/s² downwards

(*b*) 4 m/s² upwards

(*c*) 4 m/s² downwards

(*d*) 8 m/s² downwards

25. A person standing on the floor of an elevator drops a coin. The coin reaches the floor of the elevator in time t_1 when elevator is stationary and in time t_2 if it is moving uniformly. Then

(*a*) $t_1 = t_2$

(*b*) $t_1 > t_2$

(*c*) $t_1 > t_2$

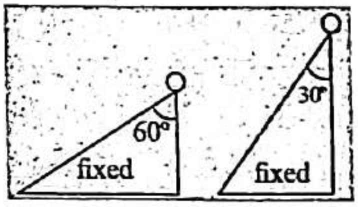
(*d*) $t_1 < t_2$ or $t_1 > t_2$ depending upon that elevator is moving upwards or downwards

26. A block of mass M is placed on a fixed smooth inclined plane of inclination θ with the horizontal. The force exerted by the plane on the block has a magnitude

(*a*) Mg (*b*) $Mg/\cos\theta$

(*c*) $Mg\cos\theta$ (*d*) $Mg\tan\theta$

27. Two particles start together from a point O and slide down along straight smooth inclined planes at 30° and 60° to the vertical and in the same vertical plane as in figure. The relative acceleration of second with respect to first will be (in magnitude and direction) as



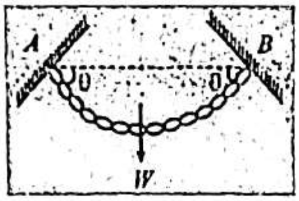
(*a*) $\frac{g}{2}$ in the vertical direction

(*b*) $\frac{g\sqrt{3}}{2}$ at 45° with vertical

(*c*) $\frac{g}{\sqrt{3}}$ inclined at 60° to vertical

(*d*) g in the vertical direction

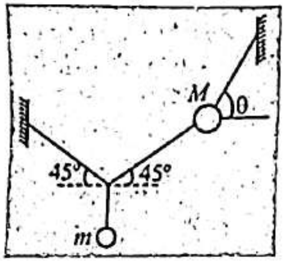
28. A flexible chain of weight W hangs between two fixed points A and B at the same level. The inclination of the chain with the horizontal at the two points of support is θ. What is the tension of the chain at the endpoint.



(*a*) $\frac{W}{2}$ cosec θ (*b*) $\frac{W}{2}$ sec θ

(*c*) $W\cos\theta$ (*d*) $\frac{W}{3}\sin\theta$

29. Two masses m and M are attached with strings as shown. For the system to be in equilibrium we have



(*a*) $\tan\theta = 1 + \frac{2M}{m}$ (*b*) $\tan\theta = 1 + \frac{2m}{M}$

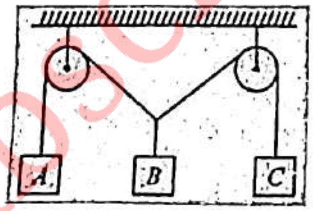
(*c*) $\tan\theta = 1 + \frac{M}{2m}$ (*d*) $\tan\theta = 1 + \frac{m}{2M}$

30. A balloon of gross weight w newton is falling vertically downward with a constant acceleration $a(<g)$. The magnitude of the air resistance is: (Neglecting buoyant force)

(*a*) w (*b*) $w\left(1+\frac{a}{g}\right)$

(*c*) $w\left(1-\frac{a}{g}\right)$ (*d*) $w\frac{a}{g}$

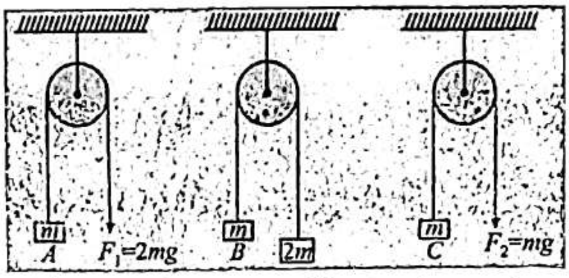
31. Three blocks A, B and C are suspended as shown in the figure. Mass of each block A and C is m. If system is in equilibrium and mass of B is M, then:



(*a*) $M = 2$ m (*b*) $M < 2$ m

(*c*) $M > 2$ m (*d*) $M = m$

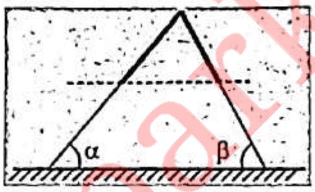
32. In the figure, the blocks A, B and C of mass m each have acceleration a_1, a_2 and a_3 respectively. F_1 and F_2 are external forces of magnitudes 2 mg and mg respectively.



(*a*) $a_1 = a_2 = a_3$ (*b*) $a_1 > a_2 > a_3$

(*c*) $a_1 = a_2, a_2 > a_3$ (*d*) $a_1 > a_2, a_2 = a_3$

33. A uniform rope of length L and mass M is placed on a smooth fixed wedge as shown. Both ends of rope are at same horizontal level. The rope is initially released from rest, then the magnitude of initial acceleration of rope is



(*a*) Zero (*b*) $M(\cos\alpha - \cos\beta)g$

(*c*) $M(\tan\alpha - \tan\beta)g$ (*d*) None of these

34. A force of magnitude F_1 acts on a particle so as to accelerate it from rest to a velocity v. The force F_1 is then replaced by another force of magnitude F_2 which decelerates it to rest.

(*a*) F_1 must be the equal to F_2

(*b*) F_1 may be equal to F_2

(*c*) F_1 must be unequal to F_2

(*d*) None of these

35. In an imaginary atmosphere, the air exerts a small force F on any particle in the direction of the particle's motion. A particle of mass m projected upward takes a time t_1 in reaching the maximum height and t_2 in the return journey to the original point. Then

(*a*) $t_1 < t_2$

(*b*) $t_1 > t_2$

(*c*) $t_1 = t_2$

(*d*) The relation between t_1 and t_2 depends on the mass of the particle.

CONSTRAINT RELATION DYNAMICS OF MULTI SYSTEM SPRING

36. A man is standing at a spring platform. Reading of spring balance is 60 kg wt. If man jumps outside platform, then reading of spring balance

(*a*) First increases then decreases to zero

(*b*) Decreases

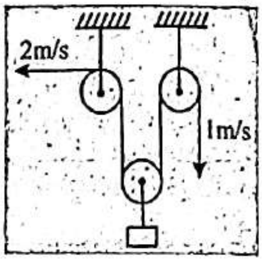
(*c*) Increases

(*d*) Remains same

37. A bird is sitting in a large closed cage which is placed on a spring balance. It records a weight of 25 N. The bird (mass $m = 0.5$ kg) flies upward in the cage with an acceleration of 2 m/s^2. The spring balance will now record a weight of

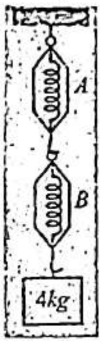
(*a*) 24 N (*b*) 25 (*c*) 26 N (*d*) 27 N

38. Find the velocity of the hanging block if the velocities of the free ends of the rope are as indicated in the figure.



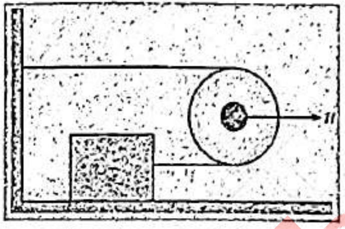
(*a*) 3/2 m/s ↑ (*b*) 3/2 m/s ↓
(*c*) 1/2 m/s ↑ (*d*) 1/2 m/s ↓

39. A block of mass 4 kg is suspended through two light spring balances *A* and *B*. Then *A* and *B* will read respectively



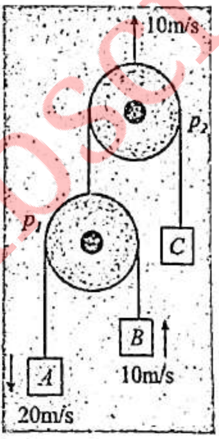
(*a*) 4 kg and zero kg
(*b*) Zero kg and 4 kg
(*c*) 4 kg and 4 kg
(*d*) 2 kg and 2 kg

40. In the figure shown, the pulley is moving with velocity *u*. Calculate the velocity of the block attached with string.



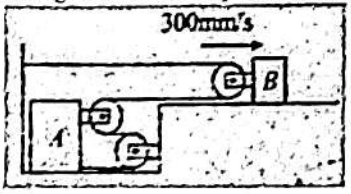
(*a*) *u* (*b*) 2*u* (*c*) 3*u* (*d*) 2.5 *u*

41. Velocities of blocks *A*, *B* and pulley p_2 are shown in figure. Find velocity of pulley p_1 and mass *C*.



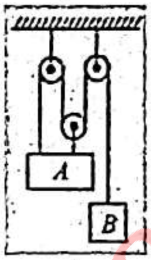
(*a*) 10 m/s (*b*) 20 m/s (*c*) 5 m/s (*d*) 25 m/s

42. If velocity of block *B* in the given arrangement is 300 mm/sec towards right. Then velocity of *A* will be



(*a*) 200 mm/sec (*b*) 100 mm/sec
(*c*) 450 mm/sec (*d*) 150 mm/sec

43. At a given instant, *A* is moving with velocity of 5 m/s upwards. What is velocity of *B* at the time

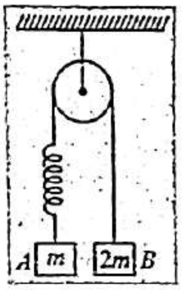


(*a*) 15 m/s↓ (*b*) 15 m/s↑ (*c*) 5 m/s↓ (*d*) 5 m/s↑

44. A block of mass *m* is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block has a magnitude.

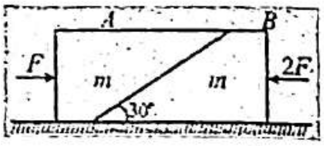
(*a*) *mg* (*b*) *mg*/cos θ (*c*) *mg*cos θ (*d*) *mg*tan θ

45. In the figure a block '*A*' of mass '*m*' is attached at one end of a light spring and the other end of the spring is connected to another block '*B*' of mass 2m through a light string. '*A*' is held and *B* is in static equilibrium. Now *A* is released. The acceleration of *A* just after that instant is '*a*'. In the next case, *B* is held and *A* is in static equilibrium. Now when *B* is released, its acceleration immediately after the release is '*b*'. The value of *a*/*b* is:



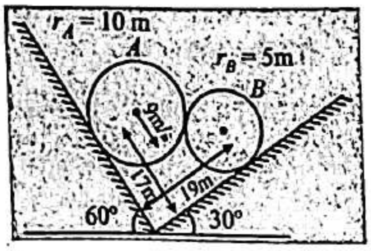
(*a*) 0 (*b*) $\frac{1}{2}$ (*c*) 2 (*d*) Undefined

46. Two blocks '*A*' and '*B*' each of mass '*m*' are placed on a smooth horizontal surface. Two horizontal forces *F* and 2*F* are applied on the two blocks '*A*' and '*B*' respectively as shown in figure. The block *A* does not slide on block *B*. Then the normal reaction acting between the two blocks is:



(*a*) *F* (*b*) *F*/2 (*c*) $\frac{F}{\sqrt{3}}$ (*d*) 3*F*

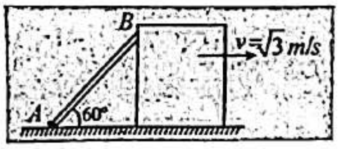
47. System is shown in the figure. Velocity of sphere A is 9 m/s. Then speed of sphere B will be



(*a*) 9 m/s (*b*) 12 m/s

(c) $9\times\frac{5}{4}$ m/s (*d*) None of these

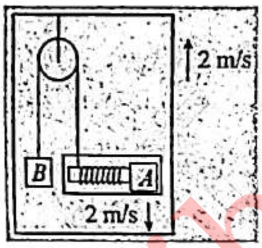
48. A rod AB is shown in figure. End A of the rod is fixed on the ground. Block is moving with velocity $\sqrt{3}$ m/s towards right. The velocity of end B of rod when rod makes an angle of 60° with the ground is:



(*a*) $\sqrt{3}$ m/s (*b*) 2 m/s

(c) $2\sqrt{3}$ m/s (*d*) 3 m/s

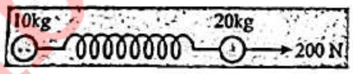
49. In the figure shown the velocity of lift is 2 m/s while string is winding on the motor shaft with velocity 2 m/s and block A is moving downwards with a velocity of 2 m/s, then find out the velocity of block B.



(*a*) 2 m/s ↑ (*b*) 2 m/s ↓

(c) 4 m/s ↑ (*d*) 6 m/s ↑

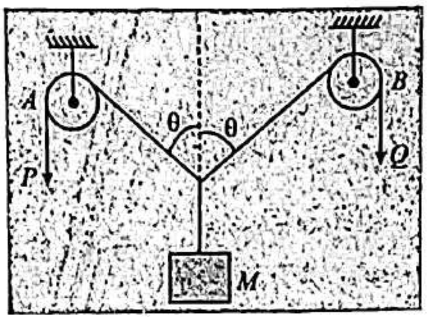
50. Two masses of 10 kg and 20 kg respectively are connected by a massless spring as shown in figure. A force of 200 N acts on the 20 kg mass at the instant when the 10 kg mass has an acceleration of 12 ms^{-2} towards right, the acceleration of the 20 kg mass is:



(*a*) 2 ms^{-2} (*b*) 4 ms^{-2}

(c) 10 ms^{-2} (*d*) 20 ms^{-2}

51. In the arrangement shown in figure the ends P and Q of an unstretchable string move downwards with uniform speed U. Pulleys A and B are fixed. Mass M moves upwards with a speed.



(*a*) $2U\cos\theta$ (*b*) $U\cos\theta$

(c) $\frac{2U}{\cos\theta}$ (*d*) $\frac{U}{\cos\theta}$

NON-INERTIAL REFERENCE FRAME AND PSEUDO FORCE

52. A person is standing in an elevator. In which situation he finds his weight less than actual when

(*a*) The elevator moves upward with constant acceleration

(*b*) The elevator moves downward with constant acceleration.

(*c*) The elevator moves upward with uniform velocity

(*d*) The elevator moves downward with uniform velocity

53. A body of mass 4 kg weighs 4.8 kg when suspended in a moving lift. The acceleration of the lift is

(*a*) 9.80 ms^{-2} downwards (*b*) 9.80 ms^{-2} upwards

(c) 1.96 ms^{-2} downwards (*d*) 1.96 ms^{-2} upwards

54. The mass of a lift is 500 kg. When it ascends with an acceleration of 2 m/s^2, the tension in the cable will be [$g = 10$ m/s^2]

(*a*) 6000 N (*b*) 5000 N

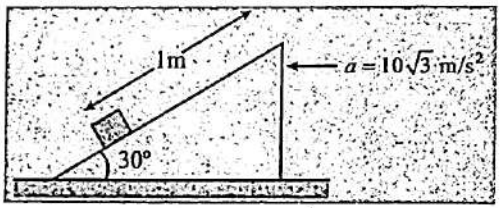
(c) 4000 N (*d*) 50 N

55. A plumb line is suspended from a ceiling of a car moving with horizontal acceleration of a. What will be the angle of inclination with vertical

(*a*) $\tan^{-1}(a/g)$ (*b*) $\tan^{-1}(g/a)$

(c) $\cos^{-1}(a/g)$ (*d*) $\cos^{-1}(g/a)$

56. In the figure, the wedge is pushed with an acceleration of $10\sqrt{3}$ m/s^2. It is seen that the block starts climbing up on the smooth inclined face of wedge. What will be the time taken by the block to reach the top?



(*a*) $\frac{2}{\sqrt{5}}$s (*b*) $\frac{1}{\sqrt{5}}$s

(c) $\sqrt{5}$ s (*d*) $\frac{\sqrt{5}}{2}$s

STATIC FRICTION

57. The coefficient of friction μ and the angle of friction λ are related as

(*a*) $\sin\lambda = \mu$ (*b*) $\cos\lambda = \mu$

(*c*) $\tan\lambda = \mu$ (*d*) $\tan\mu = \lambda$

58. A force of 98 N is required to just slide body of mass 100 kg over ice. The coefficient of static friction is

(*a*) 0.6 (*b*) 0.4

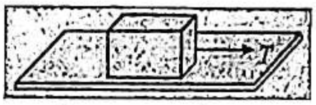
(*c*) 0.2 (*d*) 0.1

59. Maximum value of static friction is called

(*a*) Limiting friction (*b*) Rolling friction

(*c*) Normal reaction (*d*) Coefficient of friction

60. In the figure shown, a block of weight 10 N resting on a horizontal surface. The coefficient of static friction between the block and the surface is $\mu_s = 0.4$. A force of 3.5 N will keep the block in uniform motion, once it has been set in motion. A horizontal force of 3 N is applied to the block, then the block will



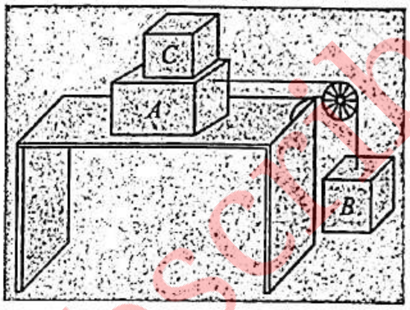
(*a*) Move over the surface with constant velocity

(*b*) Move having accelerated motion over the surface

(*c*) Not move

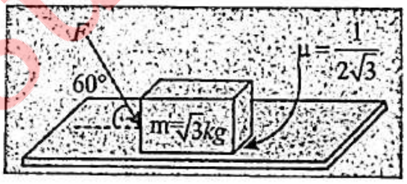
(*d*) First it will move with a constant velocity for some time and then will have accelerated motion

61. Two masses *A* and *B* of 10 kg and 5 kg respectively are connected with a string passing over a frictionless pulley fixed at the corner of a table as shown. The coefficient of static friction of *A* with table is 0.2. The minimum mass of *C* that may be placed on *A* to prevent it from moving is



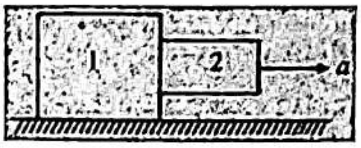
(*a*) 15 kg (*b*) 10 kg (*c*) 5 kg (*d*) 12 kg

62. What is the maximum value of the force *F* such that the block shown in the arrangement, does not move?



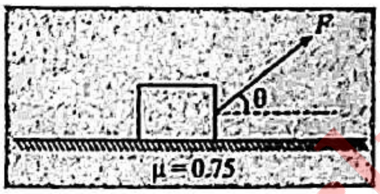
(*a*) 20 N (*b*) 10 N (*c*) 12 N (*d*) 15 N

63. The coefficient of static friction between the two blocks is 0.363. What is the minimum acceleration of block 1 so that block 2 does not fall



(*a*) 6 ms^{-2} (*b*) 12 ms^{-2} (*c*) 18 ms^{-2} (*d*) 27 ms^{-2}

64. A block of mass $m = 10$ kg is to be pulled on a horizontal rough surface with the minimum force.



(i) The block should be pulled at an angle = ...

(ii) The magnitude of the force *F* is equal to ....

(*a*) 45°, 30 N (*b*) 37°, 30 N

(*c*) 37°, 60 N (*d*) 53°, 30 N

KINETIC FRICTION

65. A car is moving along a straight horizontal road with a speed v_0. If the coefficient of friction between the tyres and the road is μ, the shortest distance in which the car can be stopped is

(*a*) $\frac{v_0^2}{2\mu g}$ (*b*) $\frac{v_0}{\mu g}$

(*c*) $\left(\frac{v_0}{\mu g}\right)^2$ (*d*) $\frac{v_0}{\mu}$

66. A body *B* lies on a smooth horizontal table and another body *A* is placed on *B*. The coefficient of friction between *A* and *B* is μ. What acceleration given to *B* will cause slipping to occur between *A* and *B*

(*a*) μg (*b*) g/μ (*c*) μ/g (*d*) $\sqrt{\mu g}$

67. A 500 kg horse pulls a cart of mass 1500 kg along a level road with an acceleration of 1 ms^{-2}. If the coefficient of sliding friction is 0.2, then the force exerted by the horse on cart in forward direction is

(*a*) 3000 N (*b*) 4000 N

(*c*) 5000 N (*d*) 4500 N

68. A horizontal force of 129.4 N is applied on a 10 kg block which rests on a horizontal surface. If the coefficient of friction is 0.3, the acceleration should be

(*a*) 9.8 m/s^2 (*b*) 10 m/s^2

(*c*) 12.6 m/s^2 (*d*) 19.6 m/s^2

69. If μ_s, μ_k and μ_r are coefficients of static friction, sliding friction and rolling friction, then

(*a*) $\mu_s < \mu_k < \mu_r$ (*b*) $\mu_k < \mu_r < \mu_s$

(*c*) $\mu_r < \mu_k < \mu_s$ (*d*) $\mu_r = \mu_k = \mu_s$

70. The coefficient of friction between a body and the surface of an inclined plane at 45° is 0.5. If $g = 9.8$ m/s^2, the acceleration of the body downwards in m/s^2 is

(*a*) $\frac{4.9}{\sqrt{2}}$ (*b*) $4.9\sqrt{2}$

(*c*) $19.6\sqrt{2}$ (*d*) 4.9

71. The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A body starting from the rest at top comes back to rest at the bottom if the coefficient of friction for the lower half is given by

(*a*) $\mu = \sin\theta$ (*b*) $\mu = \cot\theta$

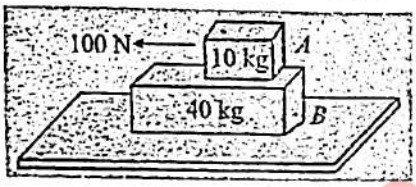
(*c*) $\mu = 2\cos\theta$ (*d*) $\mu = 2\tan\theta$

72. The time taken by a body to slide down a rough 45° inclined plane is twice that required to slide down a smooth 45° inclined plane. The coefficient of kinetic friction between the object and rough plane is given by

(*a*) $\frac{1}{3}$ (*b*) $\frac{3}{4}$ (*c*) $\sqrt{\frac{3}{4}}$ (*d*) $\sqrt{\frac{2}{3}}$

TWO AND THREE BLOCKS PROBLEMS

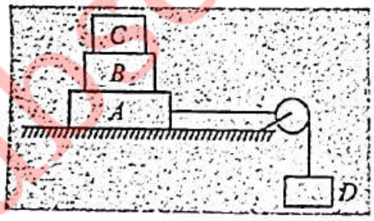
73. A 40 kg slab rests on a frictionless floor as shown in the figure. A 10 kg block rests on the top of the slab. The static coefficient of friction between the block and slab is 0.60 while that of the kinetic friction is 0.40. The 10 kg block is acted upon by a horizontal force of 100 N. If $g = 9.8$ m/s^2, the resulting acceleration of the slab will be



(*a*) 0.98 m/s^2 (*b*) 1.47 m/s^2

(*c*) 1.52 m/s^2 (*d*) 6.1 m/s^2

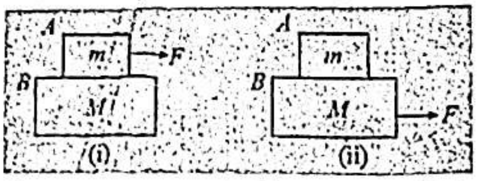
74. Three blocks A, B and C of equal masses 'm' each are placed one over the other on a frictionless table. The coefficient of friction between any two blocks is μ. Find the maximum value of mass of block D so that the blocks A, B and C move without slipping over each other.



(*a*) $\frac{3m\mu}{\mu+1}$ (*b*) $\frac{3m(1-\mu)}{\mu}$

(*c*) $\frac{3m(1+\mu)}{\mu}$ (*d*) $\frac{3m\mu}{(1-\mu)}$

75. A body A rests on B and friction coefficient between A and B is μ. Block M is placed on a frictionless surface. Then



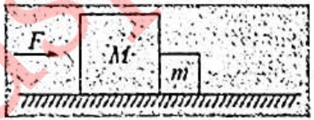
(*a*) The maximum possible value of F so that both bodies move together in case (i) is $\mu(M+m)g$.

(*b*) The maximum possible value of F so that both bodies move together in case (i) is μmg.

(*c*) The maximum possible value of F so that both bodies move together in case (ii) is $\mu(M+m)g$.

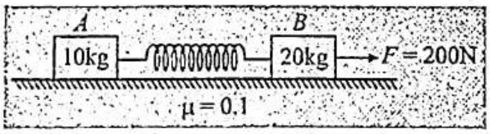
(*d*) The maximum possible value of F so that both bodies move together in case (ii) is μmg.

76. Two blocks of masses $M = 3$ kg and $m = 2$ kg are in contact on a horizontal table. A constant horizontal force $F = 5$ N is applied to block M as shown. There is a constant frictional force of 2 N between the table and the block m but no frictional force between the table and the first block M, then the acceleration of the two blocks is



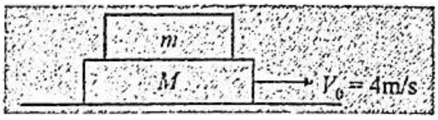
(*a*) 0.4 m/s^2 (*b*) 0.6 m/s^2 (*c*) 0.8 m/s^2 (*d*) 1 m/s^2

77. Two blocks A and B attached to each other by a massless spring, are kept on a rough horizontal surface ($\mu = 0.1$) and pulled by a force $F = 200$ N as shown in figure. If at some instant the 10 kg mass has acceleration of 12 m/s^2, what is the acceleration of 20 kg mass?



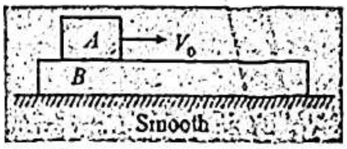
(*a*) 2.5 m/s^2 (*b*) 4.0 m/s^2 (*c*) 3.6 m/s^2 (*d*) 1.2 m/s^2

78. A stationary body of mass m is slowly lowered onto a massive platform of mass M ($M >> m$) moving at a speed $V_0 = 4$ m/s as shown in fig. How far will the body slide along the platform? ($\mu = 0.2$ and $g = 10$ m/s^2)



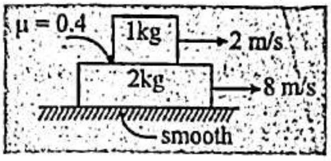
(*a*) 4 m (*b*) 6 m (*c*) 12 m (*d*) 8 m

79. A block A of mass m is placed over a plank B of mass $2m$. Plank B is placed over a smooth horizontal surface. The coefficient of friction between A and B is 0.5. Block A is given a velocity v_0 towards right. Acceleration of B relative to A is



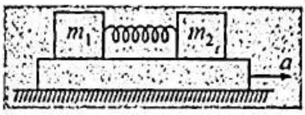
(*a*) $\frac{g}{2}$ (*b*) g (*c*) $\frac{3g}{4}$ (*d*) Zero

80. The time when relative motion between the block will stop is



(*a*) 1 s (*b*) 2 s (*c*) 3 s (*d*) 4 s

81. Two block of masses m_1 and m_2 are connected with a massless unstretched spring and placed over a plank moving with an acceleration 'a' as shown in figure. The coefficient of friction between the blocks and platform is μ.



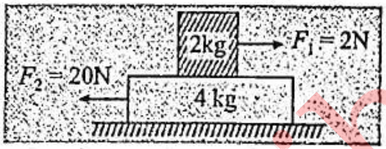
(*a*) Spring will be stretched if $a > \mu g$.

(*b*) Spring will be compressed if $a \leq \mu g$.

(*c*) Spring will neither be compressed nor be stretched only if $a \leq \mu g$.

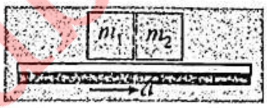
(*d*) Spring will be in its natural length under all conditions.

82. In the arrangement shown in figure, coefficient of friction between the two block is $\mu = \frac{1}{2}$. The force of friction acting between the two blocks is



(*a*) 8 N (*b*) 10 N (*c*) 6 N (*d*) 4 N

83. Two blocks of masses m_1 and m_2 are placed in contact with each other on a horizontal platform as shown in figure. The coefficient of friction between m_1 and platform is 2μ and that between block m_2 and platform is μ. The platform moves with an acceleration a. The normal reaction between the block is



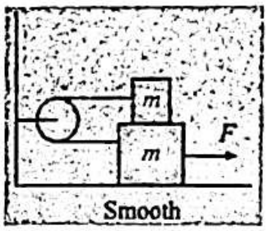
(*a*) Zero in all cases

(*b*) Zero only if $m_1 = m_2$

(*c*) Non-zero only if $a > 2\mu g$

(*d*) Non-zero only if $a > \mu g$

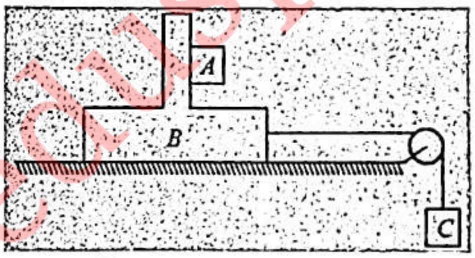
84. For the arrangement shown in figure the coefficient of friction between the two blocks is α. If both the blocks are identical and moving, then the acceleration of each block is



(*a*) $\frac{F}{2m} - 2\alpha g$ (*b*) $\frac{F}{2m}$

(*c*) $\frac{F}{2m} - \alpha g$ (*d*) Zero

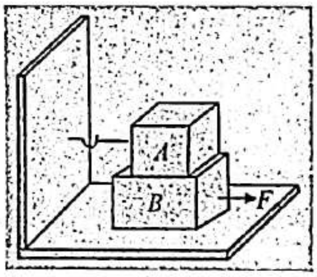
85. In the arrangement shown in the figure, mass of the block B and A is $2m$ and m respectively. Surface between B and floor is smooth. The block B is connected to the block C by means of a string pulley system. If the whole system is released, then find the minimum value of mass of block C so that A remains stationary w.r.t. B. Coefficient of friction between A and B is μ.



(*a*) $\frac{m}{\mu}$ (*b*) $\frac{2m+1}{\mu+1}$

(*c*) $\frac{3m}{\mu-1}$ (*d*) $\frac{6m}{\mu+1}$

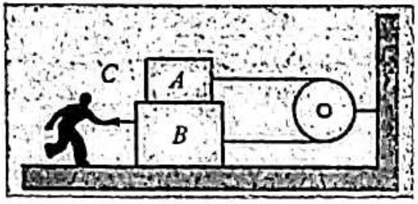
86. A block A with mass 100 kg is resting on another block B of mass 200 kg. As shown in figure a horizontal rope tied to a wall holds it. The coefficient of friction between A and B is 0.2 while coefficient of friction between B and the ground is 0.3. The minimum required force F to start moving B will be



(*a*) 900 N (*b*) 100 N

(*c*) 1100 N (*d*) 1200 N

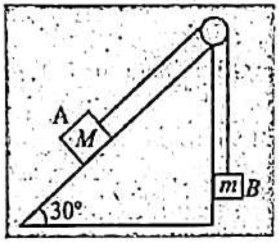
87. In the figure $m_A = m_B = m_C = 60$ kg. The coefficient of friction between C and ground is 0.5, B and ground is 0.3, A and B is 0.4. C is pulling the string with the maximum possible force without moving. Then the tension in the string connected to A will be



(*a*) 120 N (*b*) 60 N (*c*) 100 N (*d*) Zero

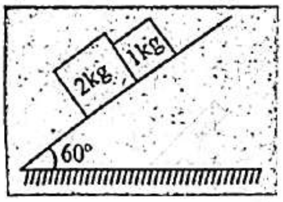
INCLINED PLANE PROBLEMS

88. Block A of mass M in the system shown in the figure slides down the incline at a constant speed. The coefficient of friction between block A and the surface is $\frac{1}{3\sqrt{3}}$. The mass of block B is



(*a*) $M/2$ (*b*) $M/3$ (*c*) $2M/3$ (*d*) $M/\sqrt{3}$

89. In the figure shown if friction coefficient of block 1kg and 2kg with inclined plane is $\mu_1 = 0.5$ and $\mu_2 = 0.4$ respectively, then



(*a*) Both block will move together

(*b*) Both block will move separately

(*c*) There is a non-zero contact force between two blocks

(*d*) None of these

90. The force (along incline) required to just move a body up the inclined plane is double the force required to just prevent the body from sliding down the plane. The coefficient of friction is μ. The inclination θ of the plane is

(*a*) $\tan^{-1}(\mu)$ (*b*) $\tan^{-1}\left(\frac{\mu}{2}\right)$

(*c*) $\tan^{-1}(2\mu)$ (*d*) $\tan^{-1}(3\mu)$

91. A body is moving down inclined plane of slope 37°. The coefficient of friction between the body and plane varies as $\mu = 0.3\,x$, where x is distance travelled down the plane. The body will have maximum speed

$\left(\sin 37° = \frac{3}{5} \text{and } g = 10\text{m/s}^2\right)$

(*a*) At $x = 1.16$ m (*b*) At $x = 2$ m

(*c*) At bottom of plane (*d*) At $x = 2.5$ m

92. A block of mass m is placed at rest on an inclined plane of inclination θ to the horizontal. If the coefficient of friction between the block and the plane is μ, then the total force the inclined plane exerts on the block is

(*a*) mg (*b*) $\mu mg\cos\theta$

(*c*) $mg\sin\theta$ (*d*) $\mu mg\tan\theta$

93. A smooth inclined plane of length L having inclination θ with the horizontal is inside a lift which is moving down with a retardation a. The time taken by a body to slide down the inclined plane from rest will be

(*a*) $\sqrt{\frac{2L}{(g+a)\sin\theta}}$ (*b*) $\sqrt{\frac{2L}{(g-a)\sin\theta}}$

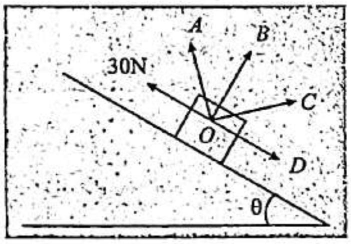
(*c*) $\sqrt{\frac{2L}{a\sin\theta}}$ (*d*) $\sqrt{\frac{2L}{g\sin\theta}}$

94. A block rests on a rough inclined plane making an angle of 30° with horizontal. The coefficient of static friction between the block and inclined plane is 0.8. If the frictional force on the block is 10 N, the mass of the block in kg is ($g = 10$ m/s^2)

(*a*) 2.0 (*b*) 4.0

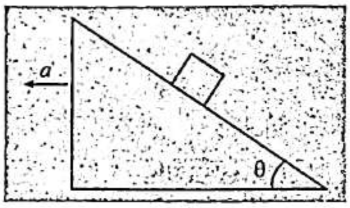
(*c*) 1.6 (*d*) 2.5

95. A body of mass 10 kg lies on a rough inclined plane of inclination $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ with the horizontal. When the force of 30 N is applied on the block parallel to and upward the plane, the total force by the plane on the block is nearly along



(*a*) OA (*b*) OB (*c*) OC (*d*) OD

96. A block of mass m is resting on a wedge of angle θ as shown in the figure. With what minimum acceleration a should the wedge move so that the mass m falls freely?



(*a*) g (*b*) $g\cot\theta$

(*c*) $g\cos\theta$ (*d*) $g\tan\theta$

DYNAMICS OF CIRCULAR MOTION

97. If the overbridge is concave instead of being convex, the thrust on the road at the lowest position will be

(a) $mg+\frac{mv^2}{r}$ (b) $mg-\frac{mv^2}{r}$

(c) $\frac{m^2v^2g}{r}$ (d) $\frac{v^2g}{r}$

98. A motor cyclist moving with a velocity of 72 km/hour on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 meters. The acceleration due to gravity is 10 m/sec^2. In order to avoid skidding, he must not bend with respect to the vertical plane by an angle greater than

(a) $\theta=\tan^{-1}6$ (b) $\theta=\tan^{-1}2$

(c) $\theta=\tan^{-1}25.92$ (d) $\theta=\tan^{-1}4$

99. A ball of mass 0.25 kg attached to the end of a string of length 1.96 m is moving in a horizontal circle. The string will break if the tension is more than 25 N. What is the maximum speed with which the ball can be moved

(a) 14 m/s (b) 3 m/s (c) 3.92 m/s (d) 5 m/s

100. A car of mass M when passes through a convex bridge with speed v and radius of curvature 'r' exerts a force on it which is equal to

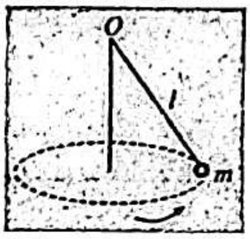
(a) $Mg+\frac{Mv^2}{r}$ (b) $\frac{Mv^2}{r}$

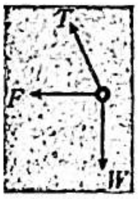
(c) Mg (d) $Mg-\frac{Mv^2}{r}$

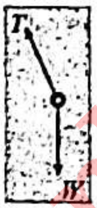
101. An unbanked curve has a radius of 60 m. The maximum speed at which a car can make a turn if the coefficient of static friction is 0.75, is

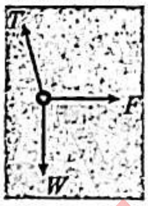
(a) 2.1 m/s (b) 14 m/s (c) 21 m/s (d) 7 m/s

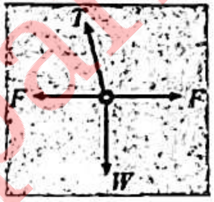
102. A point mass m is suspended from a light thread of length l, fixed at O, is whirled in a horizontal circle at constant speed as shown. From your point of view, stationary with respect to the mass, the forces on the mass are



(a)


(b)


(c)


(d)


103. A stone of mass 0.5 kg tied with a string of length 1 metre is moving in a horizontal circular path with a speed of 4 m/sec. The tension acting on the string in newton is

(a) 2 (b) 8 (c) 0.2 (d) 0.8

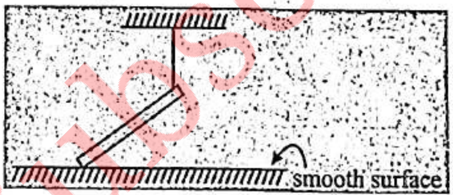
104. When the road is dry and the coefficient of friction is μ, the maximum speed of a car in a circular path is 10 m/s, if the road becomes wet and coefficient of friction becomes $\frac{\mu}{2}$. What is the maximum speed permitted?

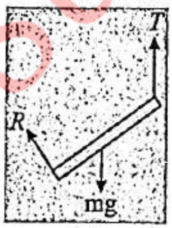
(a) 5 m/s (b) 10 m/s

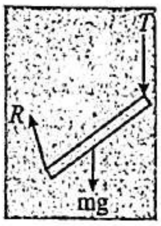
(c) $10\sqrt{2}$ m/s (d) $5\sqrt{2}$ m/s

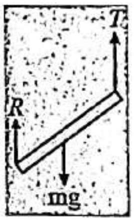
Prabal (JEE Main Level)

1. Which figure represents the correct F.B.D. of rod of mass m as shown in figure:



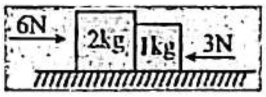
(a)


(b)


(c)


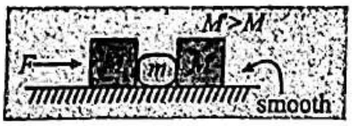
(d) None of these

2. Two forces of 6 N and 3 N are acting on the two blocks of 2 kg and 1 kg kept on frictionless floor. What is the force exerted on 2 kg block by 1 kg block?



(a) 1 N (b) 2 N (c) 4 N (d) 5 N

3. A constant force F is applied in horizontal direction as shown. Contact force between M and m is N and between m and M' is N' then



(a) $N = N'$ (b) $N > N'$

(c) $N' > N$ (d) Cannot be determined

4. In which of the following cases the net force is not zero?

(a) A kite skillfully held stationary in the sky

(b) A ball freely falling from a height

(c) An aeroplane rising upwards at an angle of 45° with the horizontal with a constant speed

(d) A cork floating on the surface of water

5. Two persons are holding a rope of negligible weight tightly at its ends so that it is horizontal. A 15 kg weight is attached to the rope at the mid point which now no longer remains horizontal. The minimum tension required to completely straighten the rope is

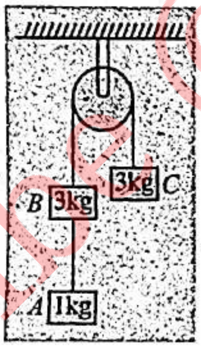
(a) 15 kg

(b) $\frac{15}{2}$ kg

(c) 5 kg

(d) Infinitely large (or not possible)

6. In the system shown in the figure, the acceleration of the 1 kg mass and the tension in the string connecting between A and B is



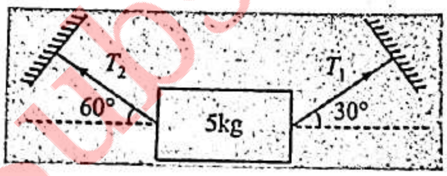
(a) $\frac{g}{4}$ downwards, $\frac{8g}{7}$

(b) $\frac{g}{4}$ upwards, $\frac{g}{7}$

(c) $\frac{g}{7}$ downwards, $\frac{6}{7}g$

(d) $\frac{g}{2}$ upwards, g

7. A body of mass 5 kg is suspended by the strings making angles 60° and 30° with the horizontal



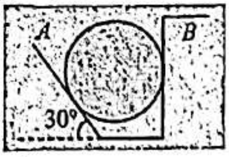
A. $T_1 = 25$ N B. $T_2 = 25$ N

C. $T_1 = 25\sqrt{3}$ N D. $T_2 = 25\sqrt{3}$ N

(a) A, B (b) A, D

(c) C, D (d) B, C

8. The 50 kg homogeneous smooth sphere rests on the 30° incline A and bears against the smooth vertical wall B. Calculate the contact forces at A and B.



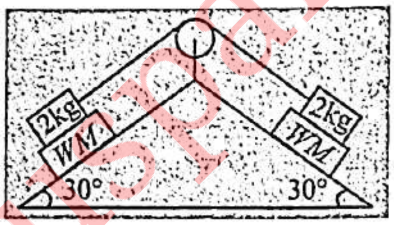
(a) $N_B = \frac{1000}{\sqrt{3}}\text{N}, N_A = \frac{500}{\sqrt{3}}\text{N}$

(b) $N_A = \frac{1000}{\sqrt{3}}\text{N}, N_B = \frac{500}{\sqrt{3}}\text{N}$

(c) $N_A = \frac{100}{\sqrt{3}}\text{N}, N_B = \frac{500}{\sqrt{3}}\text{N}$

(d) $N_A = \frac{1000}{\sqrt{3}}\text{N}, N_B = \frac{50}{\sqrt{3}}\text{N}$

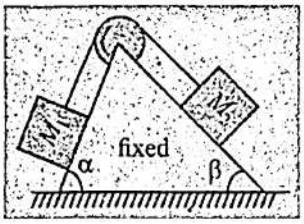
9. Find out the reading of the weighing machine in the following cases.



(a) $10\sqrt{3}$ (b) $10\sqrt{2}$

(c) $20\sqrt{3}$ (d) $30\sqrt{3}$

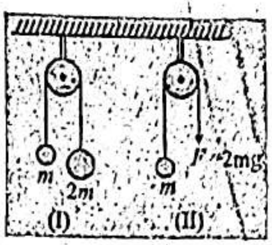
10. Two masses M_1 and M_2 are attached to the ends of a light string which passes over a massless pulley attached to the top of a double inclined smooth plane of angles of inclination α and β. If $M_2 > M_1$ and $\beta > \alpha$ then the acceleration of block M_2 down the incline will be:



(a) $\frac{M_2 g(\sin\beta)}{M_1 + M_2}$ (b) $\frac{M_1 g(\sin\alpha)}{M_1 + M_2}$

(c) $\left[\frac{M_2 \sin\beta - M_1 \sin\alpha}{M_1 + M_2}\right]g$ (d) Zero

11. The pulley arrangements shown in figure are identical, the mass of the rope being negligible. In case-I, the mass m is lifted by attaching a mass $2m$ to the other end of the rope. In case-II, the mass m is lifted by pulling the other end of the rope with a constant downward force $F = 2mg$, where g is acceleration due to gravity. The acceleration of mass in case-I is



(*a*) Zero

(*b*) More than that in case-II

(*c*) Less than that in case-II

(*d*) Equal to that in case-II

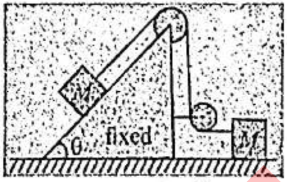
12. A fireman wants to slide down a rope. The rope can bear a tension of $\frac{3}{4}$*th* of the weight of the man. With what minimum acceleration should the fireman slide down:

(*a*) $\frac{g}{3}$ (*b*) $\frac{g}{6}$ (*c*) $\frac{g}{4}$ (*d*) $\frac{g}{2}$

13. A rope of mass 5 kg is moving vertically with an upwards force of 100 N acting at the upper end and a downwards force of 70 N acting at the lower end. The tension at midpoint of the rope is

(*a*) 100 N (*b*) 85 N (*c*) 75 N (*d*) 105 N

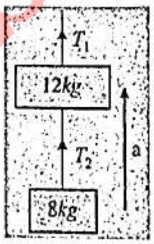
14. Two blocks, each having mass M, rest on frictionless surfaces as shown in the figure. If the pulleys are light and frictionless, and M on the incline is allowed to move down, then the tension in the string will be:



(*a*) $\frac{2}{3}Mg\sin\theta$ (*b*) $\frac{3}{2}Mg\sin\theta$

(*c*) $\frac{Mg\sin\theta}{2}$ (*d*) $2\,Mg\sin\theta$

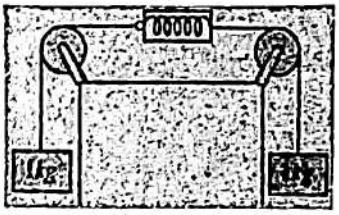
15. A body of mass 8 kg is hanging from another body of mass 12 kg. The combination is being pulled by a string with an acceleration of 2.2 ms^{-2}. The tension T_1 and T_2 will be respectively: (use $g = 9.8\text{m/s}^2$)



(*a*) 200 N, 80 N (*b*) 20 N, 90 N

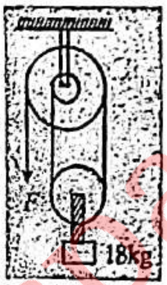
(*c*) 240 N, 96 N (*d*) 260 N, 96 N

16. In the given figure, what is the reading of the spring balance?



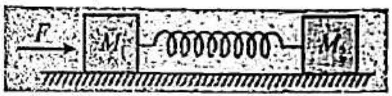
(*a*) 10 N (*b*) 20 N (*c*) 5 N (*d*) Zero

17. In the figure at the free end a force F is applied to keep the suspended mass of 18 kg at rest. The value of F is-



(*a*) 180 N (*b*) 90 N (*c*) 60 N (*d*) 30 N

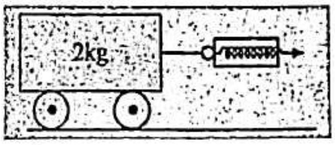
18. Two blocks of masses M_1 and M_2 are connected to each other through a light spring as shown in figure. If we push mass M_1 with force F and cause acceleration a_1 in right direction in mass M_1, what will be the magnitude of acceleration in M_2?



(*a*) F/M_2 (*b*) $F/(M_1 + M_2)$

(*c*) a_1 (*d*) $(F - M_1a_1)/M_2$

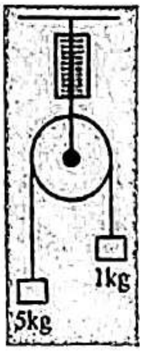
19. A massless spring balance is attached to 2 kg trolley and is used to pull the trolley along a flat surface as shown in the fig. The reading on the spring balance remains at 10 kg during the motion. The acceleration of the trolley is (Use $g = 9.8$ ms^{-2})



(*a*) 4.9 ms^{-2} (*b*) 9.8 ms^{-2}

(*c*) 49 ms^{-2} (*d*) 98 ms^{-2}

20. In the figure a smooth pulley of negligible weight is suspended by a spring balance. Weights of 1 kg and 5 kg are attached to the opposite ends of a string passing over the pulley and move with acceleration because of gravity. During the motion, the spring balance reads a weight of



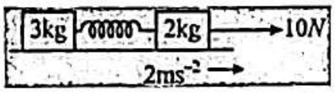
(*a*) 6 kg

(*b*) Less than 6 kg

(*c*) More than 6 kg

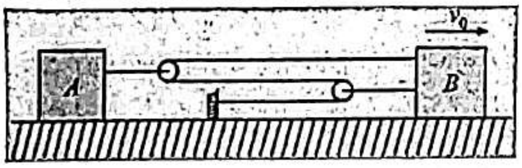
(*d*) May be more or less than 6 kg

21. Find the acceleration of 3 kg mass when acceleration of 2 kg mass is 2 ms^{-2} as shown in figure.



(*a*) 3 ms^{-2} (*b*) 2 ms^{-2} (*c*) 0.5 ms^{-2} (*d*) Zero

22. Block *B* moves to the right with a constant velocity v_0. The velocity of block *A* relative to *B* is



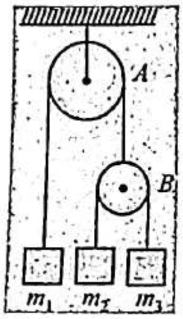
(*a*) $\frac{v_0}{2}$, towards left

(*b*) $\frac{v_0}{2}$, towards right

(*c*) $\frac{3v_0}{2}$, towards left

(*d*) $\frac{3v_0}{2}$, towards right

23. In the arrangement shown in figure, pulleys are massless and frictionless and threads are light and inextensible. Block of mass m_1 will remain at rest if:



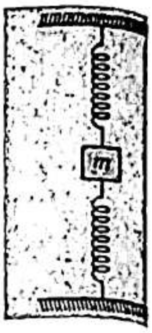
(*a*) $\frac{1}{m_1}=\frac{1}{m_2}+\frac{1}{m_3}$

(*b*) $\frac{4}{m_1}=\frac{1}{m_2}+\frac{1}{m_3}$

(*c*) $m_1=m_2+m_3$

(*d*) $\frac{1}{m_3}=\frac{2}{m_2}+\frac{3}{m_1}$

24. A block tied between two springs is in equilibrium. If upper spring is cut then the acceleration of the block just after cut is 6 m/s^2 downwards. Now, if instead of upper spring, lower spring is cut then the magnitude of acceleration of the block just after the cut will be: (Take $g = 10$ m/s^2)



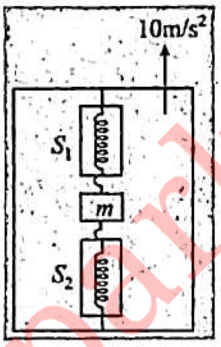
(*a*) 16 m/s^2

(*b*) 4 m/s^2

(*c*) Cannot be determined

(*d*) None of these

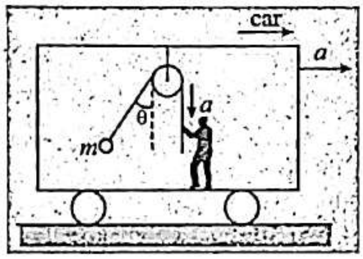
25. Reading shown in two spring balances S_1 and S_2 is 90 kg and 30 kg respectively when lift is accelerating upwards with acceleration 10 m/s^2. The mass is stationary with respect to lift. Then the mass of the block will be:



(*a*) 60 kg (*b*) 30 kg

(*c*) 120 kg (*d*) None of these

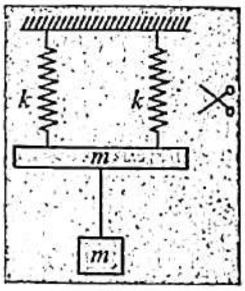
26. A bob is hanging over a pulley inside a car through a string. The second end of the string is in the hand of a person standing in the car. The car is moving with constant acceleration '*a*' directed horizontally as shown in figure. Other end of the string is pulled with constant acceleration '*a*' (relative to car) vertically. The tension in the string is equal to



(*a*) $m\sqrt{g^2+a^2}$ (*b*) $m\sqrt{g^2+a^2}-ma$

(*c*) $m\sqrt{g^2+a^2}+ma$ (*d*) $m(g+a)$

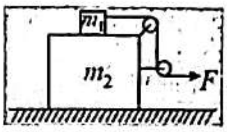
27. System shown in figure is in equilibrium. The magnitude of change in tension in the string just before and just after, when one of the spring is cut. Mass of both the blocks is same and equal to *m* and spring constant of both springs is *k*. (Neglect any effect of rotation)



(*a*) $\frac{mg}{2}$ (*b*) $\frac{mg}{4}$

(*c*) $\frac{3mg}{4}$ (*d*) $\frac{3mg}{2}$

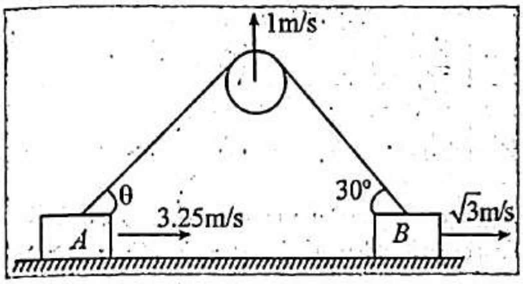
28. In the arrangement shown in the figure all surfaces are frictionless, pulley and string are light. The masses of the block are $m_1 = 20$ kg and $m_2 = 30$ kg. The accelerations of masses m_1 and m_2 will be if $F = 180$ N is applied according to figure.



(a) $a_{m_1} = 9\,\text{m/s}^2$, $a_{m_2} = 0$ (b) $a_{m_1} = 9\,\text{m/s}^2$, $a_{m_2} = 9\,\text{m/s}^2$

(c) $a_{m_1} = 0$, $a_{m_2} = 9\,\text{m/s}^2$ (d) None of these

29. In the figure shown, find out the value of θ at this instant [assume string to be tight]



(a) $\tan^{-1}\frac{3}{4}$ (b) $\tan^{-1}\frac{4}{3}$

(c) $\tan^{-1}\frac{3}{8}$ (d) None of these

30. A body is placed on a rough inclined plane of inclination θ. As the angle θ is increased from 0° to 90° the contact force between the block and the plane.

(a) Remains constant

(b) First remains constant then decreases

(c) First decreases then increases

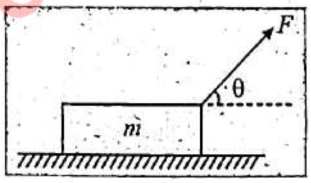
(d) First increases then decreases

31. A chain is lying on a rough table with a fraction $1/n$ of its length hanging down from the edge of the table. If it is just on the point of siding down from the table, then the coefficient of friction between the table and the chain is-

(a) $\frac{1}{n}$ (b) $\frac{1}{(n-1)}$

(c) $\frac{1}{(n+1)}$ (d) $\frac{n-1}{(n+1)}$

32. A wooden block of mass m resting on a rough horizontal table (coefficient of friction = μ) is pulled by a force F as shown in figure. The acceleration of the block moving horizontally is:



(a) $\frac{F\cos\theta}{m}$ (b) $\frac{\mu F\sin\theta}{M}$

(c) $\frac{F}{m}(\cos\theta + \mu\sin\theta) - \mu g$ (d) None of these

33. A block of mass 2 kg rests on a rough inclined plane making an angle of 30^0 with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is:

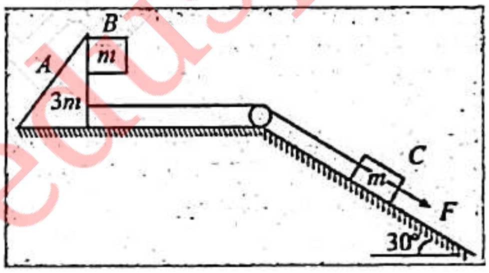
(a) 9.8 N (b) $0.7 \times 9.8\,\sqrt{3}$ N

(c) 9.8×7 N (d) 0.8×9.8 N

34. A 60 kg body is pushed horizontally with just enough force to start it moving across a floor and the same force continues to act afterwards. The coefficient of static friction and sliding friction are 0.5 and 0.4 respectively. The acceleration of the body is:

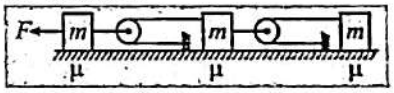
(a) $6\,\text{m/s}^2$ (b) $4.9\,\text{m/s}^2$ (c) $3.92\,\text{m/s}^2$ (d) $1\,\text{m/s}^2$

35. In the arrangement shown in the figure. The mass of wedge A and that of the block B are $3m$ and m respectively. Friction exists between A and B only. The mass of the block C is m. The force $F = 19.5\ \text{m} \times g$ is applied on the block C as shown in the figure. The minimum coefficient of friction (μ) between A and B so that B remains stationary with respect to wedge A will be



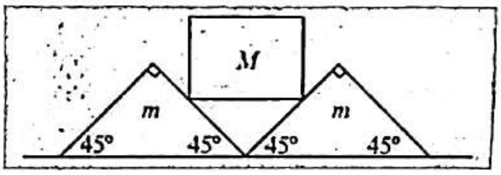
(a) $\frac{2}{5}$ (b) $\frac{1}{10}$ (c) $\frac{2}{5}$ (d) $\frac{1}{4}$

36. On a table, three blocks (including the first block) are placed as shown in the figure. Mass of each block is m and coefficient of friction for each block is μ. A force F is applied on the first block so as to move the system. The minimum value of F should be



(a) $8\mu mg$ (b) $9\mu mg$ (c) $7\mu mg$ (d) $5\mu mg$

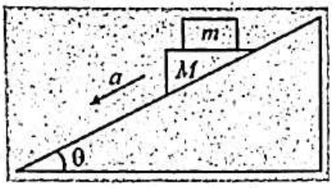
37. Two wedges, each of mass m, are placed next to each other on a flat floor. A cube of mass M is balanced on the wedges as shown below. Assume no friction between the cube and the wedges, but a coefficient of static friction $\mu < 1$ between the wedges and the floor. What is the largest M that can be balanced as shown without motion of the wedges?



(*a*) $\frac{\mu m}{\sqrt{2}}$ (*b*) $\frac{\mu m}{1-\mu}$

(*c*) $\frac{2\mu m}{1-\mu}$ (*d*) All M will balance

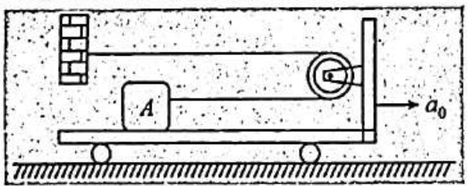
38. As shown in the figure, a wedge of mass M is placed on a smooth inclined ramp that makes an angle θ to the horizontal. An object of mass m rests on top of the wedge. The system is sliding down the ramp at acceleration a. Determine the apparent weight of the object as it slides down. Note that there is friction between the object and the wedge so that the object remains relatively at rest on the wedge.



(*a*) $mg\cos\theta$ (*b*) $mg\cos^2\theta$

(*c*) $mg\sin\theta\cos\theta$ (*d*) $mg\tan\theta$

39. A flat car is given an acceleration $a_0 = 2\ \text{m/s}^2$ starting from rest. A cable is connected to a crate A of weight 50 kg as shown. Neglect friction between the floor and the car wheels. Also the mass of the pulley is negligible. If coefficient of static friction is $\mu = 0.30$ between the crate and the floor of the car, then the tension in the cable is



(*a*) 350 N (*b*) 250 N

(*c*) 300 N (*d*) 400 N

40. A block is given certain upward velocity along the incline of elevation α. The time of ascent to upper point was found to be half the time of descent to initial point. The co-efficient of friction between block and incline is:

(*a*) 0.5 tan α (*b*) 0.3 tan α

(*c*) 0.6 tan α (*d*) 0.2 tan α

41. A plank of mass 2kg and length 1 m is placed on a horizontal floor. A small block of mass 1 kg is placed on top of the plank, at its right extreme end. The coefficient of friction between plank and floor is 0.5 and that between plank and block is 0.2. If a horizontal force = 30 N starts acting on the plank to the right, the time after which the block will fall off the plank is ($g = 10\ \text{m/s}^2$)

(*a*) (2/3) s

(*b*) 1.5 s

(*c*) Block will never fall off the plank

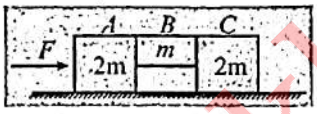
(*d*) (4/3) s

42. If the coefficient of friction between an insect and bowl is μ and the radius of the bowl is r, find the maximum height to which the insect can crawl up in the bowl.

(*a*) $r\left(1-\frac{1}{\sqrt{\mu^2+1}}\right)$ (*b*) $r\left(1-\frac{1}{\sqrt{\mu+1}}\right)$

(*c*) $r\left(1+\frac{1}{\sqrt{\mu^2+1}}\right)$ (*d*) $r\left(1+\frac{1}{\sqrt{\mu-1}}\right)$

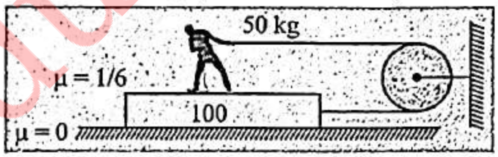
43. The system is pushed by a force F as shown in figure. All surfaces are smooth except between B and C. Friction coefficient between B and C is μ. Minimum value of F to prevent block B from downward slipping is



(*a*) $\left(\frac{3}{2\mu}\right)mg$ (*b*) $\left(\frac{5}{2\mu}\right)mg$

(*c*) $\left(\frac{5}{2}\right)\mu mg$ (*d*) $\left(\frac{3}{2}\right)\mu mg$

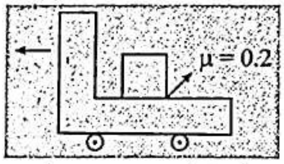
44. A man of mass 50 kg is pulling on a plank of mass 100 kg kept on a smooth floor as shown with force of 100 N. If both man and plank move together, find force of friction acting on man.

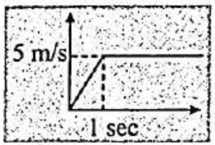


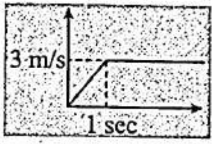
(*a*) $\frac{100}{3}$ N towards left (*b*) $\frac{100}{3}$ N towards right

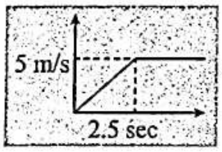
(*c*) $\frac{250}{3}$ N towards left (*d*) $\frac{250}{3}$ N towards right

45. A truck starting from rest moves with an acceleration of $5\ \text{m/s}^2$ for 1 sec and then moves with constant velocity. The velocity w.r.t. ground v/s time graph for block in truck is (Assume that block does not fall off the truck)



(*a*)


(*b*)


(*c*)


(*d*) None of these

46. On horizontal smooth surface a mass of 2 kg is whirled in a horizontal circle by means of a string at an initial angular speed of 5 revolutions per minute. Keeping the radius constant and the tension in the string is doubled. The new angular speed is nearly:

(*a*) 14 rpm (*b*) 10 rpm (*c*) 2.25 rpm (*d*) 7 rpm

47. A coin placed on a rotating turntable just slips if it is placed at a distance of 16 cm from the centre. If the angular velocity of the turntable is doubled, it will just slip at a distance of

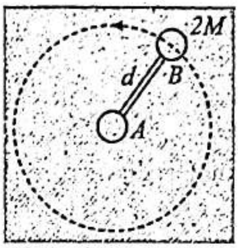
(*a*) 1 cm (*b*) 2 cm (*c*) 4 cm (*d*) 8 cm

48. A man is standing on a rough ($\mu = 0.5$) horizontal disc rotating with constant angular velocity of 5 rad/sec. At what distance from centre should he stand so that he does not slip on the disc?

(*a*) $R \le 0.2$ m (*b*) $R > 0.2$ m

(*c*) $R > 0.5$ m (*d*) $R > 0.3$ m

49. The dumbell is placed on a frictionless horizontal table. Sphere *A* is attached to a frictionless pivot so that *B* can be made to rotate about *A* with constant angular velocity. If *B* makes one revolution in time period *P*, the tension in the rod is



(*a*) $\frac{4\pi^2 Md}{P^2}$ (*b*) $\frac{8\pi^2 Md}{P^2}$

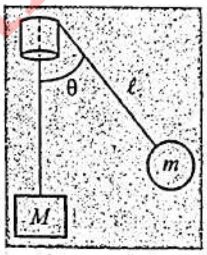
(*c*) $\frac{4\pi^2 Md}{P}$ (*d*) $\frac{2Md}{P}$

50. A cyclist is moving on a circular track of radius 80 m with a velocity of 72 km/hr. He has to lean from the vertical approximately through an angle

(*a*) $\tan^{-1}(1/4)$ (*b*) $\tan^{-1}(1)$

(*c*) $\tan^{-1}(1/2)$ (*d*) $\tan^{-1}(2)$

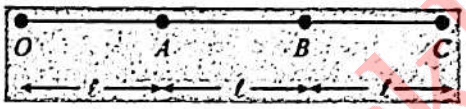
51. A large mass *M* hangs stationary at the end of a light string that passes through a smooth fixed ring to a small mass *m* that moves around in a horizontal circular path. If ℓ is the length of the string from *m* to the top end of the tube and θ is angle between this part and vertical part of the string as shown in the figure, then time taken by *m* to complete one circle is equal to



(*a*) $2\pi\sqrt{\frac{\ell}{g\sin\theta}}$ (*b*) $2\pi\sqrt{\frac{\ell}{g\cos\theta}}$

(*c*) $2\pi\sqrt{\frac{m\ell}{gM\sin\theta}}$ (*d*) $2\pi\sqrt{\frac{\ell m}{gM}}$

52. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving on a smooth horizontal plane about point *O*. If the speed of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is ($T_{BC} : T_{AB} : T_{DA}$): (Assume that the string remains straight)



(*a*) 3 : 5 : 7 (*b*) 3 : 4 : 5

(*c*) 7 : 11 : 6 (*d*) 3 : 5 : 6

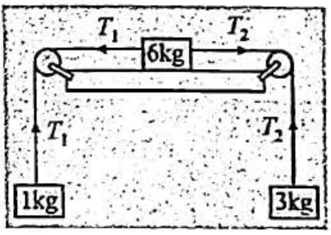
53. A Toy cart attached to the end of an unstretched string of length a, when revolved in circle of radius 2*a* with a time period *T*. Now the toy cart is speeded up until it moves in a circle of radius 3a with a period *T'*. If Hook's law holds then (Assume no friction):

(*a*) $T' = \sqrt{\frac{3}{2}}\,T$ (*b*) $T' = \left(\frac{\sqrt{3}}{2}\right)T$

(*c*) $T' = \left(\frac{3}{2}\right)T$ (*d*) $T' = T$

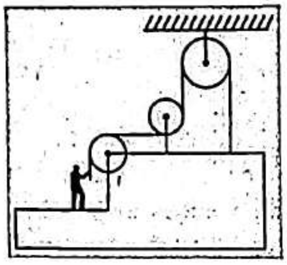
INTEGER TYPE QUESTIONS

54. Three masses of 1 kg, 6 kg and 3 kg are connected to each other with threads and are placed on table as shown in figure. What is the acceleration (in ms^{-2}) with which the system is moving? Take $g = 10$ ms^{-2}.



55. What will be the displacement (in cm) of a block in first 0.2s if the block is kept on the floor of an elevator at rest. Suddenly elevator starts descending with an acceleration of 13 m/s². Take $g = 10$ m/s².

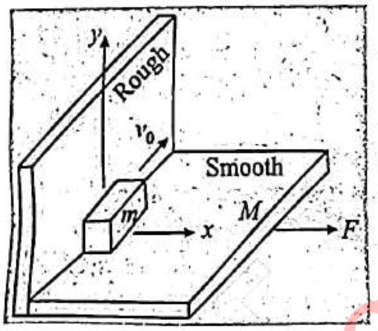
56. A system is shown in the figure. A man standing on the block is pulling the rope. Velocity of the point of string in contact with the hand of the man is 2 m/s downwards. The velocity of the block in ms^{-1} will be: [Assume that the block does not rotate]



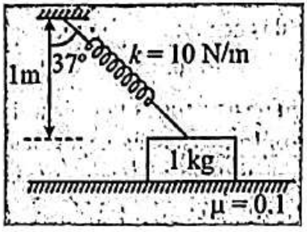
57. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6 If the acceleration of the truck is 5 m/s^2, the frictional force acting on the block is x N. Find the value of x.

58. You pour a quantity of flour of volume $V = 225$ cm^3 onto a board, where it forms a conical pile. The coefficient of static friction between flour grains is $\mu_s = \sqrt{1.60}$. Find the maximum height (in cm) of the pile to nearest integer. (Take $\pi = 3.15$)

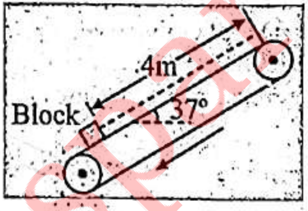
59. Figure shows a block placed on a bracket. Bracket is placed on a smooth floor, it is pulled by a force $F = 6$N horizontally. Block is projected with velocity v_0 relative to bracket as shown in figure. Find time in second after which it stops relative to bracket. Horizontal surface of bracket is smooth while vertical surface is rough (Given: $m = 1$kg, $M = 5$kg, $v_0 = 5$ m/s, $\mu = 0.5$) (Round off nearest integer)



60. The spring shown in the figure has a natural length of 1m. What is the initial acceleration (in cm/sec^2) of the block when released?



61. The following figure shows an accelerating conveyor belt inclined at an angle 37° above horizontal. The coefficient of friction between the belt and block is '1'. The least time in which block can reach the top, starting from rest at the bottom is t seconds. Find the value of t. (Take $g = 10$ ms^{-2})



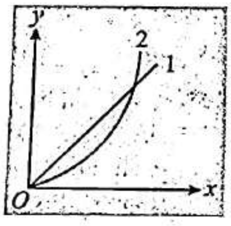
62. A road is banked at an angle of 30° to the horizontal for negotiating a curve of radius $10\sqrt{3}$ m. At what velocity (in km/hr) will a car experience no friction while negotiating the curve?

63. A string can bear a maximum tension of 100 Newton without breaking. A body of mass 1 kg is attached to one end of 1 m length of thin string and it is revolved in a horizontal plane. The maximum linear velocity which can be imparted to the body without breaking the string is x ms^{-1}. Find the value of x.

Parikshit (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. A particle is resting on a smooth horizontal floor. At $t = 0$, a horizontal force starts acting on it. Magnitude of the force increases with time according to law $F = \alpha.t$, where α is a constant. For the figure shown which of the following statements is/are correct?



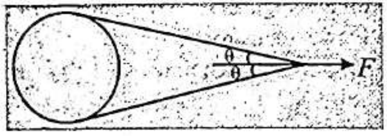
(*a*) Curve 1 shows acceleration against time.

(*b*) Curve 2 shows velocity against time.

(*c*) Curve 2 shows velocity against acceleration.

(*d*) None of these.

2. A string is wrapped round a log of wood and it is pulled with a constant force F as shown in the figure.



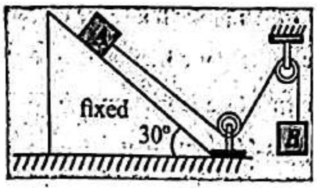
(*a*) Tension T in the string increases with increase in θ

(*b*) Tension T in the string decreases with increase in θ

(*c*) Tension $T > F$ if $\theta > \pi/3$

(*d*) Tension $T > F$ if $\theta > \pi/4$

3. Two blocks A and B of equal mass m are connected through a massless string and arranged as shown in figure. Friction is absent everywhere. When the system is released from rest.



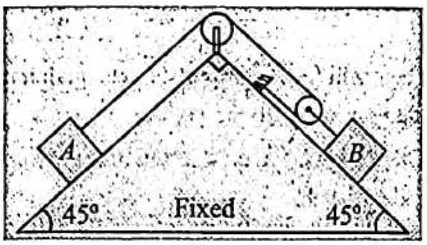
(*a*) Tension in string is $\frac{mg}{2}$

(*b*) Tension in string is $\frac{mg}{4}$

(*c*) Acceleration of A is $\frac{g}{2}$

(*d*) Acceleration of A is $\frac{3}{4}g$

4. Two blocks A and B of mass 10 kg and 40 kg are connected by an ideal string as shown in the figure. Neglect the masses of the pulleys and effect of friction. ($g = 10$ m/s^2)



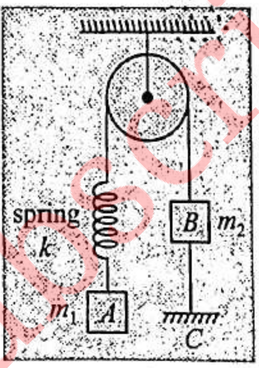
(*a*) The acceleration of block A is $\frac{5}{\sqrt{2}}$ ms^{-2}

(*b*) The acceleration of block B is $\frac{5}{2\sqrt{2}}$ ms^{-2}

(*c*) The tension in the string is $\frac{125}{\sqrt{2}}$ N

(*d*) The tension in the string is $\frac{150}{\sqrt{2}}$ N

5. In the system shown in the figure $m_1 > m_2$. System is held at rest by thread BC. Just after the thread BC is burnt:



(*a*) Acceleration of m_2 will be upwards

(*b*) Magnitude of acceleration of both blocks will be equal to $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$

(*c*) Acceleration of m_1 will be equal to zero

(*d*) Magnitude of acceleration of two blocks will be non-zero and unequal

6. A contact force exerted by one body on horizontal surface is equal to the normal force (≠ 0) between them. It can be said that:

(*a*) The contact surfaces must be frictionless.

(*b*) The force of friction between the contact surfaces is zero.

(*c*) The magnitude of normal force equals that of friction.

(*d*) It is possible that the bodies are rough and they do not slip on each other.

7. A heavy particle is tied to the end A of a string of length 1.6 m. Its other end O is fixed. It revolves as a conical pendulum with the string making 60° with the vertical. Then ($g = 9.8$ m/s^2)

(*a*) Its period of revolution is $\frac{4\pi}{7}$ sec.

(*b*) The tension in the string is double the weight of the particle

(*c*) The speed of the particle = $2.8\sqrt{3}$ m/s

(*d*) The centripetal acceleration of the particle is $9.8\sqrt{3}$ m/s^2.

8. A particle moving with constant speed u inside a fixed smooth hemispherical bowl of radius a describes a horizontal circle at a distance $\frac{a}{2}$ below its centre. Then

(*a*) The radius of the circular motion is $\frac{a\sqrt{3}}{2}$

(*b*) The value of u is $\sqrt{\frac{3ag}{2}}$

(*c*) The normal reaction of the spherical surface on the particle is $\frac{mg}{2}$

(*d*) The magnitude of the resultant force acting on the particle is zero, in an inertial frame.

9. A particle of mass m suspended by a string of length l revolves in a horizontal circle (θ is the angle the string makes with vertical). Which of the following statement(s) is/are correct?

(*a*) Tension in string = $mg\cos\theta$

(*b*) Speed v of particle = $\sqrt{\frac{lg\sin^2\theta}{\cos\theta}}$

(*c*) Period $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

(*d*) Centripetal acceleration $a = \frac{F\sin\theta}{m}$ where F is tension

10. For a curved track of radius R, banked at angle θ :

(Take $v_0 = \sqrt{Rg\tan\theta}$)

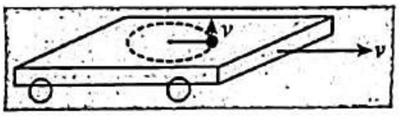
(*a*) a vehicle moving with a speed v_0 is able to negotiate the curve without calling friction into play at all

(*b*) a vehicle moving with any speed $V > V_0$ is always able to negotiate the curve, with friction called into play

(*c*) a vehicle moving with any speed $V < V_0$ must have the force of friction into play

(*d*) the minimum value of the angle of banking for a vehicle parked on the banked road can stay there without slipping, is given by $\theta = \tan^{-1}\mu_0$ (μ_0 = coefficient of static friction)

11. On a train moving along east with a constant speed v, a boy revolves a bob with string of length ℓ on smooth surface of a train, with equal constant speed v relative to train. Mark the correct option(s).



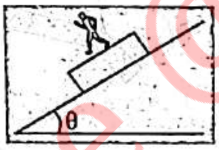
(*a*) Maximum speed of bob is $2v$ in ground frame.

(*b*) Tension in string connecting bob is $\frac{4mv^2}{\ell}$ at an instant.

(*c*) Tension in string is $\frac{mv^2}{\ell}$ at all the moments

(*d*) Minimum speed of bob is zero in ground frame.

12. Ram is running down with some acceleration on a plank kept on a fixed inclined plane. Which of following cases are possible.



(*a*) plank may accelerate down

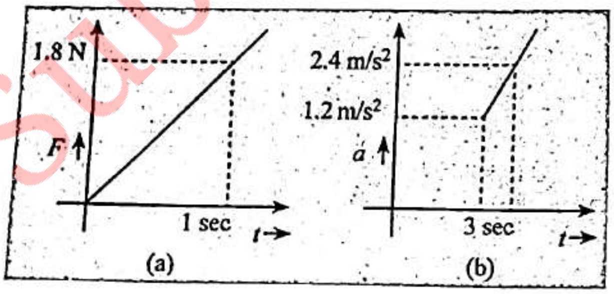
(*b*) Plank may accelerate up

(*c*) Plank may be at rest

(d) Friction on plank due to Ram may be down the plane.

13. 1.5 kg box kept on horizontal rough ground is pushed by a horizontal force whose F - t graph is shown in figure (a). It's $a - t$ graph is show in figure (b).

[Take $g = 10\text{m/s}^2$]



(a)

(b)

Then which of the following is/are correct

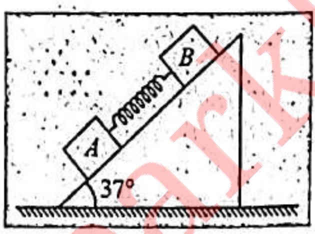
(*a*) Coefficient of static friction is 0.36

(*b*) Coefficient of static friction is 0.48

(*c*) Time when acceleration is 2.4 m/s² is 5sec

(*d*) Coefficient of kinetic friction is 0.24

14. Two blocks A and B of mass 4 kg and 2 kg respectively connected by a spring of force constant 100 N/m are placed on a fixed inclined plane of inclination 37° as shown in fig. If the system is released from rest with spring at natural length, which one of the following statement(s) is/are correct



(*a*) There will be no compression/elongation in the spring if all surfaces are smooth

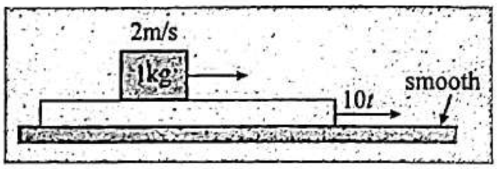
(*b*) There will be some extension in the spring if $\mu_A = 0.8$ and $\mu_B = 0.9$.

(*c*) There will be some compression in the spring if $\mu_A = 0.8$ and $\mu_B = 0.7$.

(*d*) There will be some compression in the spring if $\mu_A = 0.1$ and $\mu_B = 0.1$.

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 15 to 16): A small block of mass 1 kg starts moving with constant velocity 2 m/s on a smooth long plank of mass 10 kg which is also pulled by a horizontal force $F = 10t$ N where t is in seconds and F is in newtons. (the initial velocity of the plank is zero).



15. The time ($t \neq 0$) at which displacement of block and plank with respect to ground is same will be

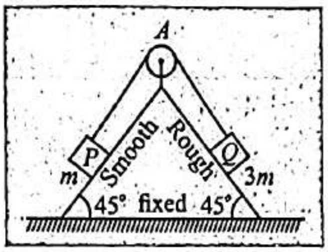
(*a*) 12 s (*b*) $2\sqrt{3}$ s

(*c*) $3\sqrt{3}$ s (*d*) $\sqrt{3}/2$ s

16. Relative velocity of plank with respect to block when acceleration of plank is 4 m/s² will be

(*a*) Zero (*b*) 10 m/s (*c*) 6 m/s (*d*) 8 m/s

Comprehension (Q. 17 to 18): A fixed wedge with both surface inclined at 45° to the horizontal as shown in the figure. A particle P of mass m is held on the smooth plane by a light string which passes over a smooth pulley A and attached to a particle Q of mass 3m which rests on the rough plane. The system is released from rest. Given that the acceleration of each particle is of magnitude $\frac{g}{5\sqrt{2}}$ then



17. In the above question the coefficient of friction between Q and the rough plane is:

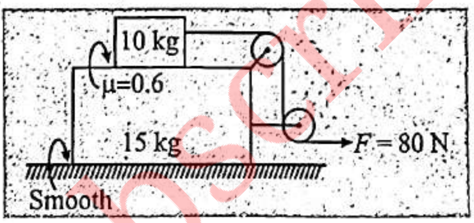
    (*a*) $\frac{4}{5}$ (*b*) $\frac{1}{5}$ (*c*) $\frac{3}{5}$ (*d*) $\frac{2}{5}$

18. In the above question the magnitude and direction of the force exerted by the string on the pulley is:

    (*a*) $\frac{6mg}{5}$ downward (*b*) $\frac{6mg}{5}$ upward

    (*c*) $\frac{mg}{5}$ downward (*d*) $\frac{mg}{4}$ downward

Comprehension (Q. 19 to 20): A block of mass 15 kg is placed over a frictionless horizontal surface. Another block of mass 10 kg is placed over it, that is connected with a light string passing over two pulleys fastened to the 15 kg block. A force F = 80 N is applied horizontally to the free end of the string. Friction coefficient between two blocks is 0.6. The portion of the string between 10 kg block and the upper pulley is horizontal as shown in figure Pulley string and connecting rods are massless. (Take g = 10 m/s^2)



19. The magnitude of acceleration of the 10 kg block is:

    (*a*) 3.2 m/s^2 (*b*) 2.0 m/s^2

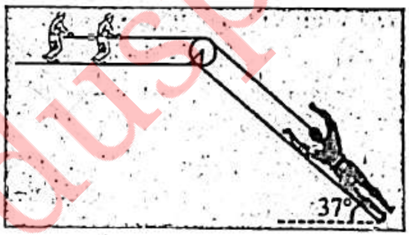
    (*c*) 1.6 m/s^2 (*d*) 0.8 m/s^2

20. The magnitude of acceleration of the 15 kg block is:

    (*a*) 4.2 m/s^2 (*b*) 3.2 m/s^2

    (*c*) 16/3 m/s^2 (*d*) 2.0 m/s^2

Comprehension (Q. 21 to 23): It was one of those chilly winter mornings. It had been snowing all night and as a result all the streets were covered with a thick layer of ice. As I was driving down the road, I saw a kid with a real desperate look on his face signaling me to stop. As I pulled my car along the side, he ran up to my window with a plea. " Sir, my father has fallen down the valley and is unconscious. Please help me to rescue him." As I looked down the valley on the side of the road, I could see a steep valley of inclination 37° covered with ice. I had a rope on me. I tied one end of it to my waist and carefully climbed down the valley to reach the man. I tied the other end of the rope to his waist and climbed back. Although the hill was covered with ice, I could easily climb up without slipping. Now when I tried to pull the man up with the rope, I just could not do it. I kept on slipping. I called the kid, "Hey, lad. Help me pull your papa up". He also pulled the rope along with me and we were able to slowly pull the man up the slope. It was no mean task because I could feel that all the time we both were on the verge of slipping. As I came up I realized he was a sturdy man of mass 100 kg. The kid had a mass of 35 kg and my mass was 120 kg at that time. The friction coefficient was same for all of us. [Take g = 10m/s^2]



21. The fact that the author could climb up and down the slope without slipping means that minimum coefficient of static friction is

    (*a*) 0.75 (*b*) 0.8 (*c*) 1.33 (*d*) 0.6

22. What is the value of friction coefficient?

    (*a*) 0.8 (*b*) 0.6 (*c*) 1.0 (*d*) 12/31

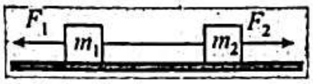
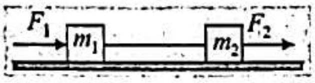
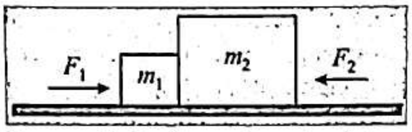
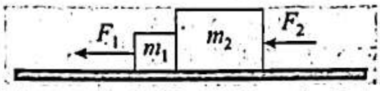
23. If while pulling the man up, the kid had released the rope what would have been the acceleration of the author and the man being pulled up?

    (*a*) $\frac{124}{220}$ m/s^2 (*b*) $\frac{4}{220}$ m/s^2

    (*c*) zero (*d*) $\frac{64}{220}$ m/s^2

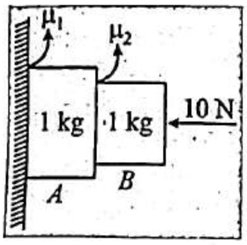
MATCH THE COLUMN TYPE QUESTIONS

24. Column-I gives four different situations involving two blocks of mass m_1 and m_2 placed in different ways on a smooth horizontal surface as shown. In each of the situations horizontal forces F_1 and F_2 are applied on blocks of mass m_1 and m_2 respectively ad also $m_2 F_1 < m_1 F_2$. Match the statements in column-I with corresponding results in column-II.

	Column-I		Column-II
A.	 Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is	p.	$\frac{m_1 m_2}{m_1+m_2}\left(\frac{F_1}{m_1}-\frac{F_2}{m_2}\right)$
B.	 Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is	q.	$\frac{m_1 m_2}{m_1+m_2}\left(\frac{F_1}{m_1}+\frac{F_2}{m_2}\right)$
C.	 The magnitude of normal reaction between the blocks is	r.	$\frac{m_1 m_2}{m_1+m_2}\left(\frac{F_2}{m_2}-\frac{F_1}{m_1}\right)$
D.	 The magnitude of normal reaction between the blocks is	s.	$m_1 m_2\left(\frac{F_1+F_2}{m_1+m_2}\right)$

(*a*) A-(q); B-(r); C-(q); D-(r)
(*b*) A-(p); B-(q); C-(r); D-(s)
(*c*) A-(r); B-(p); C-(s); D-(q)
(*d*) A-(q); B-(r); C-(p); D-(r)

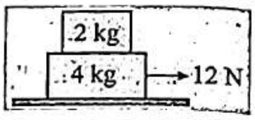
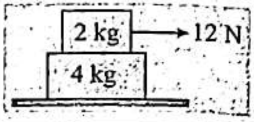
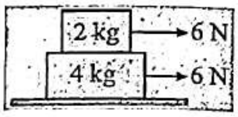
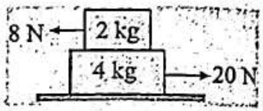
25. In the given figure find the accelerations of blocks *A* and *B* for the following cases ($g = 10$ m/s^2)



	Column-I		Column-II
A.	$\mu_1 = 0$ and $\mu_2 = 0.1$	p.	$a_A = a_B = 9.5$ m/s^2
B.	$\mu_2 = 0$ and $\mu_1 = 0.1$	q.	$a_A = 9$ m/s^2, $a_B = 10$ m/s^2
C.	$\mu_1 = 0.1$ and $\mu_2 = 1.0$	r.	$a_A = a_B = g = 10$ m/s^2
D.	$\mu_1 = 1.0$ and $\mu_2 = 0.1$	s.	$a_A = 1$, $a_B = 9$ m/s^2

(*a*) A-(r); B-(s); C-(p); D-(q)
(*b*) A-(r); B-(q); C-(p); D-(s)
(*c*) A-(s); B-(q); C-(r); D-(s)
(*d*) A-(q); B-(s); C-(p); D-(r)

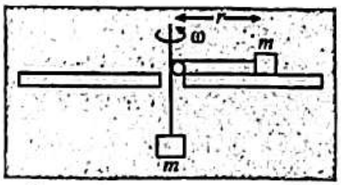
26. Column-II gives certain situations involving two blocks of mass 2 kg and 4 kg. The 4 kg block lies on a smooth horizontal table. There is sufficient friction between both the block and there is no relative motion between both the blocks in all situations. Horizontal forces act on one or both blocks as shown. Column-I gives certain statement related to figures given in column-II. Match the statements in column-I with the figure in column-II.

	Column-I		Column-II
A.	Magnitude of frictional force is maximum.	p.	
B.	Magnitude of friction force is least.	q.	
C.	Friction force on 2 kg block is towards right.	r.	
D.	Friction force on 2 kg block is towards left.	s.	

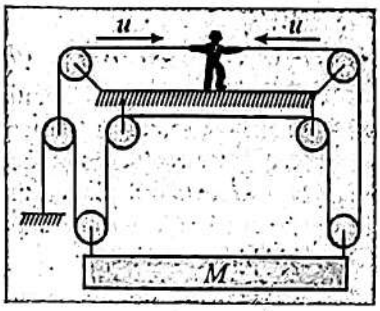
(*a*) A-(r); B-(q, r); C-(p); D-(p, s)
(*b*) A-(r); B-(q); C-(p); D-(s)
(*c*) A-(s); B-(r); C-(p, s); D-(q, r)
(*d*) A-(r); B-(q, r); C-(p); D-(s)

INTEGER TYPE QUESTIONS

27. Two equal masses *m* are attached by a string. One mass lies at radial distance *r* from the centre of a horizontal turntable which rotates with constant angular velocity $\omega = 2$ rad s^{-1}, while the second hangs from the string inside the turntable's hollow spindle (see fig.). The coefficient of static friction between the turntable and the mass lying on it $\mu_s = 0.5$. The maximum and minimum values such that the mass lying on the turntable does not slide are r_{max}, r_{min}. Then $(r_{max} + r_{min})$ (in meter).

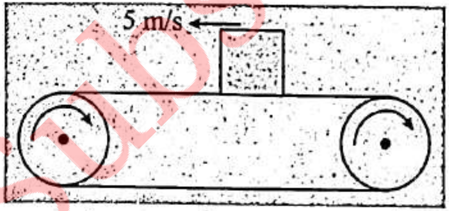


28. System is shown in the figure and man is pulling the rope from both sides with constant speed 'u'. Then the speed of the block will be xu. Find the value of $100x$ (M moves vertical):

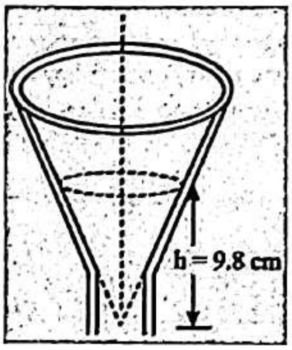


29. Five persons A, B, C, D and E are pulling a cart of mass 100 kg on a smooth surface and cart is moving with acceleration 3 m/s^2 in east direction. When person 'A' stops pulling, it moves with acceleration 1m/s^2 in the west direction. When only person 'B' stops pulling, it moves with acceleration 24 m/s^2 in the north direction. The magnitude of acceleration (in ms^{-2}) of the cart when only A and B pull the cart keeping their directions same as the old directions, is:

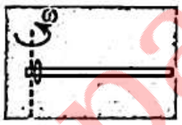
30. A block lying on a long horizontal conveyor belt moving at a constant velocity receives a velocity 5 m/s at $t = 0$ sec. relative to the ground in the direction opposite to the direction of motion of the conveyor. After $t = 4$ sec, the velocity of the block becomes equal to the velocity of the belt. The coefficient of friction between the block and the belt is 0.2. Then the velocity (in ms^{-1}) of the conveyor belt is: ($g = 10$ m/s^2)



31. A small particle moves along the inner surface of a smooth conical funnel, along a horizontal circle. (Figure) If the plane of the circular path be at a distance of 9.8 cm from the vertex, then the speed of the particle is x cms^{-1}. Find the value of x.

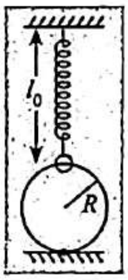


32. A rod is rotating with constant angular velocity in a horizontal plane about a vertical axis. A ring is attached to the axis as shown. The ring is released and it slides out on the rod without friction. At the time the ring comes out of the rod, what angle (in degree) does its velocity (as seen by an observer on ground) make with radially outward direction?



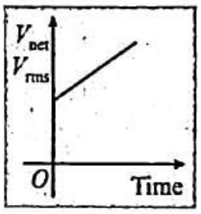
33. A bead of mass m is in unstable equilibrium at top of a smooth circular wire frame which is fixed. The bead is slightly disturbed and slowly brought to equilibrium position where spring is along tangent to the circular wire frame. If the spring is initially at natural length, what is spring constant k (in N/m)?

Take $R = 3.75$ m, $l_0 = 6$ m, $m = 1.3$ kg.



SINGLE CORRECT TYPE QUESTIONS

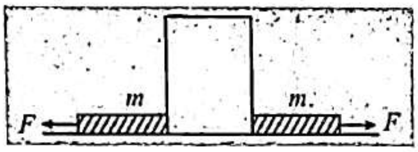
34. A particle is observed from two frames S_1 and S_2. The graph of relative velocity of S_1 with respect to S_2 is shown in figure. Let F_1 and F_2 be the pseudo forces on the particle when seen from S_1 and S_2 respectively. Which one of the following is not possible?



(a) $F_1 = 0, F_2 \neq 0$ (b) $F_1 \neq 0, F_2 = 0$

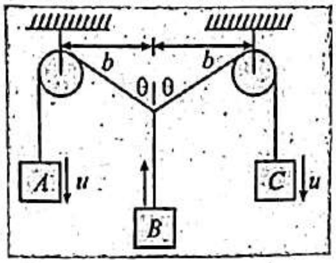
(c) $F_1 \neq 0, F_2 \neq 0$ (d) $F_1 = 0, F_2 = 0$

35. A heavy block kept on a frictionless surface and being pulled by two ropes of equal mass m as shown in figure. At $t = 0$, the force on the left rope is withdrawn but the force on the right end continues to act. Let F_1 and F_2 be the magnitudes of the forces by the right rope and the left rope on the block respectively.



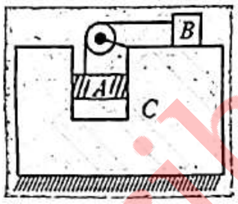
(a) $F_1 = F_2 = F$ for $t < 0$ (b) $F_1 = F_2 = F + mg$ for $t < 0$
(c) $F_1 = F, F_2 = F$ for $t > 0$ (d) $F_1 < F, F_2 = F$ for $t > 0$

36. In the figure shown the blocks A and C are pulled down with constant velocities u. Acceleration of block B is:



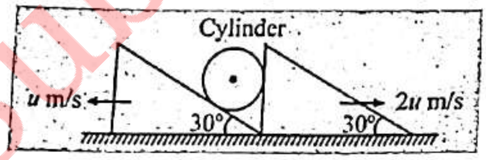
(a) $\frac{u^2}{b}\tan^2\theta\sec\theta$ (b) $\frac{u^2}{b}\tan^3\theta$
(c) $\frac{u^2}{b}\sec^2\theta\tan\theta$ (d) Zero

37. In the system shown in figure $m_A = 4m$, $m_B = 3m$ and $m_C = 8m$. Friction is absent everywhere. String is light and inextensible. If the system is released from rest find the acceleration of block B



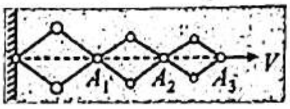
(a) $\frac{g}{8}$ (leftward) (b) $\frac{g}{2}$ (leftward)
(c) $\frac{g}{6}$ (rightward) (d) $\frac{g}{4}$ (rightward)

38. System is shown in the figure. Assume that cylinder remains in contact with the two wedges. The velocity of cylinder is-



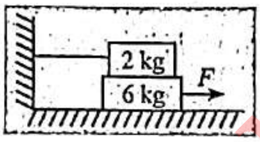
(a) $\sqrt{19-4\sqrt{3}}\,\frac{u}{2}$ m/s (b) $\frac{\sqrt{13}u}{2}$ m/s
(c) $\sqrt{3}u$ m/s (d) $\sqrt{7}u$ m/s

39. A hinged construction consists of three rhombus with the ratio of sides (5 : 3 : 2). Vertex A_3 moves in the horizontal direction with velocity V. Velocity of A_2 will be:



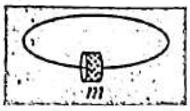
(a) 2.5 V (b) 1.5V
(c) (2/3)V (d) 0.8 V

40. If the coefficient of friction at all surfaces is 0.4, then the force required to pull out the 6 kg block with an acceleration of 1.5 m/s² will be



(a) 49 (b) 59 (c) 41 (d) 40

41. A small collar of mass m is given an initial velocity of magnitude v_0 on the horizontal circular track fabricated from a slender rod. If the coefficient of kinetic friction is μ_k, determine the distance travelled before the collar comes to rest. (Recognize that the friction force depends on the net normal force).



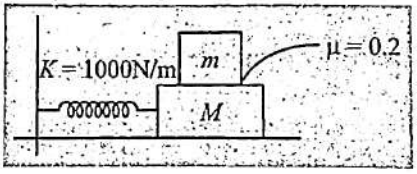
(a) $\frac{r}{\mu_k}\ln\left[\frac{v_0^2+\sqrt{v_0^4+r^2g^2}}{rg}\right]$

(b) $\frac{r}{4\mu_k}\ln\left[\frac{v_0^2+\sqrt{v_0^4+r^2g^2}}{rg}\right]$

(c) $\frac{2r}{\mu_k}\ln\left[\frac{v_0^2+\sqrt{v_0^4+r^2g^2}}{rg}\right]$

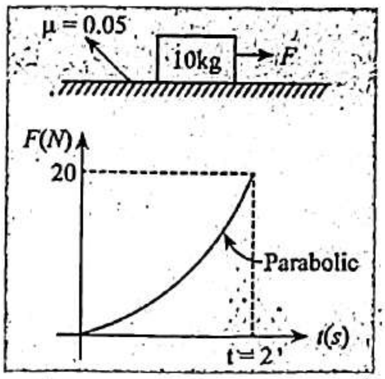
(d) $\frac{r}{2\mu_k}\ln\left[\frac{v_0^2+\sqrt{v_0^4+r^2g^2}}{rg}\right]$

42. A block of mass $M = 4$kg is kept on a smooth horizontal plane. A bar of mass $m = 1$kg is kept on it. They are connected to a spring as shown & the spring is compressed. Then what is the maximum compression in the spring for which the bar will not slip on the block when released if coefficient of friction between them is 0.2 & spring constant $k = 1000$ N/m.



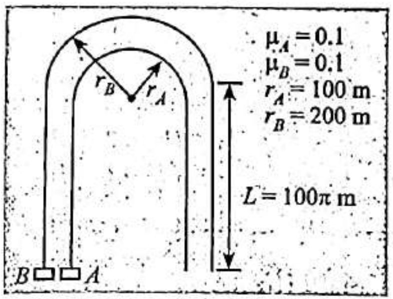
(a) 1 cm (b) 1 m
(c) 1.25 cm (d) 10 cm

43. A force F is applied to the initially stationary cart. The variation of force with time is shown in figure such that $\frac{dF}{dt} = 0$ at $t = 0$. Find the speed (m/s) of cart at $t = 2$s.



(a) 4 (b) 0.67 (c) 3.75 (d) 2

44. Two cars A and B start racing at the same time on a flat race track which consists of two straight sections each of length 100π m and one circular section as shown in Fig. The rule of the race is that each car must travel at constant speed at all times without ever skidding.



(a) car A completes its journey before car B

(b) both cars complete their journey in same time

(c) velocity of car A is greater than that of car B

(d) car B completes its journey before car A.

45. A piece of string of length ℓ which can support a maximum tension T, is used to whirl a particle of mass m in a circular path as an a conical pendulum. What is maximum speed with which the particle may be whirled as a conical pendulum.

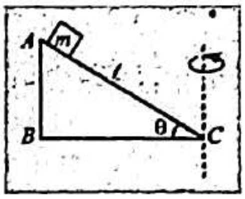
(a) $u = \left(\frac{T\ell}{2m}\right)\left[1 - \left(\frac{mg}{T}\right)^2\right]^{1/2}$

(b) $u = \left(\frac{T\ell}{m}\right)^{1/2}\left[1 - \left(\frac{2mg}{T}\right)^2\right]^{1/2}$

(c) $u = \left(\frac{T\ell}{m}\right)^{1/2}\left[1 - \left(\frac{mg}{T}\right)^2\right]^{1/2}$

(d) $u = \left(\frac{2T\ell}{m}\right)^{1/2}\left[1 - \left(\frac{2mg}{T}\right)^2\right]^{1/2}$

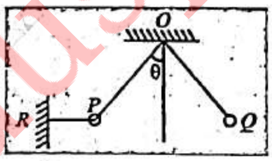
46. A block of mass m is placed at the top of a smooth wedge ABC. The wedge is rotated about an axis passing through C as shown in the figure. The value of angular speed such that the block does not slip on the wedge is



(a) $\left(\sqrt{\frac{g\sin\theta}{\ell}}\right)\sec\theta$ (b) $\left(\sqrt{\frac{g}{\ell}}\right)\cos\theta$

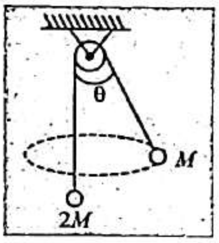
(c) $\left(\sqrt{\frac{g}{\ell\cos\theta}}\right)\cos\theta$ (d) $\left(\sqrt{\frac{g\sin\theta}{\ell}}\right)$

47. A ball of mass 1 kg is at rest in position P by means of two light strings OP and RP as shown in the figure. The string RP is now cut and the ball swings upto a position Q. If $\theta = 60°$ and tension in the string in position OP and OQ are T_1 (Just before cutting) and T_2 respectively, then $\frac{T_1}{T_2}$ is equal to



(a) 1 (b) 2 (c) 3 (d) 4

48. In the diagram shown, the mass $2M$ is hanging vertically down and the other mass M is revolving around it. The value of the angle θ is equal to

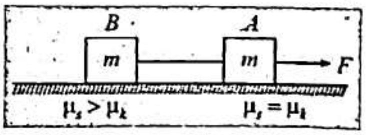


(a) 30° (b) 60° (c) 15° (d) 45°

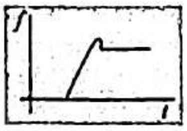
49. A uniform circular ring of mass per unit length λ and radius R is rotating with angular velocity ω about its own axis in a gravity free space. Tension in the ring is

(a) zero (b) $2\lambda R^2\omega^2$ (c) $\lambda R^2\omega^2$ (d) $\lambda R\omega^2$

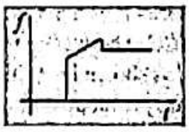
50. A force $F = t$ is applied to block A as shown in figure. The force is applied at $t = 0$ seconds when the system was at rest and string is just straight without tension. Which of the following graphs gives the friction force between B and horizontal surface as a function of time 't'.



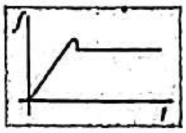
(*a*)



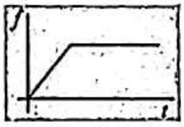
(*b*)



(*c*)

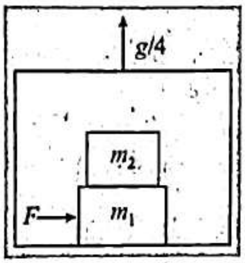


(*d*)



51. A plank of mass $m_1 = 8$ kg with a bar of mass $m_2 = 2$ kg placed on its rough surface, lie on a smooth floor of elevator ascending with an acceleration $g/4$. The coefficient of friction is $\mu = 1/5$ between m_1 and m_2. *A* horizontal force $F = 30$ N is applied to the plank. Then the acceleration of bar and the plank in the reference frame of elevator are:

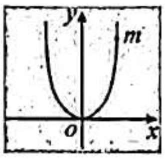
(take $g = 10$ ms^{-2})



(*a*) 3.5 m/s², 5 m/s² (*b*) 5 m/s², $\frac{50}{8}$ m/s²

(*c*) 2.5 m/s², $\frac{25}{8}$ m/s² (*d*) 4.5 m/s², 4.5m/s²

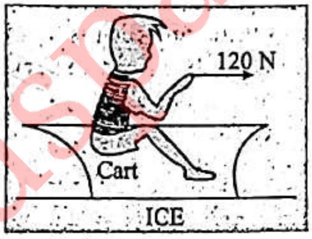
52. A bead of mass *m* is located on a parabolic wire with its axis vertical and vertex directed downward as in figure and whose equation is $x^2 = ay$. If the coefficient of friction is μ, the highest distance above the *x*-axis at which the particle will be in equilibrium is



(*a*) μa (*b*) $\mu^2 a$

(*c*) $\frac{1}{4}\mu^2 a$ (*d*) $\frac{1}{2}\mu a$

53. A child of mass 30 kg was sitting on a snow cart of mass 10 kg. The friction between the cart and the snow was negligible. If a force of 120 N was applied to the child, as shown in the figure. The minimum coefficient of friction required between the child and cart to keep the child from slipping off is



(*a*) 0.1 (*b*) 0.2

(*c*) 0.4 (*d*) 0.3

PYQ's (Past Year Questions)

LAWS OF MOTION

1. A cricket player catches a ball of mass 120 g moving with 25 m/s speed. If the catching process is completed in 0.1 s then the magnitude of force exerted by the ball on the hand of player will be (in SI unit): **[01 Feb, 2024 (Shift-II)]**

(*a*) 24 (*b*) 12 (*c*) 25 (*d*) 30

2. A body of mass 4 kg experiences two forces $\vec{F_1} = 5\hat{i} + 8\hat{j} + 7\hat{k}$ and $\vec{F_2} = 3\hat{i} - 4\hat{j} - 3\hat{k}$.

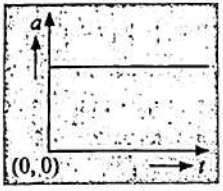
The acceleration acting on the body is: **[01 Feb, 2024 (Shift-II)]**

(*a*) $-2\hat{i} - \hat{j} - \hat{k}$ (*b*) $4\hat{i} + 2\hat{j} + 2\hat{k}$

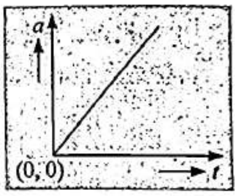
(*c*) $2\hat{i} + \hat{j} + \hat{k}$ (*d*) $2\hat{i} + 3\hat{j} + 3\hat{k}$

3. A wooden block, initially at rest on the ground, is pushed by a force which increases linearly with time *t*. Which of the following curve best describes acceleration of the block with time: **[04 April, 2024 (Shift-I)]**

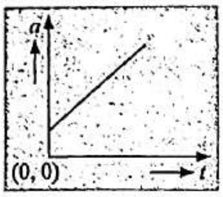
(*a*)



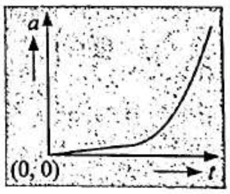
(*b*)



(*c*)



(*d*)



4. A particle moves in x-y plane under the influence of a force $\vec{F}$ such that its linear momentum is $\vec{P}(t) = \hat{i}\cos(kt) - \hat{j}\sin(kt)$. If *k* is constant, the angle between $\vec{F}$ and $\vec{P}$ will be: **[05 April, 2024 (Shift-II)]**

(*a*) $\pi/2$ (*b*) $\pi/6$

(*c*) $\pi/4$ (*d*) $\pi/3$

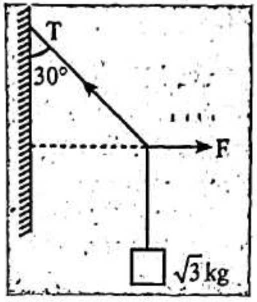
5. A player caught a cricket ball of mass 150 g moving at a speed of 20 m/s. If the catching process is completed in 0.1 s, the magnitude of force exerted by the ball on the hand of the player is: **[08 April, 2024 (Shift-I)]**

(*a*) 150 N (*b*) 3 N

(*c*) 30 N (*d*) 300 N

6. A block of $\sqrt{3}$ kg is attached to a string whose other end is attached to the wall. An unknown force F is applied so that the string makes an angle of 30° with the wall. The tension T in the string is:

(Given $g = 10\ \text{ms}^{-2}$) **[30 Jan, 2023 (Shift-II)]**



(*a*) 20 N (*b*) 25 N

(*c*) 10 N (*d*) 15 N

7. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): An electric fan continues to rotate for some time after the current is switched off.

Reason (R): Fan continuous to rotate due to inertia of motion.

In the light of above statements, choose the most appropriate answer from the options given below. **[10 Apr, 2023 (Shift-II)]**

(*a*) (A) is correct but (R) is not correct.

(*b*) Both (A) and (R) are correct and R is the correct explanation of (A).

(*c*) A is not correct but (R) is correct.

(*d*) Both (A) and (R) are correct but (R) is not the correct explanation of (A).

8. A particle of mass m is moving in the xy-plane such that its velocity at a point (x, y) is given as $\vec{v} = \alpha(y\hat{x} + 2x\hat{y})$, where a is a non-zero constant. What is the force $\vec{F}$ acting on the particle? **[JEE Adv, 2023]**

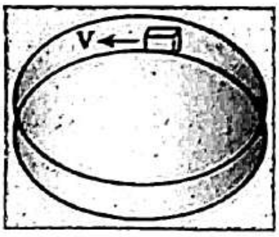
(*a*) $\vec{F} = 2m\alpha^2(x\hat{x} + y\hat{y})$ (*b*) $\vec{F} = m\alpha^2(y\hat{x} + 2x\hat{y})$

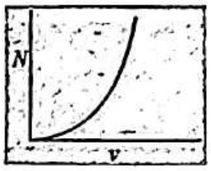
(*c*) $\vec{F} = 2m\alpha^2(y\hat{x} + x\hat{y})$ (*d*) $\vec{F} = m\alpha^2(x\hat{x} + 2y\hat{y})$

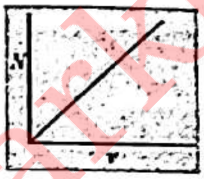
9. The velocity of the bullet becomes one third after it penetrates 4 cm in a wooden block. Assuming that bullet is facing a constant resistance during its motion in the block. The bullet stops completely after travelling at $(4 + x)$ cm inside the block. The value of x is: **[27 July, 2022 (Shift-II)]**

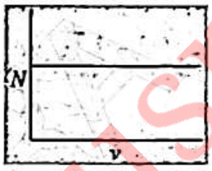
(*a*) 2.0 (*b*) 1.0 (*c*) 0.5 (*d*) 1.5

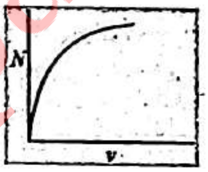
10. A smooth circular groove has a smooth vertical wall as shown in figure. A block of mass m moves against the wall with a speed v. Which of the following curve represents the correct relation between the normal reaction on the block by the wall (N) and speed of the block (v)? **[29 July, 2022 (Shift-I)]**



(*a*)


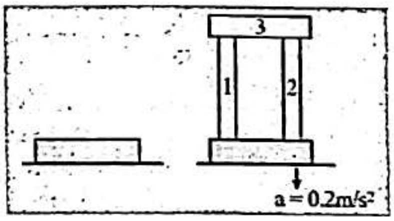
(*b*)


(*c*)


(*d*)


11. A steel block of 10 kg rests on a horizontal floor as shown. When three iron cylinders are placed on it as shown, the block and cylinders go down with an acceleration 0.2m/s^2. The normal reaction R' by the floor if mass of the iron cylinders are equal and of 20kg each, is _________N.

[Take $g = 10\text{m/s}^2$ and $\mu_s = 0.2$] **[20 July, 2021 (Shift-I)]**



(*a*) 714 (*b*) 684

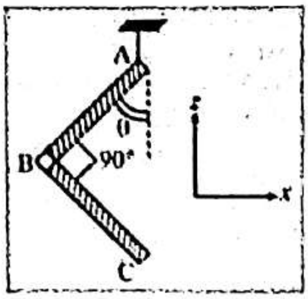
(*c*) 716 (*d*) 686

12. A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})$ m. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})$ m will be $\sqrt{x}$ N - m. The value of x is **[05 Sep, 2020 (Shift-I)]**

(*a*) 195 (*b*) 165

(*c*) 105 (*d*) 135

13. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then: **[09 Jan, 2019 (Shift-I)]**

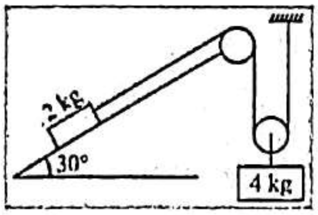


(*a*) $\tan\theta = \frac{1}{2\sqrt{3}}$ (*b*) $\tan\theta = \frac{1}{2}$

(*c*) $\tan\theta = \frac{2}{\sqrt{3}}$ (*d*) $\tan\theta = \frac{1}{3}$

"APPLICATION OF FORCE/IMPULSE, STATICS AND DYNAMICS INVOLVING SINGLE SYSTEM"

14. All surfaces shown in figure are assumed to be frictionless and the pulleys and the string are light. The acceleration of the block of mass 2 kg is: **[30 Jan, 2024 (Shift-I)]**



(*a*) g (*b*) $\frac{g}{3}$ (*c*) $\frac{g}{2}$ (*d*) $\frac{g}{4}$

15. A spherical body of mass 100 g is dropped from a height of 10 m from the ground. After hitting the ground, the body rebounds to a height of 5m. The impulse of force imparted by the ground to the body is given by: (given $g = 9.8\ m/s^2$) **[30 Jan, 2024 (Shift-I)]**

(*a*) 4.32 kg ms^{-1} (*b*) 43.2 kg ms^{-1}

(*c*) 23.9 kg ms^{-1} (*d*) 2.39 kg ms^{-1}

16. A wooden block of mass 5kg rests on soft horizontal floor. When an iron cylinder of mass 25 kg is placed on the top of the block, the floor yields and the block and the cylinder together go down with an acceleration of 0.1 ms^{-2}. The action force of the system on the floor is equal to: **[05 April, 2024 (Shift-I)]**

(*a*) 297 N (*b*) 294 N

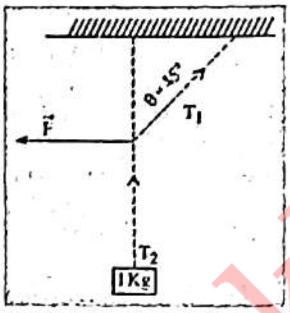
(*c*) 291 N (*d*) 196 N

17. A body of weight 200 N is suspended from a tree branch through a chain of mass 10 kg. The branch pulls the chain by a force equal to (if g = 10 m/s^2): **[06 April, 2024 (Shift-II)]**

(*a*) 150 N (*b*) 300 N

(*c*) 200 N (*d*) 100 N

18. A 1 kg mass is suspended from the ceiling by a rope of length 4m. A horizontal force 'F' is applied at the mid point of the rope so that the rope makes an angle of 45° with respect to the vertical axis as shown in figure. The magnitude of F is: **[09 April, 2024 (Shift-II)]**



(*a*) $\frac{10}{\sqrt{2}}$ N (*b*) 1 N

(*c*) $\frac{1}{10 \times \sqrt{2}}$ N (*d*) 10 N

19. Given below are two statements:

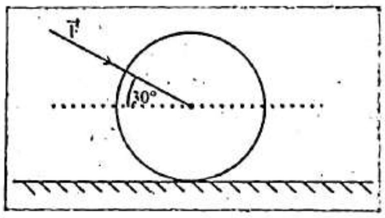
Statement-I: An elevator can go up or down with a uniform speed when its weight is balanced with the tension of its cable.

Statement-II: Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed.

In the light of the above statements, choose the correct answer from the options given below: **[24 Jan, 2023 (Shift-I)]**

(*a*) Both Statement-I and Statement-II are false
(*b*) Statement-I is true but Statement-II is false
(*c*) Both Statement-I and Statement-II are true
(*d*) Statement-I is false but Statement-II is true

20. As shown in figure, a 70kg garden roller is pushed with a force of $\vec{F} = 200\,N$ at an angle of 30° with horizontal. The normal reaction on the roller is (Given $g = 10\ ms^{-2}$)

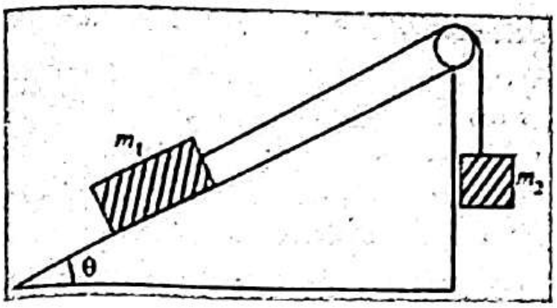


[31 Jan, 2023 (Shift-I)]

(*a*) $800\sqrt{2}\,N$ (*b*) $600\,N$

(*c*) $800\,N$ (*d*) $200\sqrt{3}\,N$

21. Two bodies of masses $m_1 = 5$kg and $m_2 = 3$kg are connected by a light string going over a smooth light pulley on a smooth inclined plane as shown in the figure. The system is at rest. The force exerted by the inclined plane on the body of mass m_1 will be: [Take $g = 10\ ms^{-2}$] **[29 July, 2022 (Shift-II)]**



(*a*) 30 N (*b*) 40 N (*c*) 50 N (*d*) 60 N

22. A person standing on a balance inside a stationary lift measures 60 kg. The weight of that person if the lift descends with uniform downward acceleration of 1.8 m/s^2 will be ________ N. [$g = 10\ m/s^2$] **[26 Feb, 2021 (Shift-I)]**

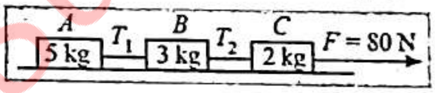
23. A particle of mass *m* is initially at rest at the origin. It is subjected to a force and starts moving along the *x*-axis. Its kinetic energy *K* changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true? **[JEE Adv, 2021]**
(*a*) The force applied on the particle is constant
(*b*) The speed of the particle is proportional to time
(*c*) The distance of the particle from the origin increases linearly with time
(*d*) The force is conservative

24. A small ball of mass *m* is thrown upward with velocity *u* from the ground. The ball experiences a resistive force mkv^2, where *v* is its speed. The maximum height attained by the ball is **[04 Sep, 2020 (Shift-II)]**
(*a*) $\frac{1}{2K}\tan^{-1}\frac{ku^2}{g}$ (*b*) $\frac{1}{K}\ln\left(1+\frac{ku^2}{2g}\right)$
(*c*) $\frac{1}{2K}\ln\left(1+\frac{ku^2}{g}\right)$ (*d*) $\frac{1}{K}\tan^{-1}\frac{ku^2}{2g}$

25. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is ($g = 10\ ms^{-2}$) **[09 Jan, 2019 (Shift-II)]**
(*a*) 200 N (*b*) 140 N (*c*) 70 N (*d*) 100 N

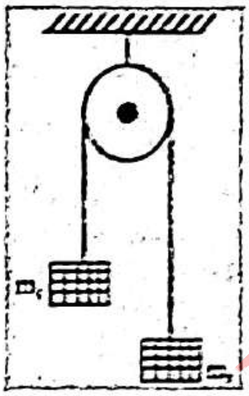
CONSTRAINT RELATION, DYNAMICS OF MULTI SYSTEM, SPRINGS

26. Three blocks *A*, *B* and *C* are pulled on a horizontal smooth surface by a force of 80 N as shown in figure



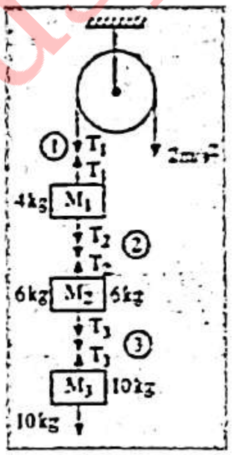
The tensions T_1 and T_2 in the string are respectively: **[30 Jan, 2024 (Shift-II)]**
(*a*) 40N, 64N (*b*) 60N, 80N
(*c*) 88N, 96N (*d*) 80N, 100N

27. A light string passing over a smooth light fixed pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is g/3, then the ratio of masses is **[31 Jan, 2024 (Shift-II)]**



(*a*) $\frac{9}{7}$ (*b*) $\frac{8}{1}$ (*c*) $\frac{4}{3}$ (*d*) $\frac{5}{3}$

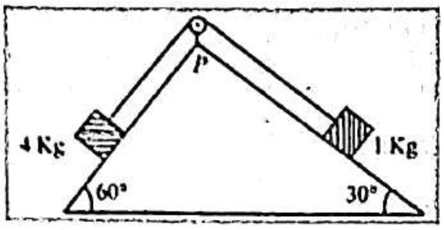
28. Three blocks M_1, M_2, M_3 having masses 4 kg, 6 kg and 10 kg respectively are hanging from a smooth pulley using rope 1, 2 and 3 as shown in figure. The tension in the rope 1, T_1 when they are moving upward with acceleration of $2ms^{-2}$ is .... N (if $g = 10\ m/s^2$) **[05 April, 2024 (Shift-I)]**



29. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (where $m_2 > m_1$). If the acceleration of the system is $\frac{g}{\sqrt{2}}$, then the ratio of the masses $\frac{m_1}{m_2}$ is: **[06 April, 2024 (Shift-I)]**
(*a*) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (*b*) $\frac{1+\sqrt{5}}{\sqrt{5}-1}$ (*c*) $\frac{1+\sqrt{5}}{\sqrt{2}-1}$ (*d*) $\frac{\sqrt{3}+1}{\sqrt{2}-1}$

30. A light unstretchable string passing over a smooth light pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is $\frac{g}{8}$, then the ratio of the masses m_2/m_1 is: **[09 April, 2024 (Shift-I)]**
(*a*) 9 : 7 (*b*) 4 : 3 (*c*) 5 : 3 (*d*) 8 : 1

31. As per given figure, a weightless pulley *P* is attached on a double inclined frictionless surface. The tension in the string (massless) will be (if $g = 10\ m/s^2$) **[24 Jan, 2023 (Shift-I)]**

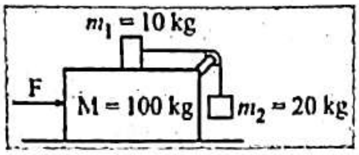


(*a*) $(4\sqrt{3}+1)N$ (*b*) $4(\sqrt{3}+1)N$

(*c*) $4(\sqrt{3}-1)N$ (*d*) $(4\sqrt{3}-1)N$

32. Three masses $M = 100$ kg, $m_1 = 10$ kg and $m_2 = 20$ kg are arranged in a system as shown in figure. All the surfaces are frictionless and strings are inextensible and weightless. The pulleys are also weightless and frictionless. A force F is applied on the system so that the mass m_2 moves upward with an acceleration of 2 ms^{-2}. The value of F is

(Take $g = 10\ \text{ms}^{-2}$) **[26 July, 2022 (Shift-I)]**



(*a*) 3360 N (*b*) 3380 N (*c*) 3120 N (*d*) 3240 N

33. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be

[12 April, 2019 (Shift-II)]

(*a*) $\frac{1}{n^2}$ (*b*) n^2 (*c*) $\frac{1}{n}$ (*d*) n

"NON-INERTIAL REFERENCE FRAME AND PSEUDO FORCE"

34. A block of mass M placed inside a box descends vertically with acceleration 'a'. The block exerts a force equal to one-fourth of its weight on the floor of the box. The value of 'a' will be: **[29 June, 2022 (Shift-II)]**

(*a*) $g/4$ (*b*) $g/2$ (*c*) $3g/4$ (*d*) g

35. A car is moving on a plane inclined at 30° to the horizontal with an acceleration of 10 ms^{-2} parallel to the plane upward. A bob is suspended by a string from the roof of the car. The angle in degrees which the string makes with the vertical is______. (Take $g = 10\ \text{ms}^{-2}$) **[31 Aug, 2021 (Shift-I)]**

36. A heavy ball of mass M is suspended from the ceiling of car by a light string of mass m (m << M). When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g is closest to: **[09 Jan, 2019 (Shift-I)]**

(*a*) $\frac{g}{30}$ (*b*) $\frac{g}{5}$ (*c*) $\frac{g}{10}$ (*d*) $\frac{g}{20}$

STATIC FRICTION

37. Given below are two statements:

Statement-I: The limiting force of static friction depends on the area of contact and independent of materials.

Statement-II: The limiting force of kinetic friction is independent of the area of contact and depends on materials.

In the light of the above statements, choose the most appropriate answer from the options given below:

[27 Jan, 2024 (Shift-II)]

(*a*) Statement-I is correct but Statement-II is incorrect
(*b*) Statement-I is incorrect but Statement-II is correct
(*c*) Both Statement-I and Statement-II are incorrect
(*d*) Both Statement-I and Statement-II are correct

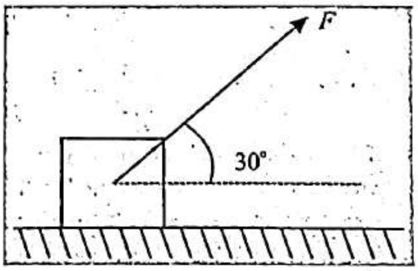
38. The vehicles carrying inflammable fluids usually have metallic chains touching the ground:

[05 April, 2024 (Shift-II)]

(*a*) To conduct excess charge due to air friction to ground and prevent sparking.
(*b*) To alert other vehicles.
(*c*) To protect tyres from catching dirt from ground.
(*d*) It is a custom.

39. As shown in the figure a block of mass 10 kg lying on a horizontal surface is pulled by a force F acting at an angle 30°, with horizontal. For $\mu_s = 0.25$, the block will just start to move for the value of F: [Given $g = 10 ms^{-2}$]

[01 Feb, 2023 (Shift-II)]



(*a*) 33.3 N (*b*) 25.2 N
(*c*) 20 N (*d*) 35.7 N

40. A uniform chain of 6 m length is placed on a table such that a part of its length is hanging over the edge of the table. The system is at rest. The co-efficient of static friction between the chain and the surface of the table is 0.5, the maximum length of the chain hanging from the table is ________m. **[25 June, 2022 (Shift-I)]**

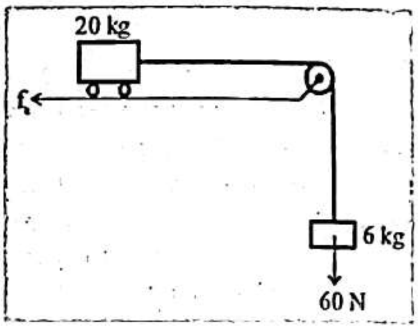
41. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall be __________N. [$g = 10\ \text{ms}^{-2}$]

[24 Feb, 2021 (Shift-I)]

KINETIC FRICTION

42. Consider a block and trolley system as shown in figure. If the coefficient of kinetic friction between the trolley and the surface is 0.04, the acceleration of the system in ms^{-2} is: (Consider that the string is massless and unstretchable and the pulley is also massless and frictionless):

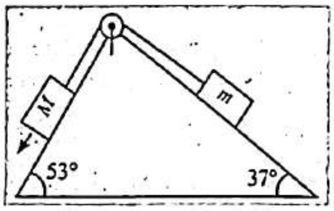
[01 Feb, 2024 (Shift-I)]



(*a*) 3 (*b*) 4 (*c*) 2 (*d*) 1.2

43. In the given arrangement of a doubly inclined plane two blocks of masses M and m are placed. The blocks are connected by a light string passing over an ideal pulley as shown. The coefficient of friction between the surface of the plane and the blocks is 0.25. The value of m, for which $M = 10$ kg will move down with an acceleration of 2 m/s^2, is: (take $g = 10$ m/s^2 and tan 37° = 3/4)

[31 Jan, 2024 (Shift-I)]



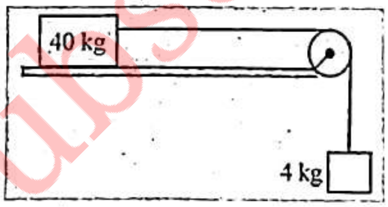
(*a*) 9 kg (*b*) 4.5 kg (*c*) 6.5 kg (*d*) 2.25 kg

44. A body of mass 10 kg is moving with an initial speed of 20 m/s. The body stops after 5 s due to friction between body and the floor. The value of the coefficient of friction is: (Take acceleration due to gravity $g = 10$ ms^{-2})

[31 Jan, 2023 (Shift-II)]

(*a*) 0.2 (*b*) 0.3 (*c*) 0.5 (*d*) 0.4

45. A block of mass 40 kg slides over a surface, when a mass of 4 kg is suspended through an inextensible massless string passing over frictionless pulley as shown below.



The coefficient of kinetic friction between the surface and block is 0.02. The acceleration of block is. (given $g = 10$ ms^{-2})

[29 June, 2022 (Shift-II)]

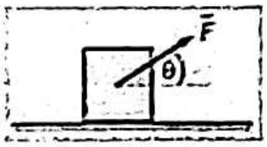
(*a*) 1 ms^{-2} (*b*) 1/5 ms^{-2}

(*c*) 4/5 ms^{-2} (*d*) 8/11 ms^{-2}

46. A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic is μ_K. Then, the block's acceleration 'a' is given by:

[16 March, 2021 (Shift-I)]

(g is acceleration due to gravity)



(*a*) $\frac{F}{m}\cos\theta + \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

(*b*) $\frac{F}{m}\cos\theta + \mu_K\left(g + \frac{F}{m}\sin\theta\right)$

(*c*) $\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

(*d*) $-\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

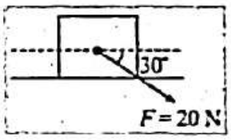
47. An insect is at the bottom of a hemispherical ditch of radius 1 *m*. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is ($g = 10$ ms^{-2})

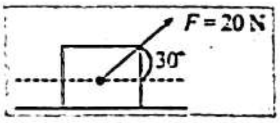
[06 Sep, 2020 (Shift-I)]

(*a*) 0.45 m (*b*) 0.80 m (*c*) 0.20 m (*d*) 0.60 m

48. A block of mass 5 kg is (i) pushed in case A and (ii) pulled in case B, by a force $F = 20$ N. Making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is $m = 0.2$. The difference between the accelerations of the block, in case B and case A will be: ($g = 10$ ms^{-2})

[12 April, 2019 (Shift-II)]

(A)


(B)


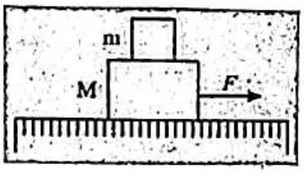
(*a*) 0 ms^{-2} (*b*) 0.8 ms^{-2}

(*c*) 0.4 ms^{-2} (*d*) 3.2 ms^{-2}

TWO AND THREE BLOCKS PROBLEMS

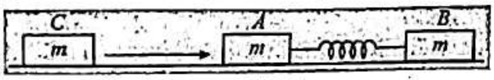
49. A system of two blocks of masses $m = 2$ kg and $m = 8$ kg is placed on a smooth table as shown in figure. The coefficient of static friction between two blocks is 0.5. The maximum horizontal force F that can be applied to the block of mass M so that the blocks move together will be:

[27 June, 2022 (Shift-I)]



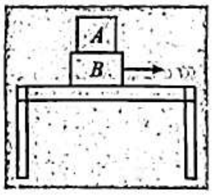
(*a*) 9.8 N (*b*) 39.2 N (*c*) 49 N (*d*) 78.4 N

50. Two identical blocks A and B each of mass m resting on a smooth horizontal floor are connected by a light spring of natural length L and spring constant K. A third block C of mass m moving with a speed v along the line joining A and B collides with A. The maximum compression in the spring is **[17 March, 2021 (Shift-II)]**



(*a*) $\sqrt{\frac{mv}{2K}}$ (*b*) $\sqrt{\frac{m}{2K}}$ (*c*) $\sqrt{\frac{mv}{K}}$ (*d*) $v\sqrt{\frac{m}{2K}}$

51. Two blocks A and B of masses $m_A = 1$ kg and $m_B = 3$ kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on horizontally, so that the block A does not slide over the block B is: (Take $g = 10$ m/s^2) **[10 April, 2019 (Shift-II)]**



(*a*) 16 N (*b*) 40 N (*c*) 12 N (*d*) 8 N

INCLINED PLANE PROBLEMS

52. A block of mass m is placed on a surface having vertical cross section given by $y = x^2/4$. If coefficient of friction is 0.5, the maximum height above the ground at which block can be placed without slipping is: **[30 Jan, 2024 (Shift-II)]**

(*a*) 1/4 m (*b*) 1/2 m (*c*) 1/6 m (*d*) 1/3 m

53. A 2 kg brick begins to slide over a surface which is inclined at an angle of 45° with respect to horizontal axis. The co-efficient of static friction between the surfaces is: **[04 April, 2024 (Shift-II)]**

(*a*) 1 (*b*) $\frac{1}{\sqrt{3}}$ (*c*) 0.5 (*d*) 1.7

54. A given object takes n times the time to slide down 45° rough inclined plane as it takes the time to slide down an identical perfectly smooth 45° inclined plane. The coefficient of kinetic friction between the object and the surface of inclined plane is: **[08 April, 2024 (Shift-II)]**

(*a*) $1-\frac{1}{n^2}$ (*b*) $1-n^2$ (*c*) $\sqrt{1-\frac{1}{n^2}}$ (*d*) $\sqrt{1-n^2}$

55. A block of mass m slides down an inclined plane inclined at angle 30° with an acceleration $\frac{g}{4}$. The value of coefficient of kinetic friction will be: **[29 Jan, 2023 (Shift-I)]**

(*a*) $\frac{2\sqrt{3}+1}{2}$ (*b*) $\frac{1}{2\sqrt{3}}$

(*c*) $\frac{\sqrt{3}}{2}$ (*d*) $\frac{2\sqrt{3}-1}{2}$

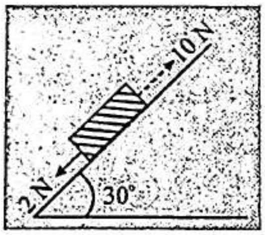
56. A block 'A' takes 2 s to slide down a frictionless incline of 30° and length 'ℓ', kept inside a lift going up with uniform velocity 'v'. If the incline is changed to 45°, the time taken by the block, to slide down the incline, will be approximately: **[27 July, 2022 (Shift-II)]**

(*a*) 2.66 s (*b*) 0.83 s (*c*) 1.68 s (*d*) 0.70 s

57. When a body slides down from rest along a smooth inclined plane making an angle of 30° with the horizontal, it takes time T. When the same body slides down from the rest along a rough inclined plane making the same angle and through the same distance, it takes time αT, where α is a constant greater than 1. The co-efficient of friction between the body and the rough plane is $\frac{1}{\sqrt{x}}\left(\frac{\alpha^2-1}{\alpha^2}\right)$ where $x =$ ________. **[01 Sep, 2021 (Shift-II)]**

58. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0. It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$. The nearest integer to I is ________. **[03 Sep, 2020 (Shift-II)]**

59. A block kept on a rough inclined plane, as shown in the figure, remains at rest up-to a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is: **[12 Jan, 2019 (Shift-II)]**



(*a*) $\frac{\sqrt{3}}{2}$ (*b*) $\frac{\sqrt{3}}{4}$

(*c*) $\frac{1}{2}$ (*d*) $\frac{2}{3}$

DYNAMICS OF CIRCULAR MOTION

60. A train is moving with a speed of 12 m/s on rails which are 1.5 m apart. To negotiate a curve radius 400 m, the height by which the outer rail should be raised with respect to the inner rail is (Given, $g = 10\ m/s^2$): **[27 Jan, 2024 (Shift-I)]**

(*a*) 6.0 cm (*b*) 5.4 cm (*c*) 4.8 cm (*d*) 4.2 cm

61. A coin is placed on a disc. The coefficient of friction between the coin and the disc is μ. If the distance of the coin from the center of the disc is *r*, the maximum angular velocity which can be given to the disc, so that the coin does not slip away, is: **[31 Jan, 2024 (Shift-I)]**

(*a*) $\frac{\mu g}{r}$ (*b*) $\sqrt{\frac{r}{\mu g}}$ (*c*) $\sqrt{\frac{\mu g}{r}}$ (*d*) $\frac{u}{\sqrt{\mu g}}$

62. A ball of mass 0.5 kg is attached to a string of length 50 cm. The ball is rotated on a horizontal circular path about its vertical axis. The maximum tension that the string can bear is 400 N. The maximum possible value of angular velocity of the ball in rad/s is: **[01 Feb, 2024 (Shift-I)]**

(*a*) 1600 (*b*) 40 (*c*) 1000 (*d*) 20

63. A man carrying a monkey on his shoulder does cycling smoothly on a circular track of radius 9m and completes 120 revolutions in 3 minutes. The magnitude of centripetal acceleration of monkey is (in m/s^2): **[05 April, 2024 (Shift-II)]**

(*a*) zero (*b*) $16\pi^2\ ms^{-2}$

(*c*) $4\pi^2\ ms^{-2}$ (*d*) $57600\ \pi^2\ ms^{-2}$

64. A car of 800 kg is taking turn on a banked road of radius 300 m and angle of banking 30°. If coefficient of static friction is 0.2 then the maximum speed with which car can negotiate the turn safely:

($g = 10\ m/s^2$, $\sqrt{3} = 1.73$) **[06 April, 2024 (Shift-II)]**

(*a*) 70.4 m/s (*b*) 51.4 m/s (*c*) 264 m/s (*d*) 102.8 m/s

65. A body of mass 200 *g* is tied to a spring of spring constant 12.5 *N*/*m*, while the other end of spring is fixed at point *O*. If the body moves about *O* in a circular path on a smooth horizontal surface with constant angular speed 5 *rad*/*s*, then the ratio of extension in the spring to its natural length will be: **[24 Jan, 2023 (Shift-II)]**

(*a*) 1 : 2 (*b*) 1 : 1 (*c*) 2 : 3 (*d*) 2 : 5

66. A boy ties a stone of mass 100 g to the end of 2 m long string and whirls it around in a horizontal plane. The string can withstand the maximum tension of 80 N. If the maximum speed with which the stone can revolve is $\frac{K}{\pi}$ rev. /min. The value of *K* is:

(Assume the string is massless and unstretchable) **[24 June, 2022 (Shift-I)]**

(*a*) 400 (*b*) 300 (*c*) 600 (*d*) 800

67. A particle is moving with uniform speed along the circumference of a circle of radius of radius *R* under the action of a central fictitious force *F* which is inversely proportional to R^3. Its time period of revolution will be given by: **[26 Feb, 2021 (Shift-I)]**

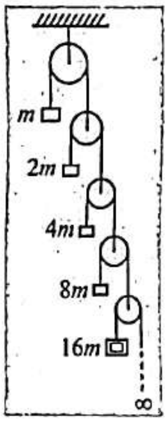
(*a*) $T \propto R^{\frac{3}{2}}$ (*b*) $T \propto R^{\frac{5}{2}}$ (*c*) $T \propto R^{\frac{4}{3}}$ (*d*) $T \propto R^2$

'FOCUS ON THE FLAME'

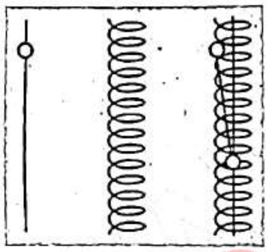
- Light a candle and stare at the flame for a few minutes, observing every flicker and movement.
- This activity will help you train your focus and build mental steadiness.

PW Challengers

1. In the infinite assembly of pulleys and blocks shown in figure below, find n, if the acceleration of block of mass m is $\frac{ng}{8}$. All the pulley are of negligible mass.



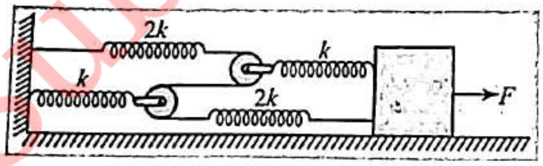
2. A bead slides down a height h on a rigid thin vertical rod in time T_1, and the same height on a rigid helix in time T_2. Now the rod is rigidly fixed along the axis of the helix and the bead that can slide on the helix is tied with a light inextensible thread to another identical bead that can slide on the rod as shown in the figure. The assembly consisting of the beads and the thread is released from rest. How long will this assembly take to slide down the same height h? Neglect friction as well as air resistance and assume radius of the helix so small as compared to the length of the thread that the thread can be assumed almost vertical.



(a) $\frac{T_1T_2}{T_1+T_2}$ (b) $\frac{T_1T_2}{2(T_1+T_2)}$

(c) $\frac{\sqrt{T_1^2+T_2^2}}{2}$ (d) $\sqrt{\frac{T_1^2+T_2^2}{2}}$

3. In the setup shown, a block is placed on a frictionless floor, the chords and pulleys are ideal and spring constant of each spring is indicated in the figure shown. If applied force is increased gradually from zero to a value F the displacement of the block is



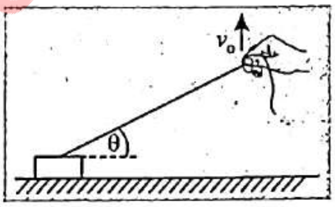
(a) $\frac{F}{k}$ (b) $\frac{2F}{3k}$ (c) $\frac{3F}{4k}$ (d) $\frac{3F}{2k}$

4. Two beetles are hanging from the lower end of a light elastic cord suspended from the ceiling. Stiffness of the cord is k, its relaxed length is l_0, mass of each beetle is m and acceleration due to gravity is g. One of the beetles starts climbing the cord with a constant velocity u relative to the ceiling while the other one keeps on hanging from the lower end. With what velocity will the lower beetle be shifted? Ignore all disturbances caused by the initial acceleration of the beetle that is climbing the rope.

(a) $\frac{mgu}{kl_0+2mg}$ (b) $\frac{mgu}{2kl_0+2mg}$

(c) $\frac{2mgu}{kl_0+2mg}$ (d) $\frac{2mgu}{2kl_0+mg}$

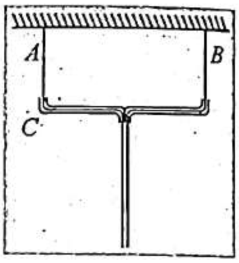
5. A light inextensible straight thread of length l is placed on a frictionless horizontal floor. A small block of mass m placed on the floor is attached at one end of the thread. With how much constant speed v_0 the free end of the thread must be lifted vertically upwards so that the block leaves the floor, when the thread makes an angle θ with the floor. Acceleration of free fall is $g = 10\text{ m/s}^2$.



(a) $\sqrt{\frac{gl}{\sin\theta}}\cos^2\theta$ (b) $\sqrt{\frac{gl}{\cos\theta}}\sin^2\theta$

(c) $\sqrt{\frac{gl}{\sin\theta}}\tan^2\theta$ (d) $\sqrt{\frac{gl}{\cos\theta}}\tan^2\theta$

6. Two identical ropes A and B of uniform mass per unit length are suspended from the ceiling and are made to pass through an almost inertia-less metallic tubular structure C as shown in the figure. The structure is made by welding two identical bent tubes. There is no friction between the ropes and the tubes. Initially the structure is held at rest and then released. The acceleration of the structure C is



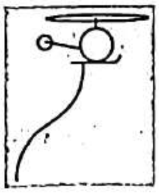
(a) 0 (b) $\frac{g}{4}$

(c) $\frac{g}{3}$ (d) $\frac{g}{2}$

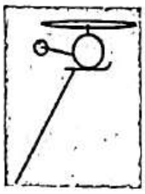
7. A helicopter is flying horizontally at constant speed. A perfectly flexible uniform cable is suspended beneath the helicopter; air friction on the cable is not negligible.

Which of the following diagrams best shows the shape of the cable as the helicopter flies through the air to the right?

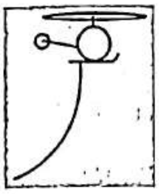
(*a*)



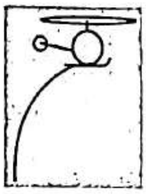
(*b*)



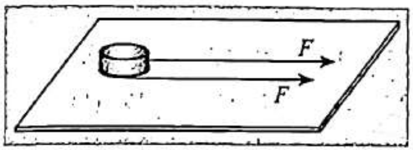
(*c*)



(*d*)

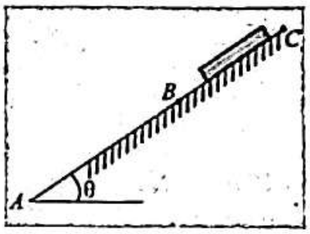


8. A uniform rope of mas m and length l is spread out on a horizontal frictionless surface, wrapping half turn around a fixed vertical frictionless cylinder of radius $r << l$. Initially, both the ends of the rope are pulled by equal forces each F in magnitude keeping both the unwrapped segments parallel to each other and equal in length as shown in the figure. Suddenly pulling force at one end is removed without any change in the pulling force at the other end. After how much time from removal of the pulling force at an end, will the rope start losing contact with the cylinder?



(*a*) $\sqrt{\dfrac{2ml}{3F}}$ (*b*) $\sqrt{\dfrac{ml}{3F}}$ (*c*) $\sqrt{\dfrac{3ml}{2F}}$ (*d*) $\sqrt{\dfrac{3ml}{F}}$

9. A uniform bar of mass m is released from an inclined plane of inclination θ. The upper part of the plane BC is rough part AB is smooth. The coefficient of static friction between the bar and upper part BC is μ.



The maximum tension in the bar while it is sliding down is

(*a*) μ mg cos θ

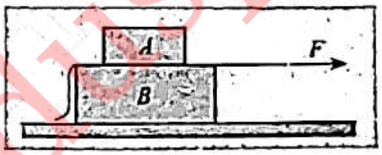
(*b*) 0.5 μ mg cos θ

(*c*) 0.25 μ mg cos θ

(*d*) 0.75 μ mg cos θ

10. On a horizontal frictionless floor, a block A of mass m rests on another stationary block B of mass M ($m < M$) with a thin paper sheet of negligible mass inserted between them. The paper sheet is wide enough to prevent direct contact between the blocks. Coefficient of friction between each block and the paper sheet is μ.

If the paper sheet slides relative to B but does not slide relative to A, then condition on F is



(*a*) $\mu mg\left[1+\dfrac{M}{m}\right] < F < 2\mu mg$

(*b*) $\mu mg\left[1+\dfrac{M}{2m}\right] < F < 2\mu mg$

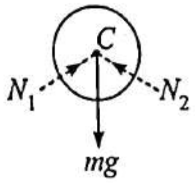
(*c*) $\mu mg\left[1+\dfrac{m}{2M}\right] < F < 2\mu mg$

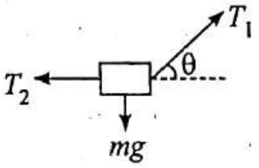
(*d*) $\mu mg\left[1+\dfrac{m}{M}\right] < F < 2\mu mg$

> *"Success demands singleness of purpose."*
>
> – **Vince Lombardi**

ANSWER KEY

CONCEPT APPLICATION

1. 

2. 

3. (*c*) 4. [240, 320, 400] 5. ($T_1 = 100$ N, $T_2 = 0$)

6. (*a*) 5, (*b*) 50, 25, 10, (*c*) 50, 25, 15, 10 7. (*a*) 15, (*b*) 125, 80, (*c*) 75, 45, 30

8. (*a*) 5, (*b*) 110, 80, (*c*) 50, 30, 20 9. (*a*) 6, (*b*) 0, (*c*) 18, 12 10. (*a*) 2, (*b*) 40, 16, (*c*) 40, 24, 16 11. $N > N'$ 12. (*b*)

13. $\dfrac{\cos\theta_2}{\cos\theta_1}$ 14. $a_1 + a_2 = 2a_3$ 15. (*c*) 16. (*c*) 17. (*b*) 18. (*a*) 19. (*c*) 20. (*b*) 21. $x = 1$

22. (*a*) 0, (*b*) 200 23. $F = 0$ 24. $(g + a)\sin\theta$ 25. (*a*) 26. (*a*) 27. (*b*) 28. (*a*)

29. (*b*) 30. (*b*) 31. (*b*) 32. (*a*) 33. (*c*) 34. (*d*) 35. (*a*) 36. (*a*) 37. (*a*, *c*) 38. (*b*, *c*, *d*)

SCHOOL LEVEL PROBLEMS

1. (*b*) 2. (*d*) 3. (*a*) 4. (*a*) 5. (*b*) 6. (*b*) 7. (*b*) 8. (*a*) 9. (*b*) 10. (*b*)

11. (*a*) 12. (*b*) 24. (i)-(*b*), (ii)-(*d*), (iii)-(*b*), (iv)-(*b*), (v)-(c) 25. (i)-(*b*), (ii)-(*b*), (iii)-(*a*), (iv)-(*d*), (v)-(*c*)

PRARAMBH (TOPICWISE)

1. (*c*) 2. (*c*) 3. (*b*) 4. (*c*) 5. (*b*) 6. (*b*) 7. (*c*) 8. (*a*) 9. (*d*) 10. (*c*)

11. (*a*) 12. (*a*) 13. (*a*) 14. (*a*) 15. (*a*) 16. (*a*) 17. (*b*) 18. (*b*) 19. (*b*) 20. (*c*)

21. (*d*) 22. (*c*) 23. (*a*) 24. (*b*) 25. (*a*) 26. (*c*) 27. (*a*) 28. (*a*) 29. (*a*) 30. (*c*)

31. (*b*) 32. (*b*) 33. (*a*) 34. (*b*) 35. (*b*) 36. (*a*) 37. (*c*) 38. (*a*) 39. (*c*) 40. (*b*)

41. (*d*) 42. (*a*) 43. (*a*) 44. (*b*) 45. (*c*) 46. (*d*) 47. (*b*) 48. (*b*) 49. (*d*) 50. (*b*)

51. (*d*) 52. (*b*) 53. (*d*) 54. (*a*) 55. (*a*) 56. (*b*) 57. (*c*) 58. (*d*) 59. (*a*) 60. (*c*)

61. (*a*) 62. (*a*) 63. (*d*) 64. (*c*) 65. (*a*) 66. (*a*) 67. (*d*) 68. (*b*) 69. (*c*) 70. (*a*)

71. (*d*) 72. (*b*) 73. (*a*) 74. (*d*) 75. (*c*) 76. (*b*) 77. (*a*) 78. (*a*) 79. (*c*) 80. (*a*)

81. (*d*) 82. (*a*) 83. (*d*) 84. (*c*) 85. (*c*) 86. (*c*) 87. (*d*) 88. (*b*) 89. (*b*) 90. (*d*)

91. (*d*) 92. (*a*) 93. (*a*) 94. (*a*) 95. (*a*) 96. (*b*) 97. (*a*) 98. (*b*) 99. (*a*) 100. (*d*)

101. (*c*) 102. (*c*) 103. (*b*) 104. (*d*)

PRABAL (JEE MAIN LEVEL)

1. (*c*) 2. (*c*) 3. (*b*) 4. (*b*) 5. (*d*) 6. (*c*) 7. (*b*) 8. (*b*) 9. (*a*) 10. (*c*)

11. (*c*) 12. (*c*) 13. (*b*) 14. (*c*) 15. (*c*) 16. (*a*) 17. (*b*) 18. (*d*) 19. (*c*) 20. (*b*)

21. (*b*) 22. (*b*) 23. (*b*) 24. (*b*) 25. (*b*) 26. (*c*) 27. (*a*) 28. (*a*) 29. (*a*) 30. (*b*)

31. (*b*) 32. (*c*) 33. (*a*) 34. (*d*) 35. (*d*) 36. (*c*) 37. (*c*) 38. (*b*) 39. (*a*) 40. (*c*)

41. (*a*) 42. (*a*) 43. (*b*) 44. (*a*) 45. (*c*) 46. (*d*) 47. (*c*) 48. (*a*) 49. (*b*) 50. (*c*)

51. (*d*) 52. (*d*) 53. (*b*) 54. [2] 55. [20] 56. [2] 57. [5] 58. [7] 59. [10] 60. [70]

61. [2] 62. [36] 63. [10]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (a, b, c) 2. (a, c) 3. (b, d) 4. (a, b, d) 5. (a, c) 6. (b, d) 7. (a,b,c,d) 8. (a, b) 9. (b, c, d) 10. (a, c)
11. (a, c, d) 12. (a,b,c,d) 13. (a, d) 14. (a, c) 15. (b) 16. (c) 17. (d) 18. (a) 19. (a) 20. (b)
21. (a) 22. (a) 23. (c) 24. (a) 25. (b) 26. (c) 27. [5] 28. [75] 29. [25] 30. [3]
31. [98] 32. [45] 33. [4] 34. (d) 35. (a) 36. (b) 37. (b) 38. (d) 39. (d) 40. (a)
41. (d) 42. (a) 43. (b) 44. (d) 45. (c) 46. (a) 47. (d) 48. (b) 49. (c) 50. (a)
51. (c) 52. (c) 53. (a)

PYQ's (PAST YEAR QUESTIONS)

1. (d) 2. (c) 3. (b) 4. (a) 5. (c) 6. (a) 7. (b) 8. (a) 9. (c) 10. (a)
11. (d) 12. (a) 13. (d) 14. (b) 15. (d) 16. (c) 17. (b) 18. (d) 19. (b) 20. (c)
21. (b) 22. [492] 23. (a, b, d) 24. (c) 25. (d) 26. (a) 27. (a) 28. [240] 29. (a) 30. (a)
31. (b) 32. (a) 33. (c) 34. (c) 35. [30] 36. (b) 37. (b) 38. (a) 39. (b) 40. [2]
41. [25] 42. (c) 43. (b) 44. (d) 45. (d) 46. (c) 47. (c) 48. (b) 49. (c) 50. (d)
51. (a) 52. (a) 53. (a) 54. (a) 55. (b) 56. (c) 57. [$\sqrt{3}$] 58. [346] 59. (a) 60. (b)
61. (c) 62. (b) 63. (b) 64. (b) 65. (c) 66. (c) 67. (d)

PW CHALLENGERS

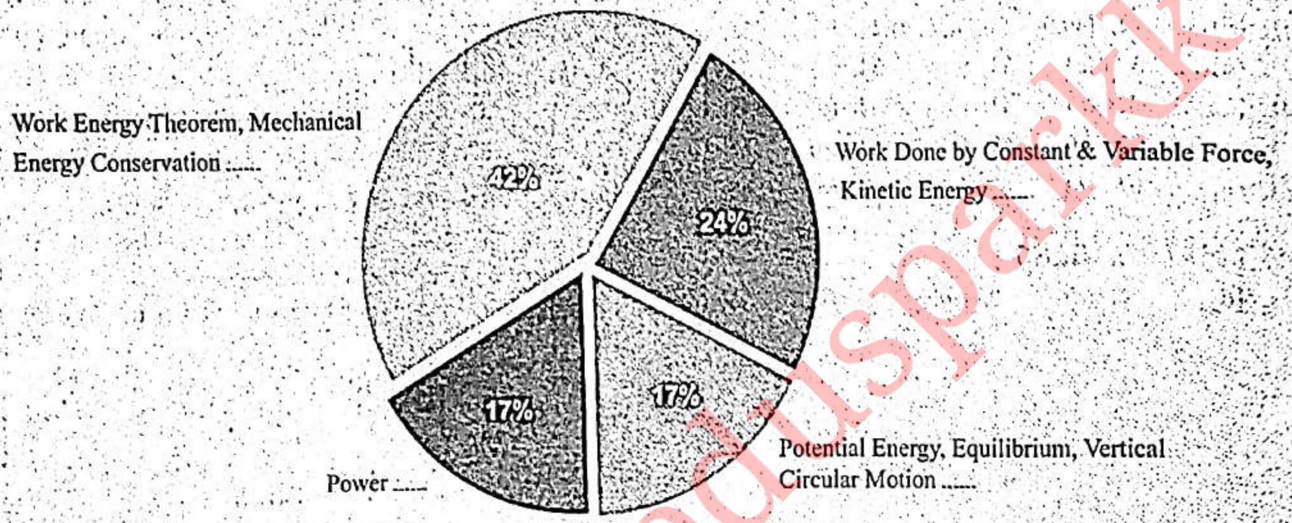
1. [6] 2. (d) 3. (a) 4. (a) 5. (a) 6. (d) 7. (b) 8. (b) 9. (c) 10. (d)

CHAPTER

6

Work, Energy & Power

Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

WORK

Work is said to be done by a force when the force is acting on a body and the body makes some displacement.

Work is the product of the applied force and the displacement of the body in the direction of force.

$W = F S \cos\theta = \vec{F}.\vec{S}$

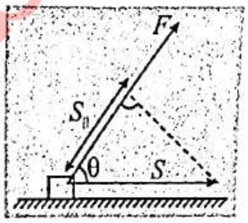
If $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{S} = x\hat{i} + y\hat{j} + z\hat{k}$

Then $W = \vec{F}.\vec{S} = F_x x + F_y y + F_z z$

Work done by a constant force may also be written as

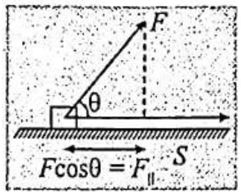
(i) $W = F(S\cos\theta) = FS_{\parallel}$

$= F \times$ component of displacement in the direction of force



(ii) $W = F\cos\theta S = F_{\parallel}S$

= Component of force along displacement × S



Notes:

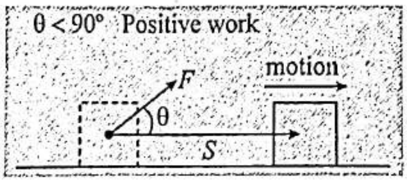
(i) **Dimension:** $M^1L^2T^{-2}$

(ii) **SI Unit:** joule, **C.G.S. Unit:** erg, 1 joule = 10^7 erg

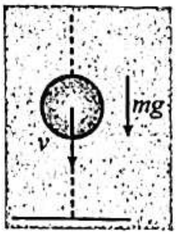
(iii) **Positive work:** $W = FS\cos\theta$

If the angle θ is acute (θ < 90°) then the work is said to be positive.

The positive work signifies that the external force favours the motion of the body.

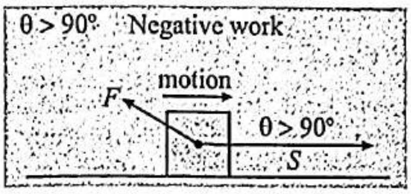


(*a*) When a body falls freely under the action of gravity (θ = 0°), the work done by gravity is positive.



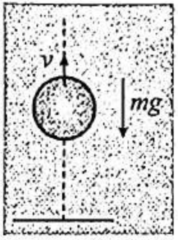
(*b*) When a spring is stretched, stretching force and the displacement both are in the same direction. So work done by stretching force is positive.

(iv) **Negative work:** If the angle θ is obtuse (θ > 90°). Then the work is said to be negative.



It signifies that the direction of force is such that it opposes the motion of the body.

(*a*) Work done by gravitational force is negative when a body moves upwards.

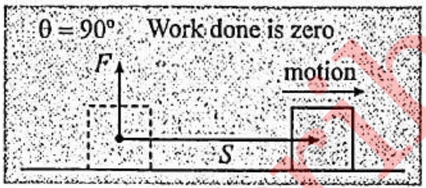


(*b*) Work done by air resistance on a moving car is negative.

(*c*) Work done by kinetic friction on a block moving on a fixed surface is negative.

(v) **Zero work:** $W = FS\cos\theta$

Work done will be zero if $F = 0$ or $S = 0$ or $\theta = 90°$



(*a*) Work done by normal reaction on a block moving on a fixed surface is zero.

(vi) **Work done by several forces:** When several forces act on a body then the net work done on the body is the algebraic sum of work done by individual forces.

$$W_{net} = \vec{F}_1.\vec{S}_1 + \vec{F}_2.\vec{S}_2 + ...\vec{F}_n.\vec{S}_n$$
$$= W_1 + W_2 + ... + W_n$$

here $\vec{S}_1, \vec{S}_2, ...\vec{S}_n$ are the displacements of points of application of forces $\vec{F}_1, \vec{F}_2, ...\vec{F}_n$ respectively.

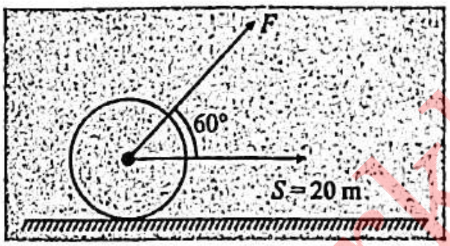
Note: Displacement depends on reference frame so work done by a force is reference frame dependent. Work done by a force is not same in different reference frames.

(vii) Work is a scalar quantity.

Train Your Brain

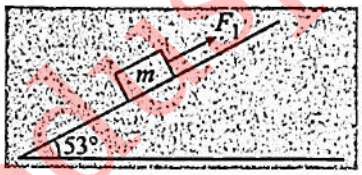
Example 1: A gardener pulls a lawn roller along the ground through a distance of 20 m. If he applies a force of 20 kg weight in a direction inclined at 60° to the ground, find the work done by him. (Take $g = 10\ \text{m/s}^2$)

Sol. $W_F = FS\cos 60° = 200 \times 20 \times \frac{1}{2}\ \text{J} = 2000\ \text{J}$



Example 2: A block of mass 20 kg is slowly slid up on a smooth incline of inclination 53° by a person. Calculate the work done by the person in moving the block through a distance of 4 m, if the driving force is (*a*) parallel to the incline (*b*) in the horizontal direction. $[g = 10\ \text{m/s}^2]$

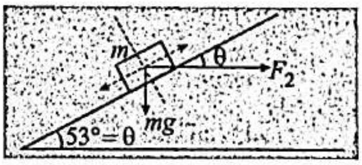
Sol.



$F_1 = mg\sin 53°$ $\quad (\because a = 0)$

$W_1 = (mg\sin 53°).(4)$

$= (20 \times 10 \times \frac{4}{5})(4) = 640\ \text{J}$

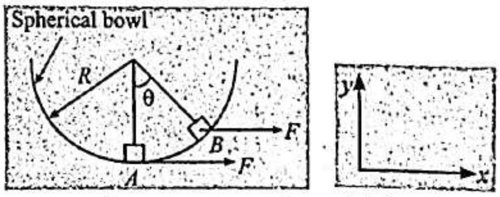


$W_{F_2} = F_2\cos\theta \times 4$

But $F_2\cos\theta = mg\sin\theta$ $\quad (\because a = 0)$

$W_2 = W_{F_2} = (mg\sin\theta) \times 4 = 640\ \text{J}$

Example 3: A block of mass *m* is taken from *A* to *B* by applying a constant horizontal force, *F* along spherical bowl as shown in figure. Find the work done by gravity and applied force *F*.



Sol. $S_x = R\sin\theta$

$S_y = R - R\cos\theta = R(1-\cos\theta)$

Work Done by gravity $= -mgS_y = -mgR(1-\cos\theta)$

Work Done by force $= FS_x = FR(\sin\theta)$

Example 4: A particle is shifted form point (0 m, 0 m, 1 m) to (1 m, 1 m, 2 m), under two forces. The forces are $\overline{F}_1 = (2\hat{i}+3\hat{j}-\hat{k})N$ and $\overline{F}_2 = (\hat{i}-2\hat{j}+2\hat{k})N$. Find the total work done by the two forces.

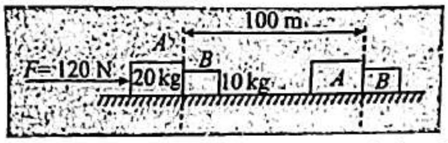
Sol. Work done by a constant force equals to dot product of the force and displacement vector.

$$W = \overline{F}_{net}\cdot\Delta\bar{r} \Rightarrow W = \left(\overline{F}_1+\overline{F}_2\right)\cdot\Delta\bar{r}$$

Substituting given values, we have

$$W = (3\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k}) = 3+1+1 = 5\,\text{J}$$

Example 5: A force $F = 120$ N acts on block A as shown in figure. Block A and B are in contact and are constrained to move on a horizontal surface.



(i) Find work done by force F on A during 100 m displacement.

(ii) Find work done by force F on B during 100 m displacement.

(iii) Find work done by normal reaction on B and A during the given displacement.

Sol. (i) $(W_F)_{\text{on }A} = F\Delta S\cos\theta$

$= 120 \times 100 \times \cos 0° = 12000$ J

(ii) $(W_F)_{\text{on }B} = 0$

$\because F$ does not act on B

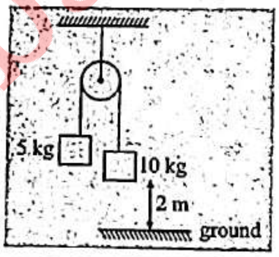
(iii) $R = \dfrac{m_A F}{m_A + m_B} = 40N$

$(W_R)_{\text{on }B} = 40\times100\times\cos0^\circ = 4000$ J

$(W_R)_{\text{on }A} = 40\times100\times\cos180^\circ = -4000$ J

Total W.D. by action-reaction pair of normal reaction on system of A and B is zero. i.e. w.d. by internal reaction on a rigid system is zero.

Example 6: In the figure shown below, system is released from rest. When 10 kg block reaches ground then find



(i) Work done by gravity on 10 kg.

(ii) Work done by gravity on 5 kg.

(iii) Work done by tension on 10 kg.

(iv) Work done by tension on 5 kg.

Sol. (i) $(W_g)_{10\text{ kg}} = 10\,g\times2 = 200$ J

(ii) $(W_g)_{5\text{kg}} = 5\,g\times2\times\cos180^\circ = -100$J

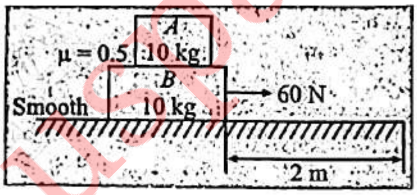
(iii) $T = \dfrac{2m_1m_2}{m_1+m_2}g = \dfrac{200}{3}$N

$(W_T)_{10\text{kg}} = \dfrac{200}{3}\times2\times\cos180^\circ = \dfrac{-400}{3}$ J

(iv) $(W_T)_{5\text{ kg}} = \dfrac{200}{3}\times2\times\cos0^\circ = \dfrac{400}{3}$ J

Net work done by tension is zero. Work done by internal tension i.e. (tension acting within system) on the system is always zero if the length of string remains constant.

Example 7: In the given figure



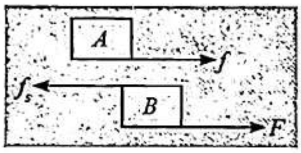
(i) Find work done by applied force during the displacement of 2 m.

(ii) Find work done by frictional force on B by A during the displacement.

(iii) Find work done by friction force on A by B during the displacement.

Sol. (i) $W_F = 60 \times 2 \times \cos 0° = 120$ J

(ii) $f_{s\max} = \mu mg = 0.5\times10\,g = 50$ N



Assuming they move together.

$60 = 20a \Rightarrow a = 3$ m/s^2

$f_s = 10 \times 3 = 30$ N

As, $f < f_{s\max}$

$\therefore$ They move together (A does not slip an B)

hence $(W_f)_{\text{on }B} = -30 \times 2 = -60$ J

$(W_f)_{\text{on }A} = 30 \times 2 = 60$ J

Note that total work done by internal static friction is zero.

(i) Work done by action reaction pair of constraint forces such as static friction, normal reaction and tension in the string is always zero when they are internal forces.

(ii) When parts of a body undergo different displacements then work done by the force is given by force × (displacement of point of application of the force), for example when a man moves up on a stair case, work done by normal reaction on him is zero because his foot in contact with the staircase does not move, though normal reaction is acting on it.

Concept Application

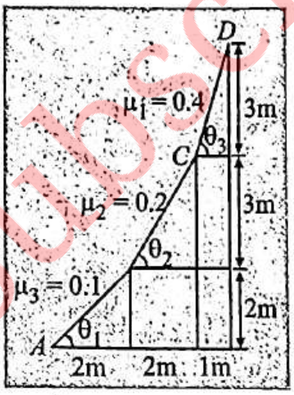
1. A small block of mass m is kept on a rough inclined surface of inclination θ fixed in an elevator. The elevator goes down with a uniform velocity v and the block does not slide on the wedge. The work done by the force of friction on the block with respect to ground in time t will be
   (a) zero
   (b) $-mgvt\cos^2\theta$
   (c) $-mgvt\sin^2\theta$
   (d) $mgvt\sin 2\theta$

2. Calculate the work done against gravity by a coolie in carrying a load of mass 10 kg on his head when he moves uniformly a distance of 5 m in the (i) horizontal direction (ii) upwards vertical direction.
   (Take $g = 10\ \text{m/s}^2$)

3. A body is constrained to move in the y-direction. It is subjected to a force $F = (-2\hat{i} + 15\hat{j} + 6\hat{k})$ Newton.
   What is the work done by this force in moving the body through a distance of 10 m in positive y direction?

4. A block of mass 500 g slides down on a rough inclined plane of inclination 53° with a uniform speed. Find the work done against the friction as the block slides through 2 m. [$g = 10\ \text{m/s}^2$]

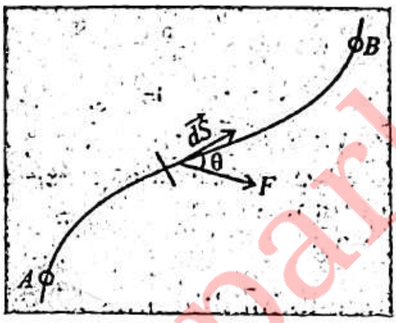
5. As shown in figure a body of mass 1 kg is shifted from A to D slowly on inclined planes by applying a force parallel to the planes, such that the block is always in contact with the plane surfaces. Neglecting the jerk experienced at points C and B, total work done by the force is
   [Take $g = 10\ \text{m/s}^2$]



   (a) 90 J
   (b) 56 J
   (c) 180 J
   (d) 0 J

6. A block of mass m is pulled on a rough horizontal surface which has a friction coefficient μ. A horizontal force F is applied which is capable of moving the body uniformly with speed v. Find the work done on the block in time t by
   (a) Weight of the block
   (b) Normal reaction by surface on the block
   (c) Friction
   (d) F

WORK DONE BY VARIABLE FORCE



When the magnitude and/or direction of a force varies with position, the work done by such a force for an infinitesimal displacement dS is given by $dW = \vec{F}.\vec{dS}$

The total work done in going from A to B is

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{S} = \int_A^B (F \cdot dS \cos\theta)$$

In terms of rectangular components

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

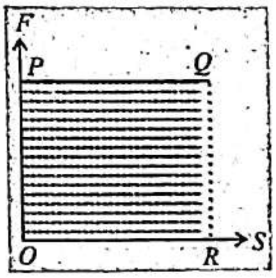
$$W = \int_A^B F_x dx + \int_A^B F_y dy + \int_A^B F_z dz$$

In one dimension $W = \int_A^B F_x dx$

GRAPHICAL INTERPRETATION OF WORK DONE

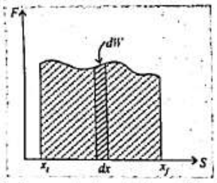
Area under F–S graph gives work. The area enclosed by the graph on displacement axis gives the amount of work done by the force.

Work done by constant force.



Work = FS = Area of $OPQR$

Work done by variable force in one dimension

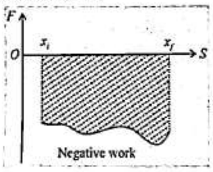


For a small displacement, dx the work done will be the area of the strip of width dx

$$W=\int_{x_i}^{x_f} dW=\int_{x_i}^{x_f} F\,dx$$

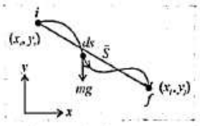
In this case work done is positive.

If area lies above X-axis work done is +ve while if the area lies below X-axis work done is –ve.



Important Cases:

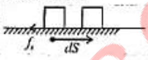
(*a*) **Work done by gravity on a body near the surface of earth:** Consider an object which moves from initial point i to final point f under the influence of gravity, mg, as shown below.



$$W=\int\vec{F}\cdot d\vec{S}=\int m\vec{g}\cdot d\vec{S}$$
$$=m\vec{g}\cdot\int d\vec{S}=m\vec{g}\cdot\vec{S}=m\vec{g}\cdot\left(\vec{r_f}-\vec{r_i}\right)$$
$$=mg\left(-\hat{j}\right)\left[\left(x_f-x_i\right)\hat{i}+\left(y_f-y_i\right)\hat{j}\right]$$
$$=-mg\,(y_f-y_i)=-mg\,(h_f-h_i)=-mg\,(\Delta h)$$
$$=-mg\times\text{change in height}$$

Work done by gravity is positive if height decreases and negative when height increases.

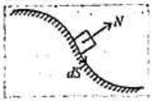
(*b*) **Work done by kinetic friction on a block moving on a fixed surface:-**



$$W_f=\int\vec{F}\cdot d\vec{S}=\int -f_k\,dS$$
$$=-f_k\int dS \qquad [\text{If } f_k \text{ is constant}]$$
$$=-f_k\times\text{distance travelled}$$
$$=-\mu_k N\times\text{distance travelled}$$

(*c*) **Work done by normal reaction on a block moving on a fixed surface:-**

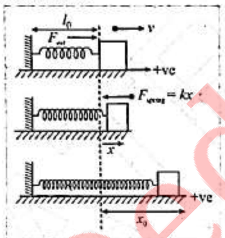
Consider a block which slides on a fixed surface.



$$W_N=\int\vec{F}\cdot d\vec{S}=\int\vec{N}\cdot d\vec{S}=0 \quad [\vec{N}\perp d\vec{S}]$$

(*d*) **Work done by a spring:**

Consider a block attached to a spring of natural length l_0 and spring stiffness k.



When spring is stretched by amount x, it applies a force kx towards left

$\therefore\ dW=-kx\,dx$

$$W=\int dW=-\int_0^{x_0} kx\,dx=-\frac{1}{2}kx_0^2$$

In general work done by the spring is given by

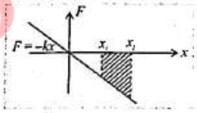
$$W=-\frac{1}{2}k(x_f^2-x_i^2)$$

where x_i and x_f are initial and final deformations of the spring.

$x_i=l_i-l_0$

$x_f=l_f-l_0$

We can also complete the work done by spring graphically as shown below



$$W=-\frac{1}{2}\left(kx_i+kx_f\right)\left(x_f-x_i\right)$$
$$=-\frac{1}{2}k\left(x_f^2-x_i^2\right)$$

Train Your Brain

Example 8: A force $\vec{F}=\left(3t\hat{i}+5\hat{j}\right)$N acts on a body due to which its position varies as $\vec{S}=\left(2t^2\hat{i}-5\hat{j}\right)$. Work done by this force in first two seconds is:

(*a*) 23 J
(*b*) 32 J
(*c*) zero
(*d*) can't be obtained

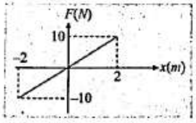
Sol. (*b*) $\dfrac{d\vec{S}}{dt}=4t\hat{i}$

$d\vec{S}=4t\,dt\,\hat{i}$

$$W=\int\vec{F}\cdot\vec{ds}$$
$$=\int(3t\hat{i}+5\hat{j})\cdot(4t\,dt\,\hat{i})$$
$$=\int_0^2 12t^2\,dt=\frac{12\left[t^3\right]_0^2}{3}=32\text{ J}$$

Example 9: A force F acting an a particle varies with the position x as shown in figure. Find the total work done by this force in displacing the particle from

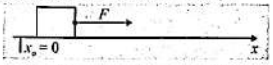
(*a*) $x=-2$ m to $x=0$
(*b*) $x=0$ to $x=2$ m

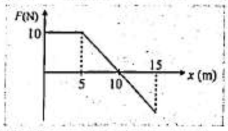


Sol. (*a*) $W=-\frac{1}{2}(2)(10)=-10$ J

(*b*) $W=\frac{1}{2}(2)(10)=10$ J

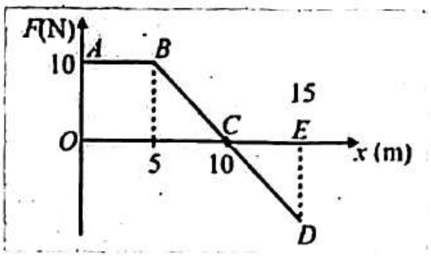
Example 10: A horizontal force F is used to pull a box placed on floor. Variation in the force with position coordinate x measured along the floor is shown in the graph.





(*a*) Calculate work done by the force in moving the box from $x=0$ m to $x=10$ m.
(*b*) Calculate work done by the force in moving the box from $x=10$ m to $x=15$ m.
(*c*) Calculate work done by the force in moving the box from $x=0$ m to $x=15$ m.

Sol. In rectilinear motion work done by a force equals to area under the force-position graph.



(a) $W_{0\to10}$ = Area of trapazium $OABC$ = 75 J

(b) $W_{10\to15}$ = –Area of triangle CDE = –25 J

(c) $W_{0\to15}$ = Area of trapazium $OABC$ – Area of triangle CDE = 50 J

Example 11: A spring of force constant 100 N/m is slowly stretched upto 5 cm by applying a variable force F. Find out work done by the force.

Sol. Spring is stretched by length $l = 5$ cm = 0.05 m

Work done for small displacement dx is

$dW = Fdx = kx\,dx$

Total work done

$$W = \int_0^l F\,dx = \int_0^l kx\,dx = \frac{1}{2}kl^2 = \frac{1}{2}100\,(0.05)^2 = 0.125\text{ J}$$

Example 12: Find the work done by $\bar{F} = (xy\hat{i} + y\hat{j})N$ on particle, when it moves from origin to point (1, 1) along the path $y = x^2$, where x and y are in meters.

Sol. $W = \int F_x dx + \int F_y dy$

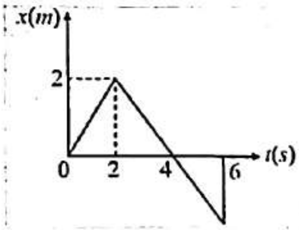
$W = \int xy\,dx + \int y\,dy$

$$\Rightarrow y = x^2 \Rightarrow \frac{dy}{dx} = 2x \text{ or } dy = 2x\,dx$$

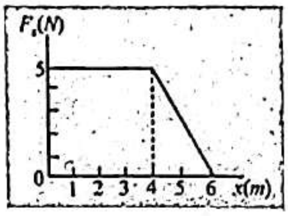
$$\Rightarrow W = \int_0^1 x^3 dx + \int_0^1 x^2\, 2x\,dx = \left.\frac{x^4}{4}\right|_0^1 + \left.\frac{2x^4}{4}\right|_0^1 = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}\text{ J}$$

Concept Application

7. A position dependent force $F = 7 - 2x + 3x^2$ acts on a small body of mass 2 kg and displaces it from $x = 0$ to $x = 5$ m. Calculate the work done in joule.
8. Position-time graph of a particle of mass 2 kg is shown in figure. Total work done on the particle from $t = 0$ to $t = 4$s is



9. Force acting on a particle varies with x as shown in figure. Calculate the work done by the force as the particle moves from $x = 0$ to $x = 6.0$ m.

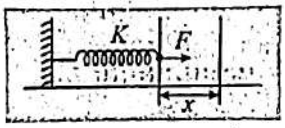


10. An object is displaced from point $A(1, 2)$ to $B(0, 1)$ by applying force $\vec{F} = x\hat{i} + 2y\hat{j}$.

Find out work done by $\vec{F}$ to move the object from point A to B.

11. A spring, which is initially in its unstretched condition, is first stretched by a length x and then again by a further length x. The work done in the first case is W_1 and in the second case is W_2. Find $\frac{W_2}{W_1}$.

12. Initially spring is relaxed. A person starts pulling the spring by applying a variable force F.



(a) Find out the work done by F to stretch the spring slowly by x.

(b) Work done by spring on wall is ________.

(c) Work done by spring force on man is ________.

Kinetic Energy

The energy associated with a particle due to its motion is called kinetic energy of the particle, $K = \frac{1}{2}mv^2$

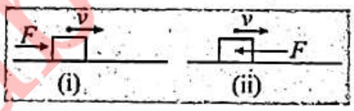
It is a scalar quantity. It is equal to work done by net external force in increasing its velocity from zero to v as is shown in next section.

Note: Momentum of a particle (P) is related to its kinetic energy (K) as,

$P = \sqrt{2mK}$

WORK ENERGY THEOREM

When a force is applied on a particle in the direction of its velocity its speed increases and work done on it is positive.



On the other hand, when force is applied opposite to its direction of motion, its speed decreases and negative work is done on it.

From this we can infer that kinetic energy increases when positive work is done on the particle and decreases when negative work is done on it.

We know, $\vec{F}_{net} = m\vec{a}$

$$W = \int \vec{F}_{net} \cdot d\vec{S} = \int m\vec{a} \cdot d\vec{S}$$

$$= \int m\frac{d\vec{v}}{dt} \cdot \vec{v}dt = \int m\vec{v} \cdot d\vec{v} = m\int_i^f (v_x dv_x + v_y dv_y + v_z dv_z)$$

$$= \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2)\Big|_{v_i}^{v_f} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= K_f - K_i = \text{change in kinetic energy}$$

Therefore, change in kinetic energy of a particle is equal to total work done. This is called work-energy theorem.

Train Your Brain

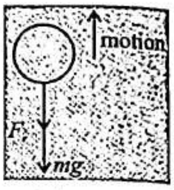
Example 13: Under the action of a force, the velocity of a body changes from $(3\hat{i} + 4\hat{j})$m/s to $(-12\hat{i} + 5\hat{j})$m/s. If mass of the body is 2 kg, find the total work done on it.

Sol. $K_i = \frac{1}{2} \times 2 \times |3\hat{i} + 4\hat{j}|^2 = 25$ J

$K_f = \frac{1}{2} \times 2 \times |-12\hat{i} + 5\hat{j}|^2 = 169$ J

Work done $= K_f - K_i = 169 - 25 = 144$ J

Example 14: A ball of mass $m = 0.4$ kg is thrown with a velocity of 20 m/s in vertically upward direction. It the ball rises to a maximum height of 19 m, find the work done by the force of air resistance that acts on it ($g = 10$ m/s^2).



Sol. $v_i = 20$ m/s

$v_f = 0$ (at maximum height)

$\Delta K = K_f - K_i = -\frac{1}{2} \times 0.4 \times 20^2 = -80$ J

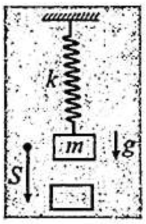
$W_{gravity} = -mg \times 19 = -76$ J

$W_{gravity} + W_R = \Delta K$

$-76 + W_R = -80$

$W_R = -4$ J

Example 15: A block of mass m is attached to a spring of spring constant k. Spring is initially relaxed. If the block is now released, find the maximum vertical displacement of the block?



Sol. At the lowest position, the velocity of block will be zero.

$K_i = 0$+

$K_f = 0$

$$W_{\text{gravity}} = mgS$$

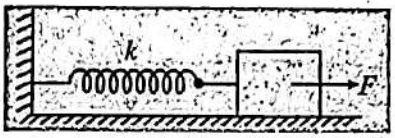
$$W_{\text{spring}} = -\frac{1}{2}kS^2$$

$$W_{\text{gravity}} + W_{\text{spring}} = K_f - K_i$$

$$mgS - \frac{1}{2}kS^2 = 0$$

$$S = \frac{2mg}{k}$$

Example 16: A block attached to a spring of spring constant k kept on a smooth surface, is pulled by a constant horizontal force as shown in the figure. Initially, the spring is in the natural state. Then the maximum positive work that the applied force F can do is [Given that spring does not break]

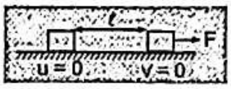


(*a*) $\frac{F^2}{k}$ (*b*) $\frac{2F^2}{k}$

(*c*) ∞ (*d*) $\frac{F^2}{2k}$

Sol. (*b*) Force will continue to do positive work till the velocity of the block becomes zero.

Applying work energy theorem on block

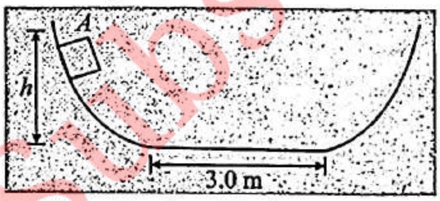


$$W_F + W_S = 0$$

$$F\ell - \frac{1}{2}k\ell^2 = 0$$

$$\therefore \ell = \frac{2F}{k} \quad \text{or work done} = F\ell = \frac{2F^2}{k}$$

Example 17: A small particle slides along a track with elevated ends and a flat central part, as shown in figure. The flat part has a length of 3 m. The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is μ = 0.2. The particle is released at point *A*, which is at a height *h* = 1.5 m above the flat part of the track. The position where the particle finally comes to rest is



(*a*) left to mid point of the flat part

(*b*) right to the mid point of the flat part

(*c*) the mid point of the flat part

(*d*) none of these

Sol. (**c**) $W_G - W_f = 0$

$$mgh = \mu mg\ell$$

$$h = \mu\ell$$

$$h = (0.2)\ell$$

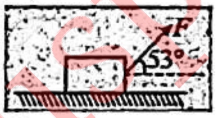
$$\Rightarrow \ell = \frac{1.5}{0.2}$$

$$\ell = 7.5\text{ m} = (3 + 3 + 1.5)\text{m}$$

So, the particle travels the flat part completely two times and then comes to rest at the mid point (1.5 m).

Concept Application

13. A block of mass m = 4 kg is dragged 2 m along a horizontal surface by a force F = 30 N acting at 53° to the horizontal. The initial speed is 3 m/s and μ_k = 1/8.



(*a*) Find the change in kinetic energy of the block.

(*b*) Find its final speed.

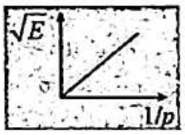
14. An open knife edge of mass '*m*' is dropped from a height '*h*' on a wooden floor. If the knife penetrates upto depth '*d*' into the wood, the average resistance offered by the wood to the knife edge is

(*a*) mg (*b*) $mg\left(1-\frac{h}{d}\right)$

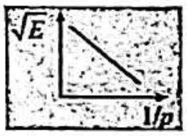
(*c*) $mg\left(1+\frac{h}{d}\right)$ (*d*) $mg\left(1+\frac{h}{d}\right)^2$

15. The graph between $\sqrt{E}$ and $\frac{1}{p}$ is (E = kinetic energy and p = momentum)

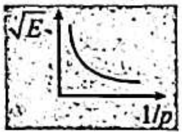
(*a*)



(*b*)



(*c*)



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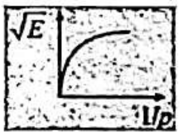

16. The displacement x of a body of mass 1 kg on horizontal smooth surface as a function of time t is given by $x = \frac{t^3}{3}$. Find the work done by the external agent on the body during first second.

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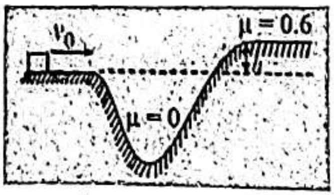
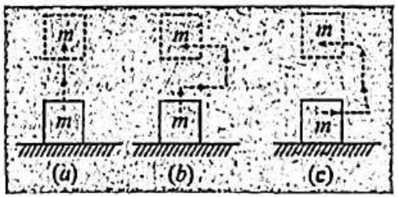
17. In the figure a block slides along a track from one level to a higher level, by moving through an intermediate valley. The track is frictionless until the block reaches the higher level. There a frictional force which stops the block in a distance d. The block's initial speed v_0 is 6m/s, the height difference h is 1.1 m and the coefficient of kinetic friction μ is 0.6. The value of d is:



(a) 1.17 m
(b) 1.71 m
(c) 7.11 m
(d) 11.7 m

CONSERVATIVE FORCE

A force is said to be conservative if work done by the force in moving a body depends only on the initial and final positions of the body and does not depend on the nature of path followed between the initial and final positions.



Consider a body of mass m being raised to a height h vertically upwards as shown in figure (a). The work done is mgh. Suppose we take the body along the path as in (b). The work done during horizontal motion is zero. Adding up the works done in the two vertical parts of the path, we get the result mgh once again. Any arbitrary path like the one shown in (c) can be broken into elementary horizontal and vertical portions. Work done along the horizontal path is zero. The work done along the vertical parts add up to mgh. Thus we conclude that the work done in raising a body against gravity is independent of the path taken. It only depends upon the initial and final positions of the body. We conclude from this discussion that the force of gravity is a conservative force.

Examples of Conservative Forces

(i) Gravitational force is a conservative force.

(ii) Elastic force in a stretched or compressed spring is a conservative force.

(iii) Electrostatic force between two electric charges is a conservative force.

Note:-Force which acts along the line joining the particles and depends on only distance between them is called a central force. Gravitational force and electrostatic force are two important examples of central forces. All central forces are conservative forces.

Properties of Conservative Forces

- Work done by a conservative force depends only on the initial and the final positions of the body.
- Work done by a conservative force does not depend upon the nature of the path between initial and final positions of the body.
- Work done by a conservative force in a round trip is zero.
- The concept of potential energy exists only in the case of conservative forces.
- The work done by a conservative force is completely recoverable.
- Complete recoverability is an important aspect of the work done by a conservative force.
- In general, a force is conservative if $\frac{\partial F_y}{\partial x} = \frac{\partial F_x}{\partial y}, \frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y}$ and $\frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x}$

NON-CONSERVATIVE FORCES

A force is said to be non-conservative if work done by the force in moving a body depends upon the path between the initial and final positions.

The frictional force is non-conservative force. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend only on the initial and final positions. Note that the work done by frictional force in a round trip is not zero.

The velocity-dependent forces such as air resistance, viscous force etc., are non-conservative forces.

Difference between Conservative and Non-conservative Forces

S.No.	Conservative forces	Non-conservative forces
1.	Work done does not depend upon path taken.	Work done depends on path taken.
2.	Work done in a round trip is zero.	Work done in a round trip is not zero.
3.	All central forces are conservative.	These forces are generally velocity-dependent and retarding in nature.
4.	When only a conservative force acts within a system, the kinetic energy and potential energy can change. However their sum, the mechanical energy of the system, does not change.	Work done against a non-conservative force may be dissipated as heat energy.
5.	Work done is completely recoverable.	Work done is not completely recoverable.

ENERGY

A body is said to possess energy if it has the capacity to do work. When a body possessing energy does some work, part of its energy is used up. Conversely if some work is done upon an object, the object is given some energy.

There are various forms of energy. Heat, electricity, light, sound and chemical energy are all familiar forms. In studying mechanics, we are however concerned chiefly with mechanical energy. This type of energy is a property of configuration or position.

1. **Kinetic Energy:** Kinetic energy (K.E.), is the capacity of a body to do work by virtue of its motion.

   If a body of mass m has velocity v, its kinetic energy is equivalent to the work which an external force would have to do to bring the body from rest up to velocity v.

   The numerical value of the kinetic energy can be calculated from the formula, K.E. $= \frac{1}{2}mv^2 = \frac{m\bar{v}.\bar{v}}{2}$

   Since both m and v^2 are always positive, K.E. is always positive and does not depend upon the direction of motion of the body.

2. **Potential Energy:** Potential energy is energy of the body by virtue of its position. A body is capable to do work by virtue of its position, configuration or state of strain.

   Relation between potential energy and work done by a conservative force is given by

   $W_C = -\Delta U$, where ΔU is change in potential energy

   There are two common forms of potential energy, gravitational and elastic.

Important points related to potential energy

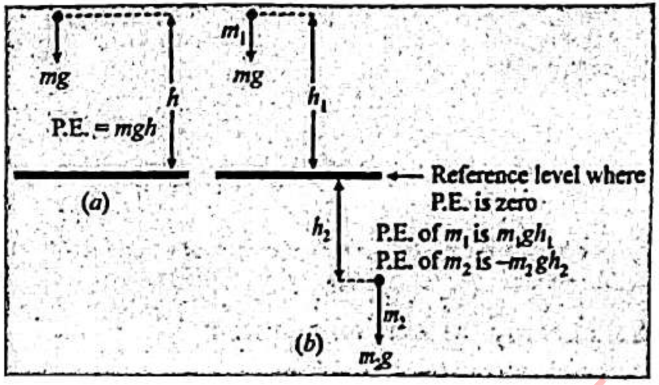
1. Potential energy is a state function (defined only for position).
2. Potential energy difference between two positions does not depend on the frame of reference.
3. Potential energy is defined only for conservative force because work done by conservative force is path independent.
4. Concept of potential energy does not exist for non-conservative forces as difference in potential energy between two states will then depend on path taken which does not make any sense.

Types of Potential Energy

(a) **Gravitational Potential Energy (GPE):** It is possessed by virtue of height.

When an object is allowed to fall from one level to a lower level it gains speed due to gravitational pull, i.e., it gains kinetic energy. Therefore, in possessing height, a body has the ability to convert its gravitational potential energy into kinetic energy. The gravitational potential energy is equal to the negative of the amount of work done by the weight of the body when body moves from reference level to the given point. If a mass m is at a height h above reference level the P.E. possessed by the mass is $(mg)(h)$. Since h is the height of an object above a specified level, an object below the specified level has negative potential energy.

Therefore GPE $= \pm mgh$


mg
P.E. = mgh
h
(a)
m_1
mg
h_1
Reference level where P.E. is zero
h_2
P.E. of m_1 is m_1gh_1
P.E. of m_2 is $-m_2gh_2$
m_2
(b)
m_2g

- The chosen level from which height is measured has no absolute position. It is important therefore to indicate clearly the zero P.E. level in any problem in which P.E. is to be calculated.
- Though we refer to it as P.E. of the object, it is actually the P.E. of earth-object system.
- GPE $= \pm mgh$ is applicable only when h is very small in comparison to the radius of the earth. We will discuss GPE in detail in the chapter of 'GRAVITATION'.

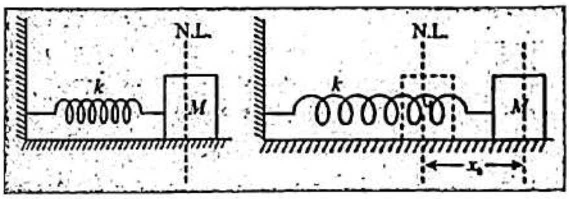
(b) **Elastic Potential Energy:** It is a property of stretched or compressed spring.

The free end of a stretched elastic spring will begin to move if it is released. The spring therefore possesses potential energy due to its elasticity (i.e., due to change in its configuration).

The amount of elastic potential energy stored in a spring of natural length l and spring constant k when it is extended by a length x (from the natural length) is equal to the amount of work necessary to produce the extension against the force exerted by the spring.

Elastic Potential Energy $= \frac{1}{2}kx^2$

It is never negative whether the spring is extended or compressed.



Consider a spring block system as shown in the figure and let us calculate work done by spring when the block is displaced by x_0 from the natural length.

At any moment if the elongation in spring is x, then the force on the block by the spring is kx towards left. Therefore, the work done by the spring when block further displaces by dx

$dW = -kx\,dx$

∴ Total work done by the spring,

$$W = -\int_0^{x_0} kx\,dx = -\frac{1}{2}kx_0^2$$

Similarly, work done by the spring when it is given a compression x_0 is $-\frac{1}{2}kx_0^2$.

$$U_f - U_i = \Delta U = -W_{\text{spring}} = \frac{1}{2}kx_0^2$$

$$\Rightarrow U = \frac{1}{2}kx_0^2$$

We have assumed zero potential energy at natural length of the spring.

Work Energy Theorem for a System

From work energy theorem, we know

$$W_{\text{total}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Now, if we consider a system of particles,

$$W_{\text{total}} = \Delta K_1 + \Delta K_2 + \ldots.. + \Delta K_n = \Delta K_{\text{system}}$$

If we split the forces into internal and external forces, we get

$$W_{\text{ext}} + W_{\text{int}} = \Delta K$$

$$\text{or } W_{\text{ext}} + W_{\text{int-}C} + W_{\text{int-}NC} = \Delta K$$

But $\Delta U = -W_{\text{int-}C}$

Therefore, $\Delta K + \Delta U = W_{\text{ext}} + W_{\text{int-}NC}$

This is work energy theorem for a system of particles.

For an isolated system $W_{\text{ext}} = 0$.

If internal non-conservative forces are also absent

$$\Delta K + \Delta U = 0$$

$$\Delta(K + U) = 0$$

$$K + U = \text{constant}$$

$$K + U = \text{Mechanical energy} = \text{constant.}$$

The principle of conservation of energy may be stated as: If all forces exerted by objects in a system are conservative and no work is done on the system by an outside agent, the mechanical energy (sum of kinetic and potential energy) does not change.

Note: While solving problems involving conservation of mechanical energy, one should note the following key points:

1. One should identify the system for which mechanical energy is conserved. That means one should be able to draw a closed surface such that what ever is inside the surface is the system and whatever is outside is the environment.
2. If friction or viscous forces (non-conservative forces) are present, mechanical energy will not be conserved.
3. The system under consideration should be isolated. The principle of mechanical energy conservation holds only for an isolated system. If external forces are present then mechanical energy will not be conserved.
4. The system changes from some initial state to some final state. One must be clear what these states are. Applying energy conservation, one should then set

$$(ME)_{\text{initial}} = (ME)_{\text{final}} \text{ or } (K_f - K_i) + (U_f - U_i) = 0$$

External Work: Let us consider a system of objects that can interact with each other by conservative forces and with other particles (objects) outside the system. Its mechanical energy will not be conserved if external forces do work on the system. Suppose, for instance, a box is lifted by a person from the ground. The force applied by the person on the box is external if the system consist of earth and the box. The work done by this force increases the mechanical energy of earth - box system. If the velocity of the box does not change, the increase in energy appears as potential energy; the box being further away from the earth. If the box's speed also increases, the increase in energy is both potential and kinetic.

The energy equation can be written as

$$\Delta K + \Delta U = W_{\text{ext}}$$

where U arises from the interaction of the objects in the system with each other and K is the kinetic energy of the system.

Friction and Thermal Energy

Let us now suppose that in addition to conservative forces, a single non-conservative force due to friction acts between the bodies of an isolated system. Then

$$\Delta K + \Delta U = W_f$$

This equation tells us that if friction force acts, the total mechanical energy is not constant, but changes by an amount equal to work done by frictional force. We can rewrite the above equation as

$$\Delta ME = ME_f - ME_i = W_f$$

But frictional force is a dissipative force. The total work done by kinetic frictional force is always negative. Therefore, presence of friction diminishes the mechanical energy of the system. The lost mechanical energy in this case appears as thermal energy (heat) and is denoted by E_{th}. This is the energy associated with random motion of individual molecules, as distinct from the motion of the macroscopic body as a whole. Just as the work done by a conservative force between bodies is the negative of the potential energy gain, the work done by frictional force is negative of the thermal (heat) energy gain. Thus

$$\Delta E_{th} = -W_f \text{ and } \Delta ME + \Delta E_{th} = 0 \quad \text{(for an isolated system)}$$

The Conservation of Energy

If we consider not only conservative forces and the force of friction but also other non-frictional non-conservative forces we can rewrite the work-energy theorem, as

$$\Delta K + \Delta U + \Delta E_{th} = W_{NC}$$

where W_{NC} is the work done by non-frictional non-conservative forces.

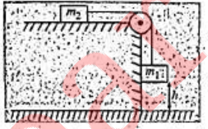
Now whatever the W_{NC} are, it is always possible to find new forms of energy whose change corresponds to negative of this work. We can then represent ΣW_{NC} by another forms of energy. As a result, we can rewrite work energy theorem as

$$\Delta K + \Delta U + \Delta E_{th} + (\text{change in other forms of energy}) = 0$$

In other words, total energy may be transformed from one form to another but the total energy for an isolated system remains constant.

Train Your Brain

Example 18: Two masses are connected by a light string which passes over a fixed pulley. The system is released from rest. Find the speed of block m_1 when it strikes the ground after falling a distance h.



Sol. Considering the system of m_1, m_2 and earth, we notice that there is no friction (non-conservative force) and system is isolated. The mechanical energy of the system is conserved. Hence $\Delta K + \Delta U = 0$

$$\Rightarrow \left[\left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right) - 0\right] - m_1gh = 0$$

in which $v_1 = v_2 = v$

The change in U of m_1 is $(-m_1gh)$ and that of m_2 is zero.

$$\Rightarrow v = \sqrt{\frac{2m_1gh}{m_1 + m_2}}$$

Example 19: If the coefficient of kinetic friction between m_2 and horizontal surface is μ in the previous problem, find the velocity of m_1 when it strikes the ground.

Sol. In this case mechanical energy will not be conserved because work done by friction (non-conservative force) is non-zero.

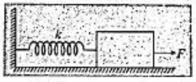
Hence, $\Delta K + \Delta U = W_{\text{friction}}$

in which $W_{\text{friction}} = -\mu Nh = -\mu m_2gh$

$$\Rightarrow \frac{1}{2}(m_1 + m_2)v^2 - m_1gh = -\mu m_2gh$$

$$\Rightarrow v = \sqrt{\frac{2(m_1 - \mu m_2)gh}{m_1 + m_2}}$$

Example 20: A block of mass m is attached to a spring of spring constant k. The other end of the spring is attached to a fixed wall (see fig.). If a force F is applied to the block as shown, find the maximum elongation of the spring



Sol. Let us consider the system consisting of the block and the spring. In this case mechanical energy will not be conserved, since external force F does some work.

Thus $\Delta K + \Delta U = \text{work done by } F$

The length of the spring will continue to increase as long as block moves to the right. Consequently, the maximum elongation of spring will occur when block comes to state of momentarily rest.

Let x_m be the maximum elongation of the spring.

Then $\Delta U_{\text{spring}} = \frac{1}{2}kx_m^{\ 2} - 0$

$\Delta K = 0 - 0 = 0$

Work done by $F = Fx_m$

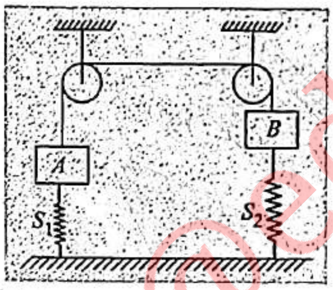
Hence, $\frac{1}{2}kx_m^2 = Fx_m \Rightarrow x_m = \frac{2F}{k}$

Notice that the block will start moving towards left after it comes to rest because force applied by the spring will be more than F at this instant.

Try yourself: Let the coefficient of kinetic friction between block and surface be μ. Find the maximum elongation of spring in this case.

[Ans: $x_m = 2\frac{(F - \mu mg)}{k}$]

Example 21: In the figure shown below masses of blocks A and B are 3 kg and 6 kg respectively. The force constants of springs S_1 and S_2 are 160 N/m and 40 N/m respectively. Length of the light string connecting the blocks is 8 m. The system is released from rest with the springs at their natural lengths. The maximum elongation of spring S_1 will be:

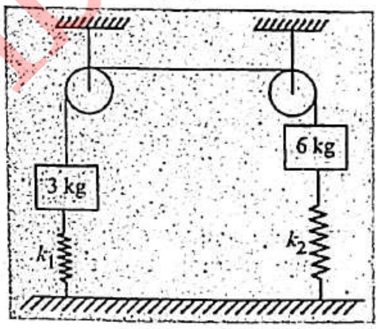


(*a*) 0.294 m (*b*) 0.490 m

(*c*) 0.588 m (*d*) 0.882 m

Sol. $\Delta K = 0 \Rightarrow \Delta U_{\text{gravity}} = -6gx + 3gx$

$\Delta U_{\text{spring}} = \frac{1}{2}k_2x^2 + \frac{1}{2}k_1x^2$



Applying, $\Delta K + \Delta U_{\text{gravity}} + \Delta U_{\text{spring}} = 0$

$6g(x) - 3g(x) = \frac{1}{2}k_2x^2 + \frac{1}{2}k_1x^2 \quad [\because \Delta K = 0]$

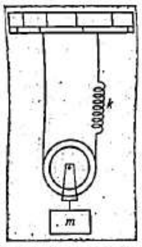
$6g = (k_1 + k_2)x, x = \frac{6 \times 9.8}{200} = \frac{3 \times 9.8}{100}$

$\Rightarrow x = 0.294$ m

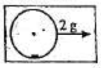
Concept Application

18. The potential energy of a particle of mass m free to move along x-axis is given by $U = 1/2kx^2$ for $x < 0$ and $U = 0$ for $x \geq 0$ (x denotes the x-coordinate of the particle and k is a positive constant). If the total mechanical energy of the particle is E, then its speed at $x = -\sqrt{\frac{2E}{k}}$ is
    (a) Zero
    (b) $\sqrt{\frac{2E}{m}}$
    (c) $\sqrt{\frac{E}{m}}$
    (d) $\sqrt{\frac{E}{2m}}$

19. In the given figure, spring, string and pulley are massless. System is released from rest when spring is in its natural length. Find maximum elongation in the spring.



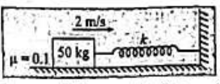
20. A block of mass m is placed inside a smooth hollow cylinder of radius R whose axis is kept horizontal. Initially system was at rest. Now cylinder is given a constant acceleration 2 g in the horizontal direction by external agent. The maximum angular displacement of the block with the vertical is



    (a) $2\tan^{-1}2$
    (b) $\tan^{-1}2$
    (c) $\tan^{-1}1$
    (d) $\tan^{-1}\left(\frac{1}{2}\right)$

21. A rod of length 1 m and mass 0.5 kg hinged at one end, is initially hanging vertical. The other end is now raised slowly until it makes an angle 60° with the vertical. The required work is: (use $g = 9.8\text{ m/s}^2$)
    (a) 1.522 J (b) 1.225 J (c) 2.125 J (d) 3.125 J

22. A block of mass 50 kg is projected horizontally on a rough horizontal floor. The coefficient of friction between the block and the floor is 0.1. The block strikes a light spring of stiffness $k = 100$ N/m with a velocity 2 m/s. The maximum compression of the spring is [Take $g = 10\text{ m/s}^2$]



    (a) 1 m (b) 2 m (c) 3 m (d) 4 m

23. A uniform string of mass M and length $2a$, is placed symmetrically over a smooth and small pulley and has particles of masses m and m' attached to its ends; show that when the string runs off the peg its velocity is $\left\{\frac{M+2(m-m')}{M+m+m'}ag\right\}$ (Assume $m > m'$).

Conservative Force and Potential Energy

$$F_s = -\frac{\partial U}{\partial s}$$

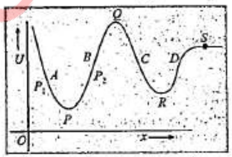
The projection of the force field, the vector $\vec{F}$, at a given point in the direction of the displacement $\vec{r}$ equals the derivative of the potential energy U with respect to a given direction, taken with the opposite sign. The designation of a partial derivative $\partial/\partial s$ emphasizes the fact of deriving with respect to a definite direction. So, having reversed the sign of the partial derivatives of the function U with respect to x, y, z, we obtain the projections F_x, F_y and F_z of the vector $\vec{F}$ on the unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$. Hence, one can readily find the vector:

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}, \text{ or } \vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)$$

The quantity in parentheses is referred to as the scalar gradient of the function U and is denoted by grad U or ∇U. We shall use the second, more convenient, designation where ∇ ("nabla") signifies the symbolic vector or operator, $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

Potential Energy Curve

- A graph plotted between the PE of a particle and its displacement from the centre of force field is called PE curve.
- Using graph, we can predict the velocity of a particle at various positions if we know the mechanical energy of the particle i.e., $\frac{1}{2}mu^2 + U = M \cdot E$
- Force on the particle is $F_{(x)} = -\frac{dU}{dx}$



Case-I: On increasing x, if U increases, force is in (–) ve x-direction i.e. attraction force.

Case-II: On increasing x, if U decreases, force is in (+) ve x-direction i.e. repulsion force.

Different Positions of a Particle

Position of Equilibrium: If net force acting on a body is zero, it is said to be in equilibrium. For equilibrium $\frac{dU}{dx} = 0$. Points P, Q, R and S are the states of equilibrium positions.

Types of Equilibrium

- **Stable equilibrium:** When a particle is displaced slightly from equilibrium position and a force acting on it tends to bring it back to the initial equilibrium position, then the position is said to be the stable equilibrium position.

  Necessary conditions: $\frac{dU}{dx} = 0$, and $\frac{d^2U}{dx^2} = +ve$

  In figure P and R points show stable equilibria.
- **Unstable equilibrium:** When a particle is displaced slightly from equilibrium position and force acting on it tries to displace the particle further away from the equilibrium position, the position is said to be unstable equilibrium.

  Condition: $\frac{dU}{dx} = 0$

  potential energy is maximum

  i.e. $\frac{d^2U}{dx^2} = -ve$

  Q point in figure shows unstable equilibrium point.
- **Neutral equilibrium:** In the neutral equilibrium, potential energy is constant. When a particle is displaced from its equilibrium position it does not experience any force acting on it and continues to be in equilibrium in the displaced position. This is said to be neutral equilibrium.

  In figure, S is the neutral point.

  Condition: $\frac{dU}{dx} = 0$, $\frac{d^2U}{dx^2} = 0$

Train Your Brain

Example 22: The potential energy between two atoms in a molecule is given by, $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are positive constants and x is the distance between the atoms. The system is in stable equilibrium when

(a) $x = 0$
(b) $x = \frac{a}{2b}$
(c) $x = \left(\frac{2a}{b}\right)^{1/6}$
(d) $x = \left(\frac{11a}{5b}\right)$

Sol. (c) Given that, $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

We, know $F = -\frac{dU}{dx} = (-12)\,a\,x^{-13} - (-6b)\,x^{-7} = 0$

or $\frac{6b}{x^7} = \frac{12a}{4x^{13}}$ or $x^6 = 12a/6b = 2a/b$ or $x = \left(\frac{2a}{b}\right)^{1/6}$

Example 23: The potential energy of a conservative system is given by $U = ax^2 - bx$ where a and b are positive constants. Find the equilibrium position and discuss whether the equilibrium is stable, unstable or neutral.

Sol. In a conservative field, $F = -\frac{dU}{dx}$

$$\therefore F = -\frac{d}{dx}(ax^2 - bx) = b - 2ax$$

For equilibrium, $F = 0$ or $b - 2ax = 0 \Rightarrow x = \frac{b}{2a}$

From the given equation we can see that $\frac{d^2U}{dx^2} = 2a$ (positive), i.e., U is minimum.

Therefore, $x = b/2a$ is the stable equilibrium position.

Concept Application

24. The potential energy U of a conservative system is given by $U = ax^2 - bxy^2$. Find the x and y components of the conservative force F_x and F_y.
25. The potential energy of a conservative system is given by $U = ax^2 - bx^4$ where a and b are positive constants. Find the position where system is in stable and unstable equilibrium.

POWER

Rate of work done by a force is called power delivered by the force.

Average power $= \frac{\text{work done}}{\text{time}}$ or $P = \frac{W}{t}$

W is the amount of the work done in total time t.

Instantaneous Power $(P) = \frac{dW}{dt}$ or $P = \frac{\vec{F}.d\vec{s}}{dt}$ $(\because dW = \vec{F}.d\vec{s})$

or $P = \vec{F}.\frac{d\vec{s}}{dt}$ or $P = \vec{F}.\vec{v}$

Notes:

(i) Dimensions: $[M^1L^2T^{-3}]$ $\left(\text{from } P = \frac{W}{t}\right)$

(ii) SI unit: watt $= \frac{\text{joule}}{\text{second}}$

(iii) Bigger unit of power:

1 kilo watt (kW) = 1000 watt = 10^3 W

1 maga watt = 1000, 000 watt or 1MW = 10^3kW

1 horse power (h.p.) = 746 W = 550 ft. lb/s

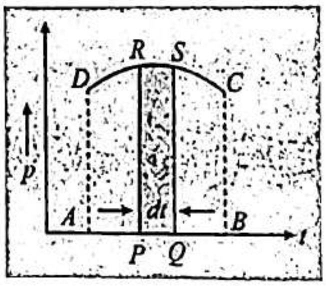
FPS Unit: $\frac{(\text{foot} \times \text{pound})}{\text{second}}$, 1W = 0.738 ft. lb/s

(iv) When an agent delivers power at a uniform rate, the average power is equal to the instantaneous power.

(v) The slope of work – time graph gives the instantaneous power

slope $= \tan\theta = \frac{dW}{dt} = P$ (instantaneous power)

(vi) Area under power – time graph gives the work done. Area under P-t graph = Area $ABCD$ = work done from A to B.



Area under power – time graph = $\int P dt = W$

Train Your Brain

Example 24: Force acting on a particle moving in a straight line varies with the velocity v of the particle as $F = \frac{K}{v}$, where K is a constant. The work done by this force in time t is

(a) $\frac{K}{v^2}t$ (b) $2Kt$

(c) Kt (d) $\frac{2Kt}{v^2}$

Sol. (c) It can be observed that power delivered to particle by force F is

$P = Fv = K.$

The power is constant.

Hence work done by force in time t is

$\Delta W = Pt = Kt$

Example 25: What is the power of an engine which can lift 20 metric ton of coal per hour from a 20 metre deep mine?

Sol. Mass, m = 20 metric ton = 20 × 1000 kg

Distance, S = 20 m

Time, t = 1 h = 3600 s

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

$$= \frac{\text{mg}\times\text{S}}{\text{t}} = \frac{20\times1000\times9.8\times20}{3600}\text{ watt}$$

$= 1.09 \times 10^3$ W

Example 26: A large family uses 8 kW of power. Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square metre. If 20% of this energy can be converted to useful electrical energy how large an area is needed to supply 8 kW?

Sol. Let A be the required area.

Useful electrical energy produced/s

$$= \frac{20}{100}(200\text{ A}) = 40.\text{ A watts}$$

But 40 A = 8000

or A = 20 m^2

Example 27: An elevator of total mass (elevator + passenger) 1800 kg is moving up with a constant speed of 2 ms^{-1}. A frictional force of 4000 N opposes its motion. Determine the minimum power delivered by the motor to the elevator. (Take g = 10 ms^{-2}).

Sol. Weight of (elevator + passenger) = mg = 1800 × 10 = 18000 N; frictional force = 4000 N

Total downward force on the elevator (18000+ 4000) = 22000 N

Clearly, the motor must have enough power to balance this force.

Now, power, $P = Fv$ = 22000 N × 2 ms^{-1}

$$= \frac{44000}{746}\text{ hp} = 58.98\text{ hp}$$

Concept Application

26. An electric motor creates a tension of 4500 N in hoisting cable and reels it at the rate of 2 m/s. What is the power of electric motor?

(a) 9 W (b) 9 kW (c) 225 W (d) 9000 h.p.

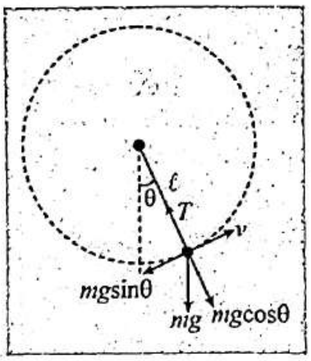
27. A man M_1 of mass 80 kg runs up a staircase in 15 s. Another man M_2 also of mass 80 kg runs up the stair case in 20 s. The ratio of the power developed by them (P_1/P_2) will be:

(a) 1 (b) 4/3

(c) 16/9 (d) None of the above

MOTION IN A VERTICAL CIRCLE

Consider the motion of a point mass tied to a string of length ℓ and whirled in a vertical circle. If at any time the body is at angular position θ, as shown in the figure, the forces acting on it are tension T in the string along the radius towards the center and the weight of the body mg acting vertically downwards.



Applying Newton's law along radial direction

$$T - \text{mg}\cos\theta = m.a_r = \frac{mv^2}{\ell}$$

$$\text{or } T = \frac{mv^2}{\ell} + mg\cos\theta \qquad \dots(i)$$

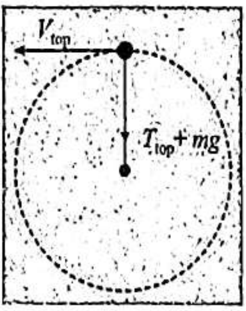
The point mass will complete the circle only and only if tension is never zero (except momentarily, if at all) if tension becomes zero at any point, string will go slack and subsequently, the only force acting on the body is gravity. Hence its subsequent motion will be that of a projectile.

From equation (i), it is evident that tension decreases with increase in θ because cos θ is a decreasing function and v. decreases with height. Hence tension is minimum at the top most point. i.e. $T_{\min} = T_{\text{topmost}}$.

$T > 0$ at all points. $\Rightarrow$ $T_{\min} > 0$.

However if tension is momentarily zero at highest point the body would still be able to complete the circle.

Hence condition for completing the circle (or looping the loop) is $T_{\min} \geq 0$ or $T_{top} \geq 0$.



$$T_{top} + mg = \frac{mv_{top}^2}{\ell} \quad \dots (ii)$$

Equation (*ii*) could also be obtained by putting θ = π in equation (*i*).

For looping the loop, $T_{top} \geq 0$.

$$\Rightarrow \frac{mv_{top}^2}{\ell} \geq mg \Rightarrow v_{top} \geq \sqrt{g\ell} \quad \dots (iii)$$

Condition for looping the loop is $v_{top} \geq \sqrt{g\ell}$.

If speed at the lowest point is u, then from conservation of mechanical energy between lowest point and top most point,

$$\frac{1}{2} mu^2 = \frac{1}{2} m v_{top}^2 + mg.2\ell.$$

using equation (iii) for v_{top} we get

$$u \geq \sqrt{5g\ell}$$

i.e., for looping the loop, velocity at lowest point must be $u \geq \sqrt{5g\ell}$.

If velocity at lowest point is just enough for looping the loop, value of various quantities are as follows (True for a point mass attached to a string or a mass moving on a smooth vertical circular track).

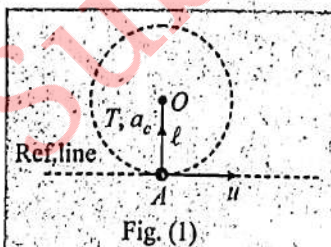


Fig. (1)

P.E. = 0

$u = \sqrt{5g\ell}$

$T - mg = \frac{mu^2}{\ell}$

[This eq. could also be obtained by putting θ = 0 in equation...(*i*)]

$\Rightarrow T = 6mg$

$a_c = 5g, a_t = 0$

$a_{\text{net}} = 5g$

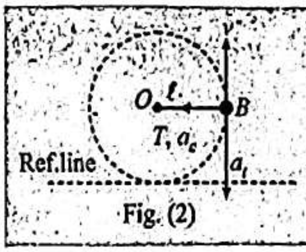


Fig. (2)

P.E. = $mg\ell$

By energy conservation,

$v = \sqrt{3g\ell}$

$T = \frac{mv^2}{\ell}$

[This eq. could also be obtained by putting θ = 90° in equation...(*i*)]

$\Rightarrow T = 3mg$

$a_c = 3g, a_t = g$

$a_{\text{net}} = g\sqrt{10}$

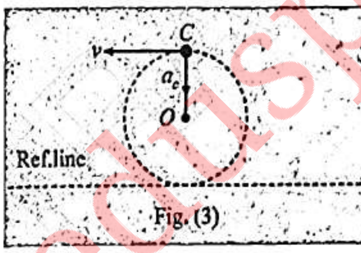


Fig. (3)

P.E. = $2mg\ell$

By energy conservation,

$v = \sqrt{g\ell}$

$T = 0$

$a_c = g, a_t = 0$

$a_{\text{net}} = g$

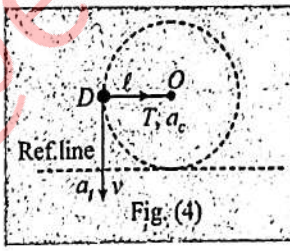


Fig. (4)

P.E. = $mg\ell$

By energy conservation,

$v = \sqrt{3g\ell}$

$T = 3mg$

$a_c = 3g, a_t = g$

$a_{\text{net}} = g$

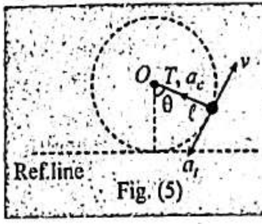


Fig. (5)

P.E. = $mg\ell(1 - \cos\theta)$

By energy conservation,

$v = \sqrt{g\ell(3 + 2\cos\theta)}$

$T = 3mg(1 + \cos\theta)$

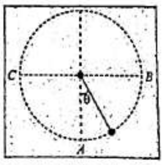
$a_c = g(3 + 2\cos\theta), a_t = g\sin\theta$

	A	B, D	C	P(general point)
Velocity	$\sqrt{5g\ell}$	$\sqrt{3g\ell}$	$\sqrt{g\ell}$	$\sqrt{g\ell(3+2\cos\theta)}$
Tension	$6mg$	$3mg$	0	$3mg(1+\cos\theta)$
Potential Energy	0	$mg\ell$	$2mg\ell$	$mg\ell(1-\cos\theta)$
Radial acceleration	$5g$	$3g$	g	$g(3+2\cos\theta)$
Tangential acceleration	0	g	0	$g\sin\theta$

Note: From above table we can see, $T_{\text{bottom}} - T_{\text{top}} = T_C - T_A = 6\,mg$, this difference in tension remain same even if $V > \sqrt{5g\ell}$

Condition for Oscillation or Leaving the Circle

In case of non uniform circular motion in a vertical plane if velocity of body at lowest point is lesser than $\sqrt{5g\ell}$, the particle will not complete the circle in vertical plane. In this case, the motion of the point mass depends on whether tension becomes zero before speed becomes zero or vice versa.



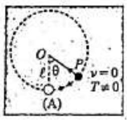
Case-I: (Speed becomes zero before tension)

In this case the ball never rises above the level of the center. O i.e. the body is confined to move within C and B, ($|\theta| < 90°$) for this the speed at A, $u < \sqrt{2g\ell}\left(\frac{1}{2}mu^2 \le mgl\right)$

In this case tension cannot be zero, since a component of gravity acts radially outwards.

Hence the string will not go slack, and the ball will reverse back as soon as its speed becomes zero.

Its motion will be oscillatory motion.



For oscillation

$0 < u < \sqrt{2gl}$

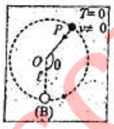
$0 < \theta < 90°$

Case-II: (Tension becomes zero before speed)

In this case the ball rises above the level of center O i.e. it goes beyond point $B(\theta > 90°)$ for this $u > \sqrt{2g\ell}\left(\frac{1}{2}mu^2 \ge mgl\right)$, but $u < \sqrt{5gl}$

In this case a component of gravity will always act towards center, hence centripetal acceleration or speed will remain nonzero. Hence tension becomes zero first.

As soon as tension becomes zero at any point, string will go slack and subsequently, the only force acting on the body is gravity. Hence its subsequent motion will be that of a projectile.



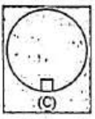
For leaving the circular path (after which motion converts into projectile motion)

$\sqrt{2g\ell} < u < \sqrt{5gl}$

$90° < \theta < 180°$

CONDITION FOR LOOPING THE LOOP IN SOME OTHER CASES

Case-I: A mass moving on a smooth vertical circular track.



For a block moving along a smooth vertical circular loop the condition to complete the loop is that normal from track at highest point should be zero. $N_{top} = 0$.

By calculation similar to that in previous section,

Minimum horizontal velocity at lowest point = $\sqrt{5gR}$

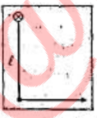
where, R = radius of the track

Case-II: A particle attached to a light rod rotated in vertical circle.

Condition for just completing the loop is that speed at the top is zero, $V_{top} = 0$ (even if tension is zero, rod won't slack, and a compressive force can appear in the rod).

By energy conservation,

velocity at lowest point = $\sqrt{4g\ell}$



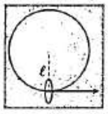
$V_{min} = \sqrt{4g\ell}$ (for just completing the circle)

Case-III: A bead attached to a ring and moving on it.

Condition for just looping the loop, velocity $v = 0$ at highest point (even if normal is zero, the bead will not lose contact with the track because normal can act radially outward too).

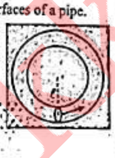
By energy conservation,

velocity at lowest point = $\sqrt{4g\ell}$



$V_{min} = \sqrt{4g\ell}$ (for just completing the circle)

Case-IV: A block rotated between smooth surfaces of a pipe.



Condition for just looping the loop, velocity $v = 0$ at highest point (even if normal is zero, the bead will not lose contact with the track, normal can act radially outward).

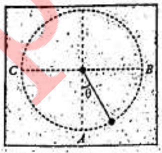
By energy conservation,

velocity at lowest point = $\sqrt{4g\ell}$

$V_{min} = \sqrt{4g\ell}$ (for completing the circle)

Train Your Brain

Example 28: Find minimum speed at A so that the ball can reach at point B as shown in figure. Also discuss the motion of particle when $T = 0$, $v = 0$ simultaneously at $\theta = 90°$.



Sol. From energy conservation.

$$\frac{1}{2}mv_A^2 + 0 = 0 + mg\ell(1-\cos\theta) \quad \text{(for minimum speed } v_B = 0)$$

$$v_{min} = \sqrt{2g\ell(1-\cos\theta)}$$

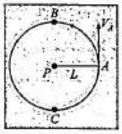
In the case when $\theta = 90°$

$V = 0$ and $T = 0$

So, ball will return back and motion will be oscillatory

Example 29: If a particle of mass M is tied to a light inextensible string fixed at point P and particle is projected at A with velocity $V_A = \sqrt{4gL}$ as shown. Find:

(i) velocity at points B and C



(ii) tension in the string at B and C

Assume particle is projected in the vertical plane.

Sol. Energy conservation between A and B,

$$\frac{1}{2}m(4gL) + 0 = \frac{1}{2}mV_B^2 + mgL$$

$$\Rightarrow V_B = \sqrt{2gL}$$

Similarly,

$$V_C = \sqrt{6gL}$$

At B, $Mg + T = \dfrac{mV_B^2}{L}$

$\Rightarrow T = Mg$

At C, $T - Mg = \dfrac{mV_C^2}{L}$

$\Rightarrow T_C = 7\,Mg$ (where $M \Rightarrow$ Mass of the particle)

Example 30: A body weighing 0.4 kg is whirled in a vertical circle by a string making 2 revolutions per second. If the radius of the circle is 1.2 m. Find the tension (i) at the top of the circle, (ii) at the bottom of the circle.

Given: $g = 10\ \text{m s}^{-2}$ and $\pi = 3.14$.

Sol. Mass, $m = 0.4$ kg;

time period $= \dfrac{1}{2}$ second, radius, $r = 1.2$ m

Angular velocity, $\omega = \dfrac{2\pi}{1/2} = 4\pi\ \text{rad s}^{-1}$

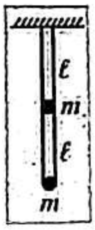
$= 12.56\ \text{rad s}^{-1}$.

(i) At the top of the circle,

$$T = \frac{mv^2}{r} - mg$$
$$= m\omega^2 r - mg$$
$$= m(r\omega^2 - g)$$
$$= 0.4\,(1.2 \times 12.56 \times 12.56 - 9.8)\ N = 71.72\ \text{N}$$

(ii) At the lowest point, $T = m(r\omega^2 + g) = 80$ N

Example 31: Two point masses both of mass m are connected to a light rod of length ℓ as shown. The rod is free to rotate in vertical plane. Calculate the minimum horizontal velocity is given to the rod so that it completes the circular motion in vertical plane.



Sol. Here tension in the rod at the top most point of circle can be zero or negative. So velocity at the top most point is zero.

From energy conservation

$$\frac{1}{2}mv^2 + \frac{1}{2}m\frac{v^2}{4} = mg(2\ell) + mg(4\ell) + 0$$

$$\Rightarrow \quad v = \sqrt{\frac{48g\ell}{5}}$$

Example 32: You may have seen in a circus a motorcyclist driving in vertical loops inside a 'death well' (a hollow spherical chamber with holes. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m?

Sol. When the motorcyclist is at the highest point of the death-well, the normal reaction R on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also acts downwards.

$$F_{net} = ma_c$$

$$\therefore R + mg = \frac{mv^2}{r}$$

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass of the motor cycle).

The minimum speed required to perform a vertical loop is given by equation (i) when $R = 0$.

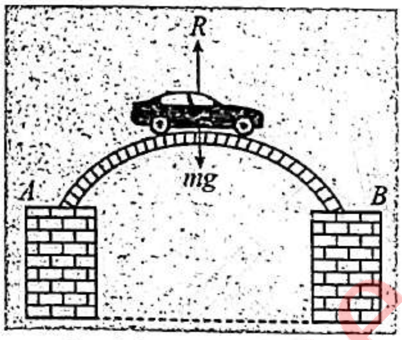
$$\therefore mg = \frac{mv_{min}^2}{r} \text{ or } v_{min}^2 = gr$$

$$\text{or } v_{min} = \sqrt{gr} = \sqrt{9.8 \times 25} \text{ ms}^{-1} = 15.65 \text{ ms}^{-1}.$$

So, the minimum speed at the top, required to perform the vertical loop is 15.65 ms^{-1}.

Example 33: Prove that a motor car moving over a convex bridge weighs lighter than the same car resting on the same bridge.

Sol. The motion of the motor car over a convex bridge AB is the motion along the segment of a circle AB (Figure);



The centripetal force is provided by the difference of weight mg of the car and the normal reaction R of the bridge.

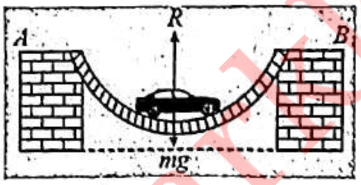
$$\therefore mg - R = \frac{mv^2}{r} \text{ or } R = mg - \frac{mv^2}{r}$$

Clearly $R <$ mg, i.e., the apparent weight of the moving car is less than the weight of the stationary car.

Example 34: Prove that a motor car moving over a concave bridge weighs heavier than the same car resting on the same bridge.

Sol. The motion of the motor car over a concave bridge AB is the motion along the segment of a circle AB (Figure);

The centripetal force is provided by the difference of normal reaction R of the bridge and weight mg of the car.



$$\therefore R - mg = \frac{mv^2}{r} \text{ or } R = mg + \frac{mv^2}{r}$$

Clearly $R > mg$, i.e., the apparent weight of the moving car is greater than the weight of the stationary car.

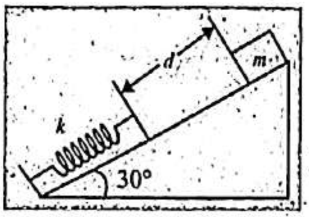
Concept Application

28. A weightless thread can support tension upto 30 N. A stone of mass 0.5 kg is tied to it and is revolved in a circular path of radius 2 m in a vertical plane. If $g = 10$ m/s^2, find the maximum angular velocity of the stone.

29. A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum angular displacement of the pendulum of the string with respect to the downward vertical.

Aarambh (Solved Examples)

1. A block of 4 kg mass starts at rest and slides a distance d down a friction less incline (angle 30°) where it runs into a spring of negligible mass. The block slides an additional 25 cm before it is brought to rest momentarily by compressing the spring. The force constant of the spring is 400 Ns^{-1}. The value of d is (take $g = 10\ ms^{-1}$)



(*a*) 25 cm (*b*) 37.5 cm

(*c*) 62.5 cm (*d*) None of the above

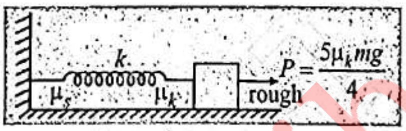
Sol. $W_{gravity} = \Delta U_{spring}$

$$mg\sin\theta\,(d + 0.25) = \frac{1}{2}k(0.25)^2$$

$$\Rightarrow d = 37.5 \text{ cm}$$

Therefore, option (*b*) is the correct answer.

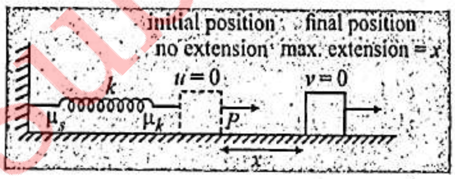
2. A block of mass m rests on a rough horizontal plane having coefficient of kinetic friction μ_k. The spring is in its natural length, when a constant force of magnitude $P = \frac{5\mu_k mg}{4}$ starts acting on the block. The spring force F is a function of extension x varies as $F = kx^3$. (where k is spring constant) Find the maximum extension in the spring (Assume that the force P is sufficient to make the block move).



(*a*) $\left(\frac{\mu_k mg}{k}\right)^{\frac{1}{2}}$ (*b*) $\left(\frac{\mu_k mg}{2k}\right)^{\frac{1}{2}}$

(*c*) $\left(\frac{\mu_k mg}{2k}\right)^{\frac{1}{3}}$ (*d*) $\left(\frac{\mu_k mg}{k}\right)^{\frac{1}{3}}$

Sol. For motion to start $\frac{5\mu_k mg}{4} > \mu_s mg$ or $5\mu_k > 4\mu_s$



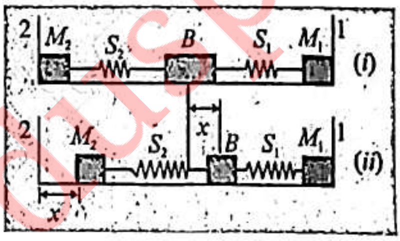
At the final position of the block extension in spring is maximum and the speed of the block is $v = 0$. Hence the net work done in taking the block from initial to final position ΔW = (Work done by force P + work done by spring force F + work done by friction) = $\Delta K = 0$

$$= Px - \int_0^x Kx^3 . dx - \mu_k mgx = \frac{5\mu_k mg}{4}x - \frac{kx^4}{4} - \mu_k mgx = 0$$

Solving we get, $x = \left(\frac{\mu_k mg}{k}\right)^{1/3}$

Therefore, option (*d*) is the correct answer.

3. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$ respectively [see Fig. (*i*)]. The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by small distance x (Fig *ii*) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ is



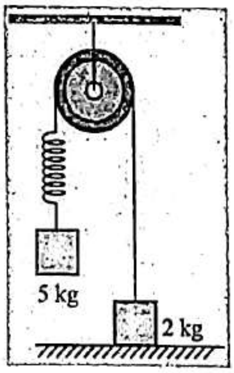
(*a*) 4 (*b*) 2 (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{4}$

Sol. From energy conservation,

$$\frac{1}{2}kx^2 = \frac{1}{2}(4k)y^2 \Rightarrow \frac{y}{x} = \frac{1}{2}$$

Therefore, option (*c*) is the correct answer.

4. The system shown in figure is released from rest with mass 2 kg in contact with the ground. Pulley and spring are massless, and friction is absent everywhere. The speed of 5 kg block when 2 kg block leaves the contact with the ground is (force constant of the spring $k = 40\ Nm^{-1}$ and $g = 10\ ms^{-2}$)



(*a*) $\sqrt{2}\ ms^{-1}$ (*b*) $2\sqrt{2}\ ms^{-1}$

(*c*) $2\ ms^{-1}$ (*d*) $\sqrt{2}\ ms^{-1}$

Sol. Let x be the extension in the spring when 2 kg block leaves the contact with ground. Then tension in the string should be equal to weight of 2 kg block.

$$kx = 2g \text{ or } x = \frac{2g}{k} = \frac{2\times10}{40} = \frac{1}{2}\text{m}$$

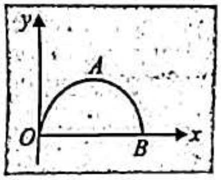
Now from conservation of mechanical energy

$$mgx = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\text{of } v = \sqrt{2gx - \frac{kx^2}{m}} = \sqrt{2\times10\times\frac{1}{2} - \frac{40}{5\times4}} = 2\sqrt{2}\text{ms}^{-1}$$

Therefore, option (b) is the correct answer.

5. Given $\bar{F} = (xy^2)\hat{i} + (x^2y)\hat{j}$ N. The work done by $\bar{F}$ when a particle is taken along the semicircular path OAB, where the coordinates of B are (4, 0), is



(a) $\frac{65}{5}$J (b) $\frac{75}{5}$J (c) $\frac{73}{4}$J (d) Zero

Sol. Given $\bar{F} = (xy^2)\hat{i} + (x^2y)\hat{j}$

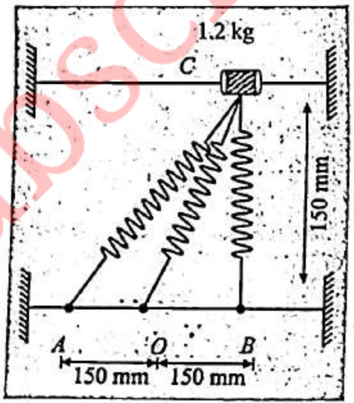
$$W = \int F_x dx + \int F_y dy = \int xy^2 dx + \int x^2 y dy$$

$$= \frac{1}{2}\int d(x^2y^2) = \left[\frac{x^2y^2}{2}\right]_{(0,0)}^{(4,0)} = 0$$

Therefore, option (d) is the correct answer.

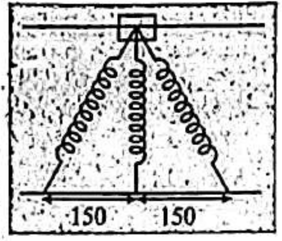
6. A 1.2 kg collar C may slide without friction along a fixed smooth horizontal rod. It is attached to three springs each of spring constant $k = 400$ N/m and 150 mm undeformed length. Knowing that the collar is released from rest in the position shown, determine the maximum velocity it will reach in its motion. [Here A, O, B are fixed points]

(a) 3.2 m/s (b) 2.0 m/s (c) 2.4 m/s (d) 1.8 m/s



Sol. Velocity will be maximum when collar is just above O and $F_t = 0$

This situation occurs for following arrangement of springs.



Natural length is $c = 150$ mm

Now, $U_i + K_i = U_f + K_f$

$$U_i = \frac{1}{2}k\{\sqrt{5}\,c - c\}^2 + \frac{1}{2}k\{\sqrt{2}\,c - c\}^2 \text{ and } K_i = 0$$

$$U_f = 2.\frac{1}{2}k\{\sqrt{2}\,c - c\}^2 \text{ and } K_f = \frac{1}{2}mv^2$$

$$\therefore \frac{1}{2}k\{\sqrt{5}\,c - c\}^2 + \frac{1}{2}k\{\sqrt{2}\,c - c\}^2$$

$$= \frac{1}{2}mv^2 + 2.\frac{1}{2}k\{\sqrt{2}\,c - c\}^2$$

Solving the equation and putting the values, we have

$$v = \left[\frac{15}{2}\left\{(\sqrt{5}-1)^2 + (\sqrt{2}-1)^2\right\}\right]^{1/2} \text{m/s} = 3.189 \text{ ms}^{-1}$$

$$= 3.2 \text{ m/s}$$

Therefore, option (a) is the correct answer.

7. Wind enters in a wind mill with a velocity of 20 m/sec. Facing area of the windmill is 10 m^2 and density of air is 1.2 kg/m^3. If wind energy is converted into electrical energy with 33.3% efficiency, then electrical power produced by the wind mill in kW is

(a) 16 kW (b) −16 kW

(c) − 15 kW (d) 15 kW

Sol. Energy entering in the windmill $= \frac{1}{2}mv^2$

$$P_{\text{in}} = \frac{dE}{dt} = \left(\frac{1}{2}v^2\right)\left(\frac{dm}{dt}\right)$$

$$P_{\text{in}} = \left(\frac{1}{2}v^2\right)(\rho Av) = \frac{1}{2}\rho Av^3$$

Electrical power output

$$P_{\text{out}} = \frac{1}{3}\left(\frac{1}{2}\rho Av^3\right)$$

$$P_{\text{out}} = \frac{1}{6}\rho Av^3 = \frac{1}{6}\times1.2\times10\times(20)^3 \Rightarrow P_{\text{out}} = 16 \text{ kW}$$

Therefore, option (a) is the correct answer.

8. An engine can pull 4 coaches at a maximum speed of 20 m/s. Mass of the engine is twice the mass of every coach. Assuming resistive forces to be proportional to the weight, maximum speeds of the engine when it pulls 12 coaches is (power of engine remains constant)

(*a*) 10 m/s (*b*) 15 m/s (*c*) 8.5 m/s (*d*) 7.5 m/s

Sol. When 4 coaches (m each) are attached with engine ($2m$)

according to question $P = K\,6mgv$ ... (*i*)

(constant power), (K being proportionality constant)

Since resistive force is proportional to weight

Now if 12 coaches are attached

$P = K14mgv_1$ ... (*ii*)

Since engine power is constant

So by equation (*i*) and (*ii*)

$$6Kmgv = 14Kmgv_1 \Rightarrow v_1 = \frac{6}{14} \times v$$

$$= \frac{6}{14} \times 20 = \frac{6 \times 10}{7} = \frac{60}{7}$$

$\Rightarrow v_1 = 8.5$ m/sec

Therefore, option (*c*) is the correct answer.

9. A pump motor is used to deliver water at a certain rate from a given pipe. To obtain "*n*" times water from the same pipe in the same time, the factor by which the power of the motor should be increased is

(*a*) n^2 (*b*) n (*c*) n^3 (*d*) $n^{3/2}$

Sol. Power $P = \vec{F} \cdot \vec{v} = Fv$

$$F = v\left(\frac{dm}{dt}\right)$$

$$= v\left\{\frac{d(\rho \times volume)}{dt}\right\}; \rho = \text{density}$$

$$= \rho v\left\{\frac{d(volume)}{dt}\right\}$$

$$= \rho v\,(Av) = \rho A v^2$$

$\therefore$ Power $P = \rho A v^3$ or $P \propto v^3$

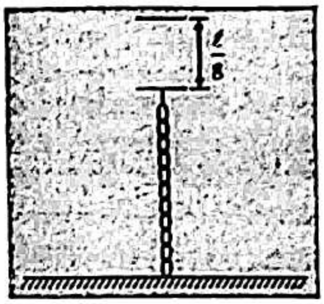
Therefore, option (*c*) is the correct answer.

10. A chain of mass M and length ℓ is held vertically such that its bottom end just touches the surface of a horizontal table. The chain is released from rest. Assume that the portion of chain on the table does not form a heap. The momentum of the portion of the chain above the table after the top end of the chain falls down by a distance $\frac{\ell}{8}$ is

(*a*) $\frac{7M}{8}\sqrt{g\ell}$ (*b*) $\frac{7M}{4}\sqrt{g\ell}$

(*c*) $\frac{7M}{16}\sqrt{g\ell}$ (*d*) $\frac{7M}{32}\sqrt{g\ell}$

Sol. After the top end of chain falls down by $\frac{\ell}{8}$, the speed of chain is



$$v = \sqrt{2g\frac{\ell}{8}} = \frac{\sqrt{g\ell}}{2}$$

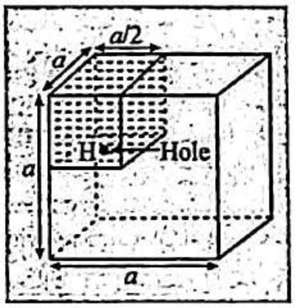
The mass of chain above table is $\frac{7}{8}M$.

$$\therefore \text{Momentum of chain is} = \frac{7}{8}M\frac{\sqrt{g\ell}}{2}$$

$$= \frac{7}{16}M\sqrt{g\ell}$$

Therefore, option (*c*) is the correct answer.

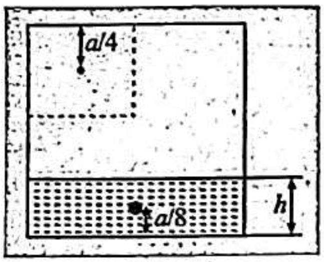
11. The figure shows a hollow cube of side '*a*' of volume *V*. There is a small chamber of volume $\frac{V}{4}$ in the cube as shown. This chamber is completely filled by *m* kg of water. Water leaks through a hole *H* and spreads in the whole cube. The work done by gravity in this process assuming that the complete water finally lies at the bottom of the cube is:



(*a*) $\frac{5}{8}mga$ (*b*) $\frac{3}{8}mga$

(*c*) $\frac{7}{8}mga$ (*d*) $\frac{5}{3}mga$

Sol. Let *h* be the height of water surface finally.



Volume of water remains the same hence,

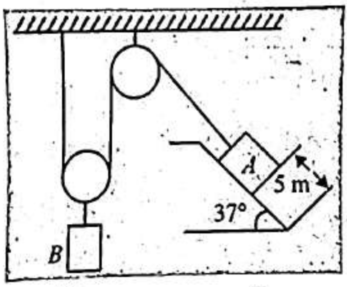
$$a^2h = a.\frac{a}{2}.\frac{a}{2} \Rightarrow h = \frac{a}{4}$$

$\therefore$ C.M gets lowered by $a - \left(\frac{a}{4} + \frac{a}{8}\right) = a - \frac{3a}{8} = \frac{5a}{8}$

$\therefore$ Work done by gravity $= mg\,\frac{5a}{8}$

Therefore, option (*a*) is the correct answer.

12. The blocks shown in figure have masses $M_A = 5$ kg and $M_B = 4$ kg. The system is released from rest. The speed of B after A has travelled a distance 1 m along the incline is



(*a*) $\frac{\sqrt{3}}{2}\sqrt{g}$ (*b*) $\frac{\sqrt{3}}{4}\sqrt{g}$

(*c*) $\frac{\sqrt{g}}{2\sqrt{3}}$ (*d*) $\frac{\sqrt{g}}{2}$

Sol. If A moves down the incline by 1 m, B shall move up by $\frac{1}{2}$ *m*.

If the speed of B is v, then the speed of A will be $2v$.

From conservation of energy,

Gain in KE = loss in PE

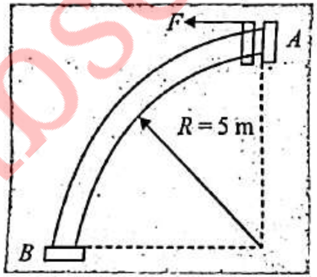
$$\frac{1}{2}m_A(2v)^2 + \frac{1}{2}m_Bv^2 = m_Ag \times \frac{3}{5} - m_Bg \times \frac{1}{2}$$

Solving, we get

$$v = \frac{1}{2}\sqrt{\frac{g}{3}}$$

Therefore, option (*c*) is the correct answer.

13. A bead of mass 1/2 kg starts from rest to move in a vertical plane along a smooth fixed quarter ring of radius 5 m, under the action of a constant horizontal force $F = 5$ N as shown. The speed of bead as it reaches point B is



(*a*) 14.14 ms^{-1} (*b*) 7.07 ms^{-1}

(*c*) 5 ms^{-1} (*d*) 25 ms^{-1}

Sol. Applying the work-energy theorem, we get

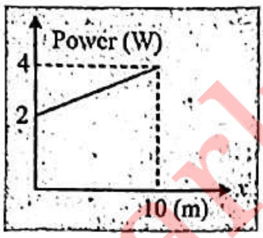
$$\frac{1}{2}mv^2 - 0 = F \times R + mg \times R$$

$$\frac{1}{2} \times \frac{1}{2} \times v^2 = 5 \times 5 + \frac{1}{2} \times 10 \times 5 = 50$$

$$v = \sqrt{200} = 14.14\ \text{ms}^{-1}$$

Therefore, option (*a*) is the correct answer.

14. A particle A of mass 10/7 kg is moving in the positive direction of x-axis. At initial position $x = 0$, its velocity is 1ms^{-1}, then its velocity at $x = 10$ m is (use the graph given)



(*a*) 4 ms^{-1} (*b*) 2 ms^{-1}

(*c*) $3\sqrt{2}$ ms^{-1} (*d*) $\frac{100}{3}$ ms^{-1}

Sol. Area under $P - x$ graph

$$= \int P\,dx = \int\left(m\frac{dv}{dt}\right)v\,dx = \int_1^v mv^2\,dv$$

$$= \left[\frac{mv^3}{3}\right]_1^v = \frac{10}{7 \times 3}(v^3 - 1)$$

From the graph, area $= \frac{1}{2}(2 + 4) \times 10 = 30$

$$\Rightarrow \frac{10}{7 \times 3}(v^3 - 1) = 30$$

$\therefore v = 4$ ms^{-1}

Alternative method:

From graph

$P = 0.2x + 2$

$$mv\frac{dv}{dx}v = 0.2x + 2$$

$$mv^2dv = (0.2x + 2)dx$$

Now integrating both sides, we get

$$\int_1^v mv^2\,dv = \int_0^{10}(0.2x + 2)dx$$

$v = 4$ ms^{-1}

Therefore, option (*a*) is the correct answer.

15. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$ where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true?

(*a*) The force applied on the particle is constant.

(*b*) The speed of the particle is proportional to time.

(*c*) The distance of the particle from the origin increases linearly with time.

(*d*) The acceleration is proportional to time.

Sol. $\frac{dK}{dt} = mv\frac{dv}{dt} = \gamma t$

$$v\,dv = \frac{\gamma}{m}\,t\,dt$$

$$\frac{v^2}{2} = \frac{\gamma}{m}\frac{t^2}{2} \Rightarrow v \propto t$$

$$\frac{dv}{dt} = \text{constant} \Rightarrow F = \text{constant}$$

$$\frac{dx}{dt} \propto t \Rightarrow x \propto t^2$$

Therefore, option (*a*, *b*) is the correct answer.

16. A force $\vec{F} = -K(y\hat{i} + x\hat{j})$ where K is a positive constant, acts on a particle moving in the *x*-*y* plane. Starting from the origin, the particle is taken along the positive *x*-axis to the point (*a*,0) and then parallel to the y-axis to the point (*a*, *a*). The total work done by the force $\vec{F}$ on the particle is

(*a*) $-2Ka^2$ (*b*) $2Ka^2$

(*c*) $-Ka^2$ (*d*) Ka^2

Sol. $dW = \vec{F}\cdot d\vec{s}$ where $d\vec{s} = dx\hat{i} + dy\hat{j}$ and $\vec{F} = -K(y\hat{i} + x\hat{j})$

$$\therefore dW = -K(ydx + xdy) = -Kd(xy)$$

$$\therefore W = \int_{(0,\,0)}^{(a,\,a)} dW = -K\int_{(0,\,0)}^{(a,\,a)} d(xy) = -\{K(xy)\}_{(0,\,0)}^{(a,\,a)}$$

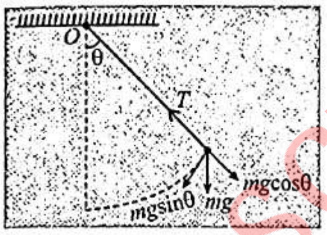
$$W = -Ka^2$$

Since $\frac{\partial F_y}{\partial x} = \frac{\partial F_x}{\partial y}$, this force is conservative and work done does not depend on the path taken.

Therefore, option (*c*) is the correct answer.

17. A simple pendulum is constructed by attaching a bob of mass *m* to a string of length *L* fixed at its upper end. The bob oscillates in a vertical circle. It is found that the speed of the bob is *v* when the string makes an angle θ with the vertical. Find the tension in the string at this instant.

Sol.



The forces acting on the bob are (figure)

(*a*) the tension *T*

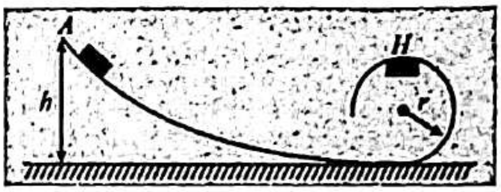
(*b*) the weight mg.

As the bob moves in a vertical circle with centre at *O*, the radial acceleration is v^2/L towards *O*. Taking the components of forces along this radius and applying Newton's second law, we get

$$T - mg\cos\theta = \frac{mv^2}{L}$$

$$\Rightarrow T = mg\cos\theta + \frac{mv^2}{L}$$

18. A small body of mass *m* is allowed to slide on an inclined frictionless track from rest position as shown in the figure.



(i) Find the minimum height *h*, so that body may successfully complete the loop of radius '*r*'.

(ii) If *h* is double of that minimum height, find the resultant force on the block at position *H*.

Sol. (i) To complete the loop minimum velocity at lowest point is $v = \sqrt{5gr}$

from energy conservation

$$\frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2}m(\sqrt{5gr})^2 = mgh \Rightarrow h = \frac{5}{2}r$$

(ii) *h* is double then velocity at position *H* is

$mg(2h) - mg(2r) = \frac{1}{2}mv^2$ (from energy conservation)

$$v = \sqrt{6gr}$$

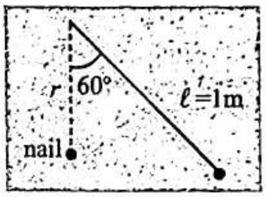
At highest point.

$$F_{net} = \frac{mv^2}{r} = \frac{m(\sqrt{6gr})^2}{r}$$

$$\Rightarrow F_{net} = 6\,mg$$

19. A nail is located at a certain distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from the position where the string makes an angle of 60° from the vertical. Calculate the distance of the nail from the point of suspension such that the bob will just perform revolution with the nail as centre. Assume the length of pendulum to be 1m.

Sol.



velocity at lowest point

$mg\,\ell\,(1 - \cos 60°) = \frac{1}{2}mv^2$ (from energy conservation)

$$v = \sqrt{g\ell}$$

for completing the loop.

$$v = \sqrt{5g(\ell - r)} = \sqrt{g\ell}$$

$$r = \frac{4}{5}\text{ m}$$

20. A particle of mass m is attached at one end of a light, inextensible string of length ℓ whose other end is fixed at the point C. The particle is hanging vertically. At the lowest point the particle is given minimum velocity to complete the circular path in the vertical plane. As it moves in the circular path the tension in the string changes with angle θ which the string make with the vertical. As θ varies from 0 to 2π (i.e. the particle completes one revolution) plot the variation of tension T against θ.

Sol. By Newton's law at B

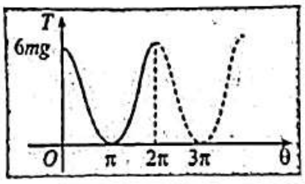
$$T - mg\cos\theta = \frac{mv^2}{\ell}$$

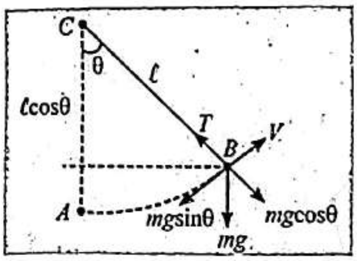
By energy conservation b/w A and B

$$mg\ell(1-\cos\theta) + \frac{1}{2}mv^2 = \frac{1}{2}m(5\ell g)$$

$$mv^2 = 5mg\ell - 2mg\ell(1-\cos\theta)$$

Thus, $T = mg\cos\theta + 5mg - 2mg(1-\cos\theta)$

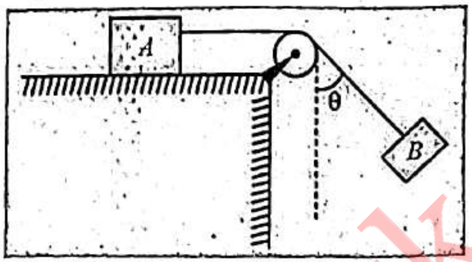




$= 3mg + 3mg\cos\theta$

$= 3mg(1+\cos\theta) = 6mg\cos^2(\theta/2)$

21. Two particles A and B each of mass m are connected by a massless string. A is placed on the rough table. The string passes over a small, smooth peg. B is left from a position making an angle θ with the vertical. Find the minimum coefficient of friction between A and the table so that A does not slip during the motion of mass B.



Sol. Block B rotates in vertical plane. Tension is maximum in string at lowest position. When block B is at lowest position and block A does not slide then it will not slide at any other position of B.

At lowest position

$$T - mg = \frac{mv^2}{\ell} \Rightarrow T = mg + \frac{mv^2}{\ell} \quad \cdots (i)$$

From energy conservation

$$mg\,\ell(1-\cos\theta) = \frac{1}{2}mv^2 \quad \cdots (ii)$$

from equation (*i*) and (*ii*)

$T = mg + 2mg(1-\cos\theta) = 3mg - 2mg\cos\theta$

For no slipping.

$T = \mu mg$

$\Rightarrow 3mg - 2mg\cos\theta \le \mu mg$

$\Rightarrow \mu \ge 3 - 2\cos\theta$

or $\mu_{\min} = 3 - 2\cos\theta$

School Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. A string is used to pull a block of mass m vertically up by a distance h at a constant acceleration $\frac{g}{3}$. The work done by the tension in the string is

(*a*) $\frac{2}{3}mgh$ (*b*) $\frac{-mgh}{3}$

(*c*) mgh (*d*) $\frac{4}{3}mgh$

2. A particle moves along X-axis from $x = 0$ to $x = 1$m under the influence of a force given by $F = (3x^2 + 2x - 10)$N. Work done by the force is

(*a*) +4 J (*b*) –4 J

(*c*) +8 J (*d*) –8 J

3. Two bodies of masses m_1 and m_2 are moving with same kinetic energy. If P_1 and P_2 are their respective momentum, the ratio $\frac{P_1}{P_2}$ is equal to

(*a*) $\frac{m_1}{m_2}$ (*b*) $\sqrt{\frac{m_2}{m_1}}$ (*c*) $\sqrt{\frac{m_1}{m_2}}$ (*d*) $\frac{m_1^2}{m_2^2}$

4. Kinetic energy acquired by a mass m in travelling a certain distance d, starting from rest, under the action of a constant force F is

(*a*) Directly proportional to $\sqrt{m}$

(*b*) Directly proportional to m

(*c*) Directly proportional to $\frac{1}{m}$

(*d*) Independent of m

5. The power of a pump, which can pump 500 kg of water to height 100 m in 10 s is
   (*a*) 75 kW (*b*) 25 kW (*c*) 50 kW (*d*) 500 kW

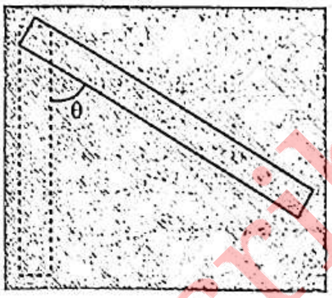
6. A pump is used to pump a liquid of density ρ continuously through a pipe of uniform cross sectional area A. If liquid is flowing with speed v, then power of pump is
   (*a*) $\frac{1}{3}\rho A v^2$ (*b*) $\frac{1}{2}\rho A v^2$ (*c*) $2\rho A v^2$ (*d*) $\frac{1}{2}\rho A v^3$

7. A particle of mass 200 g, is whirled into a vertical circle of radius 80 cm using a massless string. The speed of the particle when the string makes an angle of 60° with the vertical line is 1.5 ms^{-1}. The tension in the string at this position is
   (*a*) 1 N (*b*) 1.56 N (*c*) 2 N (*d*) 3 N

8. The total work done on a particle is equal to the change in its kinetic energy. This is applicable
   (*a*) Always
   (*b*) Only if the conservative forces are acting on it
   (*c*) Only in inertial frames
   (*d*) Only when pseudo forces are absent

9. Potential energy is defined
   (*a*) Only in conservative fields
   (*b*) As the work done by conservative forces
   (*c*) As the negative of workdone by external forces when $\Delta K = 0$
   (*d*) All of these

10. A stick of mass m and length l is pivoted at one end and is displaced through an angle θ. The increase in potential energy is



   (*a*) $mg\frac{l}{2}(1-\cos\theta)$ (*b*) $mg\frac{l}{2}(1+\cos\theta)$
   (*c*) $mg\frac{l}{2}(1-\sin\theta)$ (*d*) $mg\frac{l}{2}(1+\sin\theta)$

Directions: These questions consist of two statements each, printed as Assertion and Reason. While answering these questions, you are required to choose any one of the following four responses.

(*a*) Both Assertion and Reason are True and the Reason is a correct explanation of the Assertion.
(*b*) Both Assertion and Reason are True but Reason is not a correct explanation of the Assertion.
(*c*) Assertion is True but the Reason is False.
(*d*) Assertion and Reason are False

11. **Assertion:** The potential energy can be defined only in conservative field.
    Reason: The value of potential energy depends on the reference level (level of zero potential energy).

12. **Assertion:** When a body moves in a circle the work done by the centripetal force is always zero.
    Reason: Centripetal force is perpendicular to displacement at every instant.

VERY SHORT ANSWER TYPE QUESTIONS

13. Does KE depend upon the direction of motion?
14. Is work done by a non-conservative force always negative. Comment.
15. Can PE of a body be negative?
16. Does the work done in raising a body depend upon how fast it is raised?
17. What is the work done by centripetal force?
18. How does PE of a spring change when it is compressed or stretched?
19. A light and a heavy body have same kinetic energy, which body has more momentum?

SHORT ANSWER TYPE QUESTIONS

20. Comment on change in potential energy
    (*a*) A body is taken against the gravitational force
    (*b*) Air bubble rises up in water
    (*c*) A spring is stretched

21. A man rowing a boat upstream is at rest with respect to the shore
    (i) Comment on the work done by him
    (ii) is any work done by him when he stops rowing and moves along with the stream?

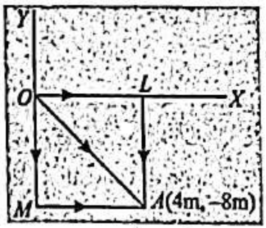
LONG ANSWER TYPE QUESTIONS

22. Compare the properties of conservative and non-conservative forces with regard to
    (i) Potential energy
    (ii) Reversibility of work done
    (iii) Dependence on the path
    (iv) Work done in a round trip. Give examples

23. A small object of weight w hangs from a string of length l. A variable horizontal force, F, which starts at zero value and increases gradually is used to pull the object very slowly until the string makes an angle θ with the vertical. Prove that the work done by the force is $wl(1 - \cos\theta)$.

CASE STUDY BASED QUESTIONS

24. A force $\vec{F}=(4\hat{i}+3\hat{j})$ N acts on a particle of mass 2 kg. The particle under the action of this force moves from the origin to a point A(4 m, –8m). Initial speed of the particle, i.e., its speed at the origin is $2\sqrt{6}\ \text{ms}^{-1}$. Figure shows three paths for the motion of the particle from O to A.



(i) Which of the following is correct?

(*a*) There is equal probability for the force being conservative or non-conservative.

(*b*) Conservative or non-conservative nature of the force cannot be predicted on the basis of the given information.

(*c*) The force is non-conservative.

(*d*) The force is conservative.

(ii) Speed of the particle at A will be nearly

(*a*) 4.0 ms^{-1}
(*b*) 2.8 ms^{-1}
(*c*) 3.6 ms^{-1}
(*d*) 5.6 ms^{-1}

(iii) If the potential energy at O is 16 J, the potential energy at A will be

(*a*) 14.5 J
(*b*) 32 J
(*c*) 24 J
(*d*) 40 J

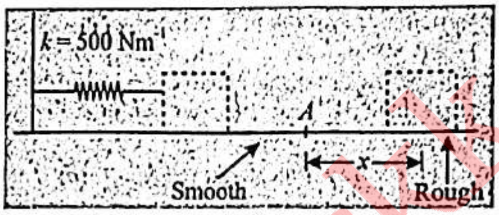
(iv) Considering the given force $\vec{F}=(4\hat{i}+3\hat{j})N$ acting on a particle of mass 2kg, What is the magnitude of the acceleration experienced by the particle?

(*a*) 2.5 m/s^2
(*b*) 0.5 m/s^2
(*c*) 2 m/s^2
(*d*) 1.5 m/s^2

(v) Calculate the work done for mass 2kg particle under the influence of force $\vec{F}=(4\hat{i}+3\hat{j})$ along path OA.

(*a*) –8 J
(*b*) 8 J
(*c*) 80 J
(*d*) –80 J

25. A small block of mass 50 g, compressing a spring (of spring constant $k=500\ \text{Nm}^{-1}$) by 10 cm is released on a horizontal surface. The surface to the right of A is rough ($\mu=0.2$). Find the following:



(i) The velocity with which the block separates from the spring is:

(*a*) 0.5 ms^{-1}
(*b*) 1 ms^{-1}
(*c*) 1.5 ms^{-1}
(*d*) None of these

(ii) The velocity (v) of the block as a function of the distance 'x' covered on the rough horizontal surface.

(*a*) $v=\sqrt{0.25+4x}$
(*b*) $v=\sqrt{0.25-4x}$
(*c*) $v=\sqrt{0.5-4x}$
(*d*) None of these

(iii) Find the maximum value of x (when the block stops)

(*a*) 1.25 cm
(*b*) 25 m
(*c*) 6.25 cm
(*d*) 2.5 m

(iv) The velocity (v) of the block as a function of time (t), when it moves on to the rough surface.

(*a*) $v=0.5+2t$
(*b*) $v=0.5-2t$
(*c*) $v=10-2t$
(*d*) None of these

(v) Calculate the potential energy stored in the spring when it is compressed?

(*a*) 0.25 J
(*b*) 0.5 J
(*c*) 1J
(*d*) 2.5 J

Prarambh (Topicwise)

DEFINITION OF WORK

1. A man pushes wall and fails to displace it. He does

(*a*) Negative work
(*b*) Positive but not maximum work
(*c*) No work at all
(*d*) Maximum work

2. If the unit of force and length each be increased by four times, then the unit of energy is increased by

(*a*) 16 times
(*b*) 8 times
(*c*) 2 times
(*d*) 4 times

WORK DONE BY CONSTANT FORCE, VARIABLE FORCE

3. You lift a heavy book from the floor of the room and keep it in the book-shelf having a height 2 m. In this process you take 5 seconds. The work done by you will depend upon

(*a*) Mass of the book and time taken
(*b*) Weight of the book and height of the book-shelf
(*c*) Height of the book-shelf and time taken
(*d*) Mass of the book, height of the book-shelf and time taken

4. A body of mass m kg is lifted by a man to a height of one metre in 30 sec. Another man lifts the same mass to the same height in 60 sec. The work done by them are in the ratio

(*a*) 1 : 2 (*b*) 1 : 1 (*c*) 2 : 1 (*d*) 4 : 1

5. The minimum work done against gravity in taking 10 kg mass to 1m height will be

(*a*) 49 J (*b*) 98 J
(*c*) 196 J (*d*) None of these

6. A particle moves from position $\vec{r}_1 = 3\hat{i} + 2\hat{j} - 6\hat{k}$ to position $\vec{r}_2 = 14\hat{i} + 13\hat{j} + 9\hat{k}$ under the action of force $4\hat{i} + \hat{j} + 3\hat{k}$ N. The work done will be

(*a*) 100 J (*b*) 50 J (*c*) 200 J (*d*) 75 J

7. A force $(\vec{F}) = 3\hat{i} + c\hat{j} + 2\hat{k}$ acting on a particle causes a displacement $(\vec{s}) = -4\hat{i} + 2\hat{j} + 3\hat{k}$. If the work done by force $\vec{F}$ is 6 J, then the value of 'c' is

(*a*) 0 (*b*) 1 (*c*) 6 (*d*) 12

8. A body of mass 6kg is moving under the action of force which causes displacement in it given by $S = \frac{t^2}{4}$ metres where t is time. The work done by the force in 2 seconds is

(*a*) 12 J (*b*) 9 J
(*c*) 6 J (*d*) 3 J

9. A cord is used to lower vertically a block of mass M by a distance d with constant downward acceleration $\frac{g}{4}$. Work done by the cord on the block is

(*a*) $Mg\frac{d}{4}$ (*b*) $3Mg\frac{d}{4}$

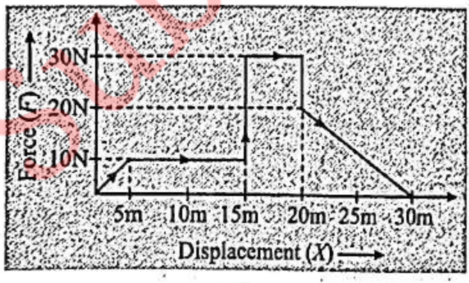
(*c*) $-3Mg\frac{d}{4}$ (*d*) Mgd

10. The force constant of a wire is k and that of another wire is $2k$. When both the wires are stretched through same distance, then the work done

(*a*) $W_2 = \frac{W_1}{2}$ (*b*) $W_2 = 2W_1$

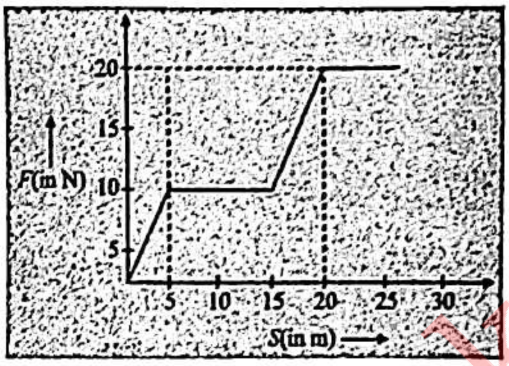
(*c*) $W_2 = W_1$ (*d*) $W_2 = 0.25\,W_1$

11. Given below is a graph between a variable force (F) (along y-axis) and the displacement (X) (along x-axis) of a particle is one dimension. The work done by the force in the displacement interval between 0m and 30 m is



(*a*) 275 J (*b*) 375 J (*c*) 400 J (*d*) 300 J

12. The work done by a force acting on a body is as shown in the graph. The total work done in covering an initial distance of 20 m is



(*a*) 225 J (*b*) 200 J (*c*) 400 J (*d*) 175 J

13. A rigid body moves a distance of 10 m along a straight line under the action of a force of 5 N. If the work done by this force on the body is 25 joules, the angle which the force makes with the direction of motion of the body is

(*a*) 0° (*b*) 30° (*c*) 60° (*d*) 90°

14. A particle moves from position $\vec{r}_1 = 6\hat{i} + 4\hat{j} - 12\hat{k}$ to position $\vec{r}_2 = 12\hat{i} + 8\hat{j} - 6\hat{k}$ under the action of force $4\hat{i} + \hat{j} + 3\hat{k}$ N. The work done by this force will be

(*a*) 46 J (*b*) 50 J (*c*) 200 J (*d*) 75 J

15. A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third second of the motion of the ball is

(*a*) 1 : 2 : 3 (*b*) 1 : 4 : 9 (*c*) 1 : 3 : 5 (*d*) 1 : 5 : 3

16. A force acting on a particle varies with the displacement x as $F = ax - bx^2$. Where $a = 1$ N/m and $b = 1$ N/m^2. The work done by this force for the first one meter (F is in newtons, x is in meters) is

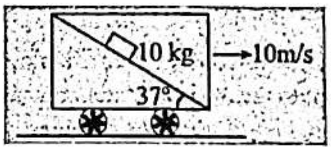
(*a*) $\frac{1}{6}$J (*b*) $\frac{2}{6}$ J

(*c*) $\frac{3}{6}$J (*d*) None of these

17. A force $\vec{F} = (3x\,\hat{i} + 4\,\hat{j})$ Newton (where x is in metres) acts on a particle which moves from a position (2 m, 3 m) to (3 m, 0 m). Then the work done is

(*a*) 7.5 J (*b*) –12 J (*c*) –4.5 J (*d*) +4.5 J

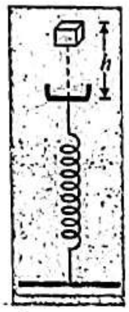
18. A block of mass 10 kg is released on a fixed wedge inside a cart which is moving with constant velocity 10 ms^{-1} towards right. There is no relative motion between block and cart. Then work done by normal reaction on block in two seconds from ground frame will be ($g = 10$ ms^{-2})



(*a*) 1320 J (*b*) 960 J (*c*) 1200 J (*d*) 240 J

WORK ENERGY THEOREM

19. A block of mass m initially at rest is dropped from a height h on to a spring of force constant k. The maximum compression in the spring is x then



(a) $mgh = \frac{1}{2}kx^2$

(b) $mg(h+x) = \frac{1}{2}kx^2$

(c) $mgh = \frac{1}{2}k(x+h)^2$

(d) $mg(h+x) = \frac{1}{2}k(x+h)^2$

20. A block of mass 3 kg starts from rest and slides down a curved path in the shape of a quarter-circle of radius 2 m and reaches the bottom of path with a 4 m/s speed. If g is 10 m/s^2, the amount of work done against friction is

(a) 60 J (b) 36 J (c) 24 J (d) 12 J

21. A truck accelerates from speed v to 2v. Work done in doing this is

(a) Three times as the work done in accelerating it from rest to v.

(b) Same as the work done in accelerating it from rest to v.

(c) Four times as the work done in accelerating it from rest to v.

(d) Less than the work done in accelerating it from rest to v.

22. If K_i and K_f are the initial and final values of kinetic energy of a body respectively, then the work done by the net force on the body is equal to

(a) $\frac{K_f K_i}{K_f - K_i}$ (b) $K_f + K_i$

(c) $\frac{K_f + K_i}{2}$ (d) $K_f - K_i$

23. A body possessing kinetic energy T moving on a rough horizontal surface is stopped in a distance y. The frictional force exerted on the body is

(a) Ty (b) $\frac{\sqrt{T}}{y}$ (c) $\frac{T}{y}$ (d) $\frac{T}{\sqrt{y}}$

24. A force of 20 N is applied on a body of mass 5 kg resting on a horizontal plane. The body gains a kinetic energy of 10 J after it moves a distance 2m. The frictional force is

(a) 10 N (b) 15 N (c) 20 N (d) 30 N

25. The work done in slowly pushing a block of mass 10 kg from bottom to the top of a frictionless inclined plane 5 m long and 3 m high is- ($g = 9.8$ m/sec^2)

(a) 392 J (b) 294 J (c) 98 J (d) 0.98 J

26. Two equal masses are attached to the two ends of a spring of spring constant k. The masses are pulled out symmetrically to stretch the spring by a length x over its natural length. The work done by the spring on each mass during the above pulling is

(a) $\frac{1}{2}kx^2$ (b) $-\frac{1}{2}kx^2$

(c) $\frac{1}{4}kx^2$ (d) $-\frac{1}{4}kx^2$

27. A particle of mass m at rest is acted upon by a force F for a time t. Its kinetic energy after an interval t is

(a) $\frac{F^2t^2}{m}$ (b) $\frac{F^2t^2}{2m}$ (c) $\frac{F^2t^2}{3m}$ (d) $\frac{Ft}{2m}$

28. A body starts from rest with uniform acceleration and acquires a velocity V in time T. The instantaneous kinetic energy of the body after any time t is proportional to:

(a) $(V/T)t$ (b) $(V^2/T)t^2$

(c) $(V^2/T^2)t$ (d) $(V^2/T^2)t^2$

29. A body of mass m accelerates uniformly from rest to a speed v_0 in time t_0. The work done on the body till any time t is

(a) $\frac{1}{2}mv_0^2\left(\frac{t^2}{t_0^2}\right)$ (b) $\frac{1}{2}mv_0^2\left(\frac{t_0}{t}\right)$

(c) $mv_0^2\left(\frac{t}{t_0}\right)$ (d) $mv_0^2\left(\frac{t}{t_0}\right)^3$

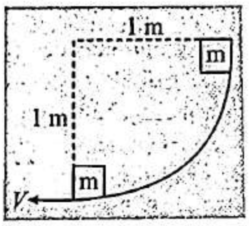
30. A body moving at 2 m/s can be stopped over a distance x. If its kinetic energy is doubled, how long will it go before coming to rest, if the retarding force remains unchanged?

(a) x (b) $2x$ (c) $4x$ (d) $8x$

31. The work done in joules in increasing the extension of a spring of stiffness 10 N/cm from 4 cm to 6 cm is:

(a) 1 (b) 10 (c) 50 (d) 100

32. A block of mass 1 kg slides down a curved track which forms one quadrant of a circle of radius 1 m as shown in figure. The speed of block at the bottom of the track is $v = 2$ ms^{-1}. The work done by the force of friction is



(a) +4 J (b) −4 J (c) −8 J (d) +8 J

POWER

33. An engine develops 10 kW of power. How much time will it take to lift a mass of 200 kg to a height of 40 m? (Take g = 10 ms^{-2}):

(a) 4s (b) 5s (c) 8s (d) 10s

34. A body of mass m accelerates uniformly from rest to v_1 in time t_1. As a function of time t, the instantaneous power delivered to the body is

(*a*) $\frac{mv_1t}{t_1}$ (*b*) $\frac{mv_1^2t}{t_1}$ (*c*) $\frac{mv_1t^2}{t_1}$ (*d*) $\frac{mv_1^2t}{t_1^2}$

35. A man is riding on a cycle with velocity 7.2 km/hr up a hill having a slope 1 in 20. The total mass of the man and cycle is 100 kg. The power of the man is

(*a*) 200 W (*b*) 175 W (*c*) 125 W (*d*) 98 W

36. An electric motor creates a tension of 4500 newton in a hoisting cable and reels it in at the rate of 2 m/sec. What is the power of electric motor

(*a*) 15 kW (*b*) 9 kW

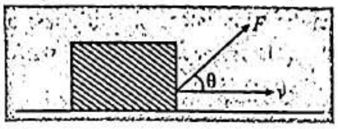
(*c*) 225 W (*d*) 9000 HP

37. A weight lifter lifts 300 kg from the ground to a height of 2 meter in 3 second. The average power generated by him is

(*a*) 5880 watt (*b*) 4410 watt

(*c*) 2205 watt (*d*) 1960 watt

38. A constant force F is acting on a body of mass m with constant velocity v as shown in the figure. The power P exerted is



(*a*) $F\cos\theta v$ (*b*) $F\cos\theta/mg$

(*c*) $F\,mg\cos\theta/v$ (*d*) $mg\sin\theta/F$

39. The power of a pump, which can pump 200 kg of water to a height of 200 m in 10 sec is ($g = 10$ m/s^2)

(*a*) 40 kW (*b*) 80 kW

(*c*) 400 kW (*d*) 960 kW

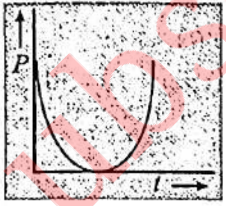
40. A force applied by an engine of a train of mass 2.05×10^6 kg changes its velocity from 5 m/s to 25 m/s in 5 minutes. The power of the engine is

(*a*) 1.025 MW (*b*) 2.05 MW

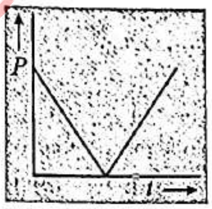
(*c*) 5 MW (*d*) 6 MW

41. A particle is projected at $t = 0$ from a point on the ground with certain velocity at an angle with the horizontal. The power of gravitation force is plotted against time. Which of the following is the best representation?

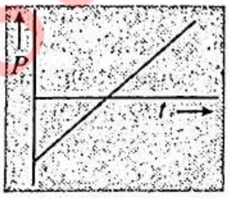
(*a*)



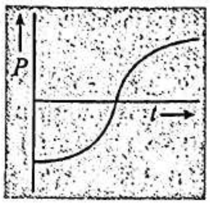
(*b*)



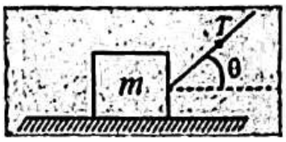
(*c*)



(*d*)



42. A block of mass m slides along the track with kinetic friction coefficient μ. A man pulls the block through a rope which makes an angle θ with the horizontal as shown in the figure. The block moves with constant speed v. Power delivered by man is



(*a*) Tv (*b*) $Tv\cos\theta$

(*c*) $(T\cos\theta - \mu mg)v$ (*d*) Zero

POTENTIAL ENERGY AND EQUILIBRIUM

43. In which case does the potential energy decrease

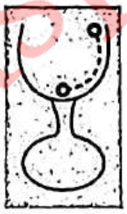
(*a*) On compressing a spring

(*b*) On stretching a spring

(*c*) On moving a body against gravitational force

(*d*) On the rising of an air bubble in water

44. A 2g ball of glass is released from the edge of a hemispherical cup whose radius is 20 cm. How much work is done on the ball by the gravitational force during the ball's motion to the bottom of the cup?



(*a*) 1.96 mJ (*b*) 3.92 mJ (*c*) 4.90 mJ (*d*) 5.88 mJ

45. A uniform chain of length L and mass m is kept on a smooth table. It is released from rest when the overhanging part was n^{th} fraction of total length. Find the kinetic energy of the chain as it completely slips off the table:

(*a*) $\frac{1}{2}mgL(1-n^2)$ (*b*) $2mgL(1-n^2)$

(*c*) $\frac{1}{2}mgL(n^2-1)$ (*d*) $2mgL(n^2-1)$

46. In the stable equilibrium position, a body has

(*a*) Maximum potential energy

(*b*) Minimum potential energy

(*c*) Minimum kinetic energy

(*d*) Zero kinetic energy

47. The potential energy of a conservative system is given by $V(x) = (x^2 - 3x)$ joule, where x is measured in metre. Then its equilibrium position is at

(*a*) 1.5 m (*b*) 2 m (*c*) 3 m (*d*) 1 m

48. When a conservative force does positive work on a body

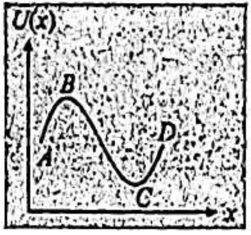
(*a*) The potential energy increases

(*b*) The potential energy decreases

(*c*) Total energy increases

(*d*) Total energy decreases

49. The potential energy of a particle varies with distance x as shown in the graph.

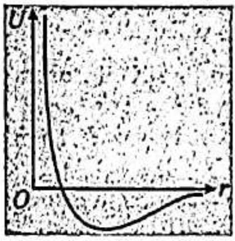


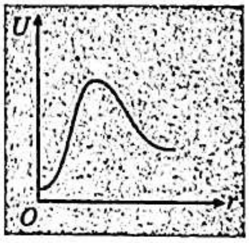
The force acting on the particle is zero at

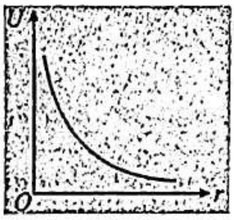
(a) C (b) B

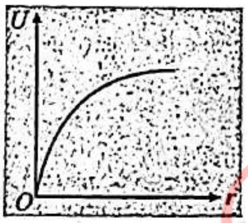
(c) B and C (d) A and D

50. The diagrams represent the potential energy U of a function of the inter-atomic distance r. Which diagram corresponds to stable molecules found in nature.

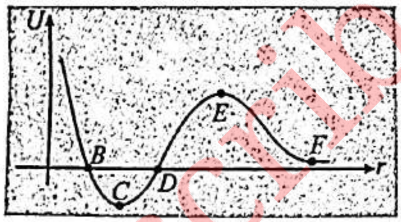
(a)


(b)


(c)


(d)


51. The given plot shows the variation of U, the potential energy of interaction between two particles with the distance separating them r.



1. B and D are equilibrium points
2. C is a point of stable equilibrium
3. The force of interaction between the two particles is attractive between points C and D and repulsive between D and E
4. The force of interaction between particles is repulsive between points E and F.

Which of the above statements are correct?

(a) 1 and 2 (b) 1 and 4

(c) 2 and 4 (d) 2 and 3

VERTICAL CIRCULAR MOTION

52. A small block is freely sliding down from top of a smooth inclined plane. The block reaches bottom of inclined plane then the block describes a vertical circle of radius 0.5 m along smooth track. The minimum vertical height of the inclined plane should be

(a) 1m (b) 1.25 m (c) 3m (d) 2.5 m

53. A simple pendulum is oscillating with an angular amplitude 60°. If mass of bob is 50 gram, the tension in the string at mean position is ($g = 10ms^{-2}$)

(a) 0.5 N (b) 1 N (c) 1.5 N (d) 2 N

54. A body of mass m is rotating in a vertical circle of radius 'r' with critical speed. The difference in its K.E. at the top and the bottom is

(a) 2 mgr (b) 4 mgr (b) 6 mgr (d) 3 mgr

55. A ball of mass 0.6 kg attached to a light inextensible string rotates in a vertical circle of radius 0.75m such that it has speed $5ms^{-1}$ when the string is horizontal. Tension in string when it is horizontal on other side is ($g = 10\ ms^{-2}$)

(a) 30N (b) 26N (c) 20N (d) 6N

56. A body of mass m is rotated at uniform speed along vertical circle with help of light string. If T_1, T_2 are tensions in the string when the body is crossing highest and lowest point of vertical circle respectively then which of following expression is correct.

(a) $T_2 - T_1 = 6\,mg$ (b) $T_2 - T_1 = 4\,mg$

(c) $T_2 - T_1 = 2\,mg$ (d) $T_2 - T_1 = mg$

57. A small bucket containing water is rotated in a vertical circle of radius R by means of a rope. V is velocity of bucket at highest point, then water will not fall down if

(a) $V \le \sqrt{gR}$ (b) $V \le \sqrt{gR/2}$

(c) $V \ge \sqrt{gR}$ (d) $V \ge \sqrt{\frac{gR}{2}}$

58. A sphere is suspended by a thread of length ℓ. What minimum horizontal velocity has to be imparted to the ball for it to reach the height of the suspension?

(a) $g\ell$ (b) $2g\ell$ (c) $\sqrt{g\ell}$ (d) $\sqrt{2g\ell}$

59. A ball is moving to and fro about the lowest point A of a smooth hemispherical bowl. If it is able to rise up to a height of 70 cm on either side of A, its speed at A must be (mass of the ball = 10g and take g = 9.8 m/s^2)

(a) 3.7 m/s (b) 5 m/s (c) 4.8 m/s (d) 6 m/s

60. A body crosses the top most point of a vertical circle with critical speed. Its centripetal acceleration, when the string is at the top point of circle is

(a) 3 g (b) g (c) 5 g (d) 6 g

61. One end of the string of length 1.0 m is tied to a body of mass 0.5 kg. It is whirled in a vertical circle with angular frequency 4 rad/s. The tension in the string when the body is at the lower most point of its motion will be equal to (take, g = 10 m/s^2)

(a) 3 N (b) 5 N (c) 8 N (d) 13 N

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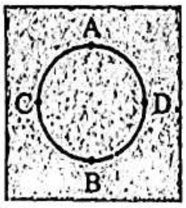
62. A stone is attached to one end of a string and rotated in a vertical circle. If string breaks at the position of maximum tension, it will break at



(*a*) A (*b*) B (*c*) C (*d*) D

63. A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angle 30° and 60° from vertical (lowest position) are T_1 and T_2 respectively, then

(*a*) $T_1 = T_2$

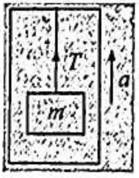
(*b*) $T_1 > T_2$

(*c*) $T_1 < T_2$

(*d*) Tension in the string always remains the same

Prabal (JEE Main Level)

1. A block of mass m is suspended by a light thread from an elevator. The elevator is accelerating upward with uniform acceleration a. The work done by tension on the block during t seconds is $(u = 0)$



(*a*) $\frac{m}{2}(g+a)\,at^2$ (*b*) $\frac{m}{2}(g-a)at^2$

(*c*) $\frac{m}{2}gat^2$ (*d*) 0

2. Two springs have their force constant as k_1 and $k_2 (k_1 > k_2)$. When they are stretched by the same force up to equilibrium

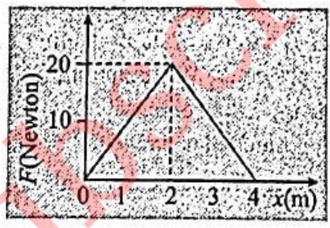
(*a*) No work is done by this force in case of both the springs

(*b*) Equal work is done by this force in case of both the springs

(*c*) More work is done by this force in case of second spring

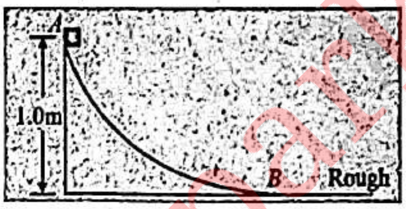
(*d*) More work is done by this force in case of first spring

3. The graph between the resistive force F acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 kg and initial velocity is 2 m/s. When the distance covered by the body is 4m, its kinetic energy would be



(*a*) 50 J (*b*) 40 J (*c*) 20 J (*d*) 10 J

4. A block weighing 10 N travels down a smooth curved track AB joined to a rough horizontal surface (figure). The rough surface has a friction coefficient of 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface, the distance it will move on the rough surface is

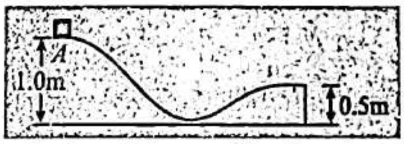


(*a*) 5.0 m (*b*) 10.0 m (*c*) 15.0 m (*d*) 20.0 m

5. A light spring of length 20 cm and force constant 2N/cm is placed vertically on a table. A small block of mass 1 kg falls on it. The length h from the surface of the table at which the ball will have the maximum velocity is

(*a*) 20 cm (*b*) 15 cm (*c*) 10 cm (*d*) 5 cm

6. Figure shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground?



(*a*) At a horizontal distance of 1 m from the end of the track.

(*b*) At a horizontal distance of 2 m from the end of the track.

(*c*) At a horizontal distance of 3 m from the end of the track.

(*d*) Insufficient information

7. A block of mass m is moving with a constant acceleration 'a' on a rough horizontal plane. If the coefficient of friction between the block and plane is μ. The power delivered by the external agent at a time t from the beginning is equal to

(*a*) ma^2t (*b*) $\mu mgat$

(*c*) $\mu m(a + \mu g)\,gt$ (*d*) $m(a + \mu g)\,at$

8. A particle moves with a velocity $\vec{v} = (5\hat{i} - 3\hat{j} + 6\hat{k})$ m/s under the influence of a constant force $\vec{F} = (10\hat{i} + 10\hat{j} + 20\hat{k})$ N. The instantaneous power applied to the particle is

(*a*) 200 J/s (*b*) 40 J/s (*c*) 140 J/s (*d*) 170 J/s

9. A man M_1 of mass 80 kg runs up a staircase in 15 s. Another man M_2 also of mass 80 kg runs up the stair case in 20 s. The ratio of the power developed by them (P_1 / P_2) will be

(*a*) 1 (*b*) 4/3

(*c*) 16/9 (*d*) None of the above

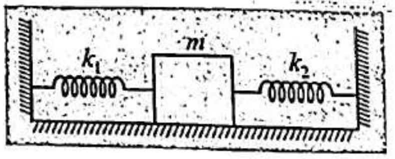
10. A pump ejects 12000 kg of water at speed of 4 m/s in 40 second. Find the average power at which the pump is working

(*a*) 0.24 kW (*b*) 2.4 W (*c*) 2.4 kW (*d*) 24 W

11. Two springs *A* and *B* ($k_A = 2k_B$) are stretched by applying forces of equal magnitudes at the four ends. If the energy stored in *A* is *E*, then the energy stored in *B* is

(*a*) *E*/2 (*b*) 2*E* (*c*) *E* (*d*) *E*/4

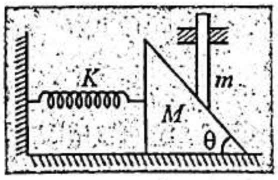
12. A block of mass *m* is attached to two unstretched springs of spring constants k_1 and k_2 as shown in figure. The block is displaced towards right through a distance *x* and is released. Find the speed of the block as it passes through the mean position shown.



(*a*) $\sqrt{\frac{k_1+k_2}{m}}x$ (*b*) $\sqrt{\frac{k_1k_2}{m(k_1+k_2)}}x$

(*c*) $\sqrt{\frac{k_1^2k_2^2}{m(k_1^2+k_2^2)}}x$ (*d*) $\sqrt{\frac{k_1^3k_2^3}{m(k_1^3+k_2^3)}}x$

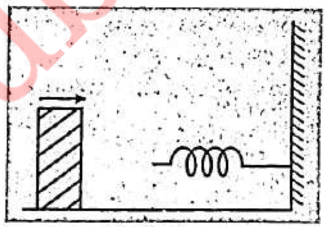
13. A wedge of mass *M* fitted with a spring of stiffness '*K*' is kept on a smooth horizontal surface. A rod of mass *m* is kept on the wedge as shown in the figure. System is in equilibrium. Assuming that all surfaces are smooth, the potential energy stored in the spring is



(*a*) $\frac{mg^2\tan^2\theta}{2K}$ (*b*) $\frac{m^2g\tan^2\theta}{2K}$

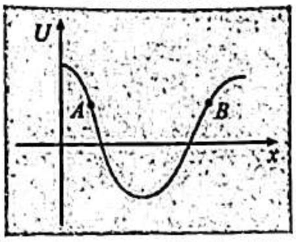
(*c*) $\frac{m^2g^2\tan^2\theta}{2K}$ (*d*) $\frac{m^2g^2\tan^2\theta}{K}$

14. A 1.0 kg block collides with a horizontal weightless spring of force constant 2.75 Nm^{-1} as shown in figure. The block compresses the spring 4.0 m from the rest position. If the coefficient of kinetic friction between the block and horizontal surface is 0.25, the speed of the block at the instant of collision is



(*a*) 0.4 ms^{-1} (*b*) 4 ms^{-1} (*c*) 0.8 ms^{-1} (*d*) 8 ms^{-1}

15. Potential energy v/s displacement curve for one dimensional conservative field is shown. Force at *A* and *B* is respectively



(*a*) Positive, Positive (*b*) Positive, Negative

(*c*) Negative, Positive (*d*) Negative, Negative

16. The potential energy for a force field $\vec{F}$ is given by $U(x, y) = \sin(x + y)$ in S.I. units. The force acting on the particle of mass *m* at $\left(0, \frac{\pi}{4}\text{m}\right)$ is

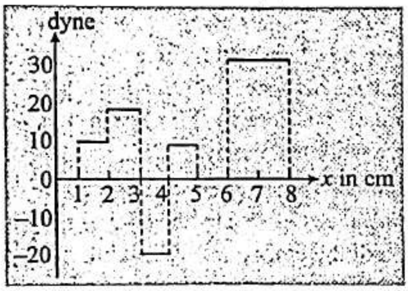
(*a*) 1N (*b*) $\sqrt{2}$ N (*c*) $\frac{1}{\sqrt{2}}$ N (*d*) 0

17. A particle is taken from point *A* to point *B* under the influence of a force field. Now it is taken back from *B* to *A* and it is observed that the work done in taking the particle from *A* to *B* is not equal to the work done in taking it from *B* to *A*. If W_{nc} and W_c is the work done by non-conservative forces and conservative forces present in the system respectively, ΔU is the change in potential energy, Δk is the change in kinetic energy, then

(*a*) $W_{nc} - \Delta U = \Delta k$ (*b*) $W_c = +\Delta U$

(*c*) $W_{nc} - W_c = \Delta k$ (*d*) $W_{nc} - \Delta U = -\Delta k$

18. The relationship between force and position is shown in fig (in one dimensional case). The work done in displacing a body from $x = 1$cm to $x = 5$ cm is



(*a*) 20 erg (*b*) 60 erg (*c*) 70 erg (*d*) 700 erg

19. $F = 2x^2 - 3x - 2$. Choose correct option

(*a*) $x = -1/2$ is position of stable equilibrium

(*b*) $x = 2$ is position of stable equilibrium

(*c*) $x = -1/2$ is position of unstable equilibrium

(*d*) $x = 2$ is position of neutral equilibrium

20. The kinetic energy of a particle continuously increases with time

(*a*) The resultant force on the particle must be parallel to the velocity at all instants.

(*b*) The resultant force on the particle must be at an angle less than 90° with the velocity all the time

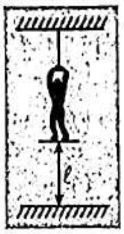
(*c*) Its height above the ground level must continuously decrease

(*d*) The magnitude of its linear momentum is increasing continuously

21. A box of mass m is initially at rest on a horizontal surface. A constant horizontal force of $mg/2$ is applied to the box directed to the right. The coefficient of friction of the surface changes with the distance pushed as $\mu = \mu_0 x$ where x is the distance from the initial location. For what distance is the box pushed until it comes to rest again

(a) $\frac{2}{\mu_0}$ (b) $\frac{1}{\mu_0}$ (c) $\frac{1}{2\mu_0}$ (d) $\frac{1}{4\mu_0}$

22. One end of a light rope is tied directly to the ceiling. A man of mass M initially at rest on the ground starts climbing the rope upto a height ℓ. From the time he starts at rest on the ground to the time he is hanging at rest at a height, how much work was done on the man by the rope?

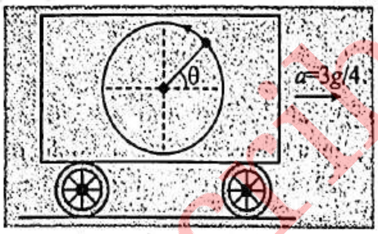


(a) 0
(b) $Mg\ell$
(c) $-Mg\ell$
(d) It depends on how fast the man goes up.

23. A particle of mass m begins to slide down a fixed smooth sphere from the top with negligible initial velocity. What is its tangential acceleration when it breaks off the sphere?

(a) $\frac{2g}{3}$ (b) $\frac{\sqrt{5}g}{3}$ (c) g (d) $\frac{g}{3}$

(For Q. 24 to 26): A bus is moving with a constant acceleration $a = 3g/4$ towards right. In the bus, a ball is tied with a rope of length ℓ and is rotated in vertical circle as shown.



24. At what value of angle θ, tension in the rope will be minimum?

(a) θ = 37° (b) θ = 53°
(c) θ = 30° (d) θ = 90°

25. Find the minimum possible speed V_{min} during whole path to complete the circular motion.

(a) $\sqrt{5g\ell}$ (b) $\frac{5}{2}\sqrt{g\ell}$ (c) $\frac{\sqrt{5g\ell}}{2}$ (d) $\sqrt{g\ell}$

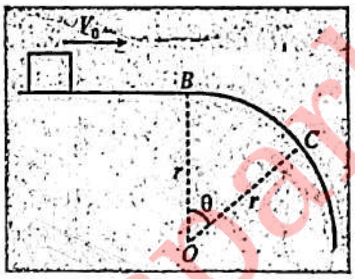
26. Find maximum tension in the string during the motion.

(a) 6 mg (b) $\frac{117}{20}mg$ (c) $\frac{15}{2}mg$ (d) $\frac{17}{2}mg$

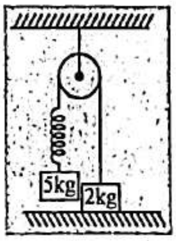
INTEGER TYPE QUESTIONS

27. A stone of mass 1 kg tied to a light inextensible string of length $L = \frac{10}{3}$ m is whirled into a circular path in the vertical plane. The ratio of maximum tension in the string to the minimum tension in the string is 4. If g is taken to be 10 m/s^2, find the speed of the stone at the highest point of the circle (in m/s).

28. A small frictionless block slides with velocity $V_0 = 0.5\sqrt{gr}$ on the horizontal surface as shown in the figure. The block leaves the surface at point C. The angle θ in the figure is $\cos^{-1}\left(\frac{a}{b}\right)$. Find $a + b$.

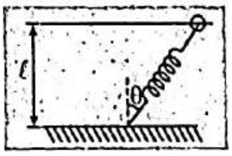


29. The system as shown in the figure is released from rest. The pulley, spring and string are ideal & friction is absent everywhere. If speed of 5 kg block when 2 kg block just leaves the contact with ground is $2\sqrt{x}$ m/s, then value of x is (spring constant $k = 40$ N/m and $g = 10$ m/s^2)

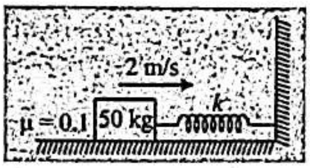


30. A spring (k = 100 Nm^{-1}) is suspended in vertical position having one end fixed at top and other end joined with a 2 kg block. When the spring is in non-deformed shape, the block is given initial velocity 2 m/s in downward direction. The maximum elongation of the spring is $\left(\frac{\sqrt{3}+1}{n}\right)$ meter. Find n.

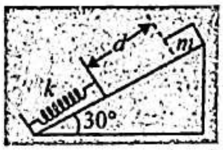
31. One end of a spring of natural length ℓ and spring constant k is fixed at the ground and the other is fitted with a smooth ring of mass m which is allowed to slide on a horizontal rod fixed at a height ℓ (figure). Initially, the spring makes an angle of θ with the vertical when the system is released from rest. If the speed of the ring when the spring becomes vertical is $(2\ell/3)\sqrt{\frac{k}{m}}$, then find the value of angle θ (in degree):



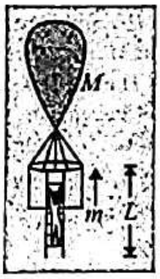
32. A block of mass 50 kg is projected horizontally on a rough horizontal floor. The coefficient of friction between the block and the floor is 0.1. The block strikes a light spring of stiffness $k = 100$ N/m with a velocity 2m/s. What is the maximum compression of the spring (in m): ($k = 100$ N/m)



33. A block of 4 kg mass starts at rest and slides a distance d down a friction less incline (angle 30°) where it runs into a spring of negligible mass. The block slides an additional 25 cm before it is brought to rest momentarily by compressing the spring. The force constant of the spring is 400 Nm^{-1}. The value of d is______ mm. (take $g = 10\ \text{ms}^{-2}$)

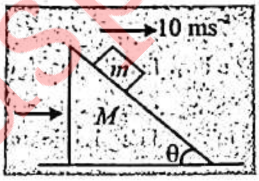


34. A rope ladder of length L = 4m is attached to a balloon of mass M. As the man of mass m climbs the ladder into the balloon basket, the balloon comes down by a vertical distance s = 0.5 m. Then find the increase in potential energy of man divided by the increase in potential energy of balloon.



35. The potential energy (in SI units) of particle of mass $m = 4$ kg freely moving along the x-axis is given by $U = \left(\frac{1}{2}\right)kx^2$ for $x < 0$ and $U = 0$ for $x \geq 0$(x denotes the x-coordinate of the particle and k is a positive constant). If the total mechanical energy of the particle is $E = 25$ J and $k = 2$ N/m then its speed at $x = -5$ m is ________ m/s.

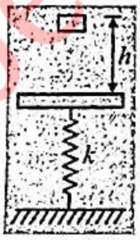
36. In the figure shown all the surfaces are frictionless, and mass of the block is $m = 100$ g. The wedge moves horizontal with an acceleration of 10 ms^{-2} by applying the force on the wedge, so that the block does not slip on the wedge. Find the work done in joules by the normal force in ground frame on the block in 1 s.



Parikshit (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. A vertical spring is fixed to one of its end and a massless plank fitted to the other end. A block is released from a height h as shown. Spring is in relaxed position. Then choose the correct statement.



(*a*) The maximum compression of the spring does not depend on h

(*b*) The maximum kinetic energy of the block does not depend on h

(*c*) The compression of the spring at maximum KE of the block does not depend on h

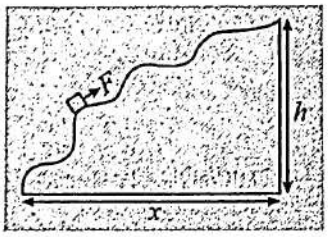
(*d*) The maximum compression of the spring does not depend on k

2. Simple pendulums P_1 and P_2 have lengths $\ell_1 = 80$ cm and $\ell_2 = 100$ cm respectively. The bobs are of mass m_1 and m_2. Initially are at rest in equilibrium position. If each of the bobs is given a displacement of 2 cm, the work done is W_1 and W_2 respectively. Then,

(*a*) $W_1 > W_2$ if $m_1 = m_2$ (*b*) $W_1 < W_2$ if $m_1 = m_2$

(*c*) $W_1 = W_2$ if $\frac{m_1}{m_2} = \frac{5}{4}$ (*d*) $W_1 = W_2$ if $\frac{m_1}{m_2} = \frac{4}{5}$

3. A body of mass m is slowly hauled up the rough hill by a force F which at each point is directed along a tangent to the hill. Work done by the force



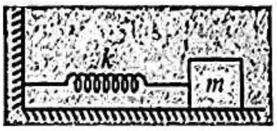
(*a*) Independent of shape of trajectory.

(*b*) Depends upon x.

(*c*) Depends upon h.

(*d*) Depends upon coefficient of friction (μ)

4. A spring block system is placed on a rough horizontal surface having coefficient of friction μ. The spring is given initial elongation $\frac{3\mu mg}{k}$ and the block is released from rest. For the subsequent motion (where m is mass of block and k is spring constant)



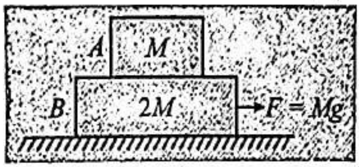
(*a*) Initial acceleration of block is $2\ \mu g$.

(*b*) Maximum compression in spring is $\mu mg/k$

(*c*) Minimum compression in spring is zero.

(*d*) Maximum speed of the block is $2\mu g\sqrt{\frac{m}{k}}$

5. In the figure shown, there is no friction between B and ground and $\mu = 2/3$ between A and B.



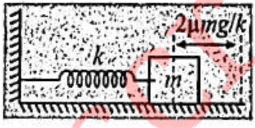
(*a*) The net work done on block A with respect to B is zero

(*b*) The net work done on block A with respect to ground for a displacement 'S' is $MgS/3$

(*c*) The net work done on block B with respect to ground for a displacement 'S' is $2MgS/3$

(*d*) The work done by friction with respect to ground on A and B is equal and opposite in sign.

6. A block of mass 'm' is attached to one end of a massless spring of spring constant 'k'. The other end of the spring is fixed to a wall. The block can move on a horizontal rough surface. The coefficient of friction between the block and the surface is μ. The block is released when the spring has a compression $\frac{2\mu mg}{k}$ then choose the **incorrect** option(s)



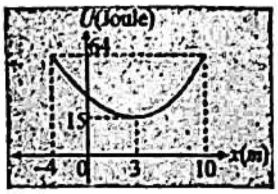
(*a*) The maximum speed of the block is $\mu g\sqrt{\frac{m}{k}}$

(*b*) The maximum speed of the block is $2\mu g\sqrt{\frac{m}{k}}$

(*c*) The block will have velocity towards left during its motion.

(*d*) The extension in the spring at the instant the velocity of block become zero for the first time after being released is $\frac{\mu mg}{k}$.

7. A single conservative force $F(x)$ acts on a particle that moves along the x-axis. The graph of the potential energy with x is given. At $x = 5$ m, the particle has a kinetic energy of 50 J and its potential energy is related to position 'x' as $U = 15+(x-3)^2$ Joule, where x is in meter.



(*a*) The mechanical energy of system is 69 J.

(*b*) The mechanical energy of system is 19J.

(*c*) At $x = 3$, the kinetic energy of particle is minimum.

(*d*) The maximum value of kinetic energy is 54 J.

8. A body of mass 1.0 kg moves in X-Y plane under the influence of a conservative force. Its potential energy is given by $U = 2x + 3y$ where (x, y) denote the coordinates of the body. The body is at rest at $(2, -4)$ initially. All the quantities have SI units. Therefore, the body

(*a*) Moves along a parabolic path

(*b*) Moves with a constant acceleration

(*c*) Never crosses the X axis

(*d*) Has a speed of $2\sqrt{13}$ m/s at time $t = 2$s.

9. A disc of mass $3\ m$ and a disc of mass m are connected by a massless spring of stiffness k. The heavier disc is placed on the ground with the spring vertical and lighter disc on top. From its equilibrium position the upper disc is pushed down by a distance δ and released. Then

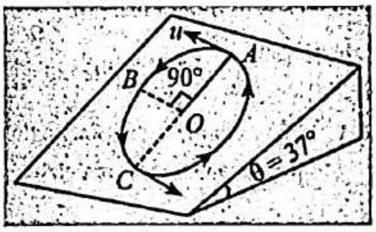
(*a*) If $\delta > \frac{3mg}{k}$, the lower disc will bounce up

(*b*) If $\delta = \frac{2mg}{k}$, maximum normal reaction from ground on lower disc = $6\ mg$

(*c*) If $\delta = \frac{2mg}{k}$, maximum normal reaction from ground on lower disc = $4\ mg$

(*d*) If $\delta > \frac{4mg}{k}$, the lower disc will bounce up

10. A small sphere of mass m is connected by a string to a nail at O and moves in a circle of radius r on the smooth plane inclined at an angle θ with the horizontal. If the sphere has a velocity u at the top position A. Mark the correct option(s).



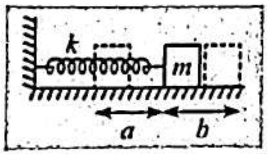
(*a*) Minimum velocity at *A* so that string does not get slack instantaneously is $\sqrt{\frac{3}{5}gr}$.

(*b*) Tension at *B* if sphere has required velocity to just complete circle is $\frac{11}{5}$ mg.

(*c*) Tension at *C* if sphere has required velocity to just complete circle is $\frac{21}{5}$ mg

(*d*) Centripetal force at point *A* is $\frac{3}{5}$ mg if it just get slack instantaneously.

11. The spring is compressed by a distance *a* and released. The block again comes to rest when the spring is elongated by a distance *b*. During this process



(*a*) work done by the spring on the block = $\frac{1}{2}k(a^2 + b^2)$

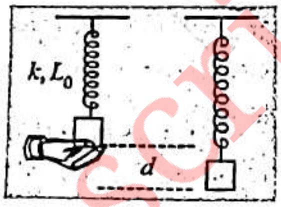
(*b*) work done by the spring on the block = $\frac{1}{2}k(a^2 - b^2)$

(*c*) coefficient of friction = $\frac{k(a-b)}{2mg}$

(*d*) coefficient of friction = $\frac{k(a+b)}{2mg}$

12. A spring having a relaxed length L_o = 25cm has a block attached to one end, while the other end is attached to a ceiling as shown. A person slowly lowers the block a distance *d* until the block just hangs without moving, as shown in the second diagram. Which statements below is/are true? [Take $g = 10\ m/s^2$]

(Use k = 2N/cm and m = 600g.)



(*a*) The distance *d* is 6cm.

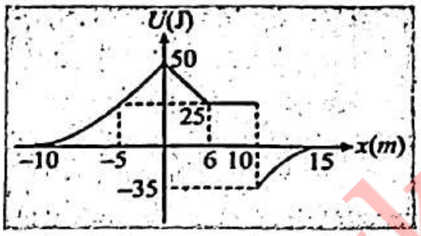
(*b*) The potential energy stored in the spring when the block is simply hanging from it is 0.09J.

(*c*) The change in gravitational potential energy of the Earth-block system is –0.18J.

(*d*) The decrease in gravitational potential energy is equal (in magnitude) to the increase in potential energy stored in the spring.

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 13 to 14): The figure shows the variation of potential energy of a particle as a function of *x*, the *x*-coordinate of the region. It has been assumed that potential energy depends only on *x*. For all other values of *x*, *U* is zero, i.e. for $x < -10$ and $x > 15$, $U = 0$. Based on above information answer the following questions.



13. If total mechanical energy of the particle is 25 J, then it can be found in the region

(*a*) $-10 < x < -5$ and $6 < x < 15$

(*b*) $-10 < x < 0$ and $6 < x < 10$

(*c*) $-5 < x < 6$

(*d*) $-10 < x < 10$

14. If total mechanical energy of the particle is – 40 J, then it can be found in region

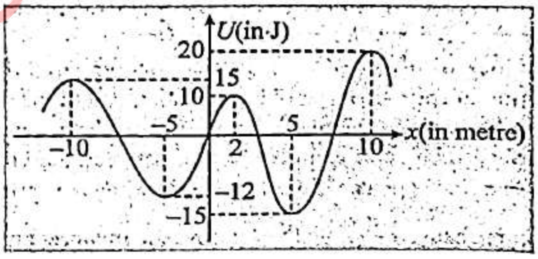
(*a*) $x < -10$ and $x > 15$

(*b*) $-10 < x < -5$ and $6 < x < 15$

(*c*) $10 < x < 15$

(*d*) It is not possible

Comprehension (Q. 15 to 16): In the figure the variation of potential energy of a particle of mass $m = 2$ kg is represented with respect to its *x*-coordinate. The particle moves under the effect of this conservative force along the *x*-axis.



15. If the particle is released at $x = 2 + \Delta x$ where $\Delta x \to 0$ (it is positive) then its maximum speed in subsequent motion will be

(*a*) $\sqrt{10}$ m/s (*b*) 5 m/s (*c*) $5\sqrt{2}$ m/s (*d*) 7.5 m/s

16. $x = -5$ m and $x = 10$ m positions of the particle are respectively are of

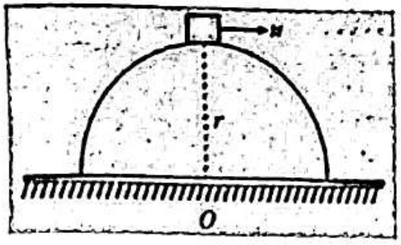
(*a*) Neutral and stable equilibrium.

(*b*) Neutral and unstable equilibrium.

(*c*) Unstable and stable equilibrium.

(*d*) Stable and unstable equilibrium.

Comprehension (Q. 17 to 18): A small block of mass *m* is projected horizontally from the top of the smooth and fixed hemisphere of radius *r* with speed *u* as shown. For values of $u \geq u_0$, ($u_0 = \sqrt{gr}$) it does not slide on the hemisphere. [i.e. leaves the surface at the top itself]



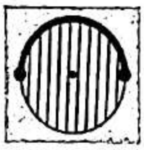
17. For $u = u_0/3$, find the height from the ground at which it leaves the hemisphere.

(*a*) $\frac{19r}{9}$ (*b*) $\frac{19r}{27}$ (*c*) $\frac{10r}{9}$ (*d*) $\frac{10r}{27}$

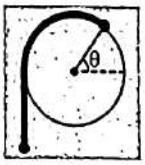
18. Find its net acceleration at the instant it leaves the hemisphere.

(*a*) $g/4$ (*b*) $g/2$ (*c*) g (*d*) $g/3$

Comprehension (Q. 19 to 21): Two identical particles are attached by an inextensible and massless string which passes over a smooth fixed cylinder of radius R as shown in figure. Initially particles lie on same horizontal line and they are slightly displaced.



19. The velocity of particles as a function of θ is given by



(*a*) $\sqrt{gR\sin\theta}$ (*b*) $\sqrt{gR(1-\cos\theta)}$

(*c*) $\sqrt{gR(1-\sin\theta)}$ (*d*) $\sqrt{gR(\theta-\sin\theta)}$

20. The angle θ at which the particle slipping on the cylinder leaves contact with the cylinder, satisfy the relation.

(*a*) $\cos\theta = 2/3$ (*b*) $\sin\theta = \theta$

(*c*) $\sin\theta = \theta/2$ (*d*) $\cos\theta = 1/3$

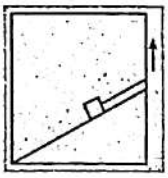
21. The tangential acceleration of particle in contact with cylinder is

(*a*) $a = g\sin\theta$ (*b*) $a = g\sin(\theta/2)$

(*c*) $a = g\sin^2\theta$ (*d*) $a = g\sin^2(\theta/2)$

MATCH THE COLUMN TYPE QUESTIONS

22. A block is placed on a rough wedge fixed on a lift as shown in figure. A string is also attached with the block. The whole system moves upwards. Block does not lose contact with wedge on the block. Match the following two columns regarding the work done (on the block).



	Column-I		Column-II
A.	Work done by normal reaction	p.	Positive
B.	Work done by gravity	q.	Negative
C.	Work done by friction	r.	zero
D.	Work done by tension	s.	Can't say anything

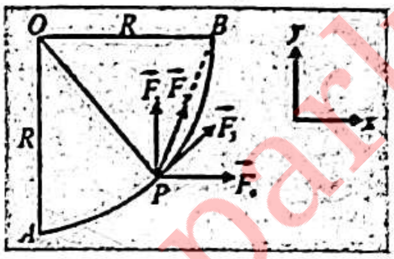
(*a*) A-(q); B-(r); C-(q); D-(r)

(*b*) A-(p); B-(q); C-(p); D-(s)

(*c*) A-(r); B-(p); C-(s); D-(q)

(*d*) A-(q); B-(r); C-(p); D-(r)

23. AB is a quarter of a smooth horizontal circular track of radius R. A particle P of mass m moves along the track from A to B under the action of following forces:



$\vec{F}_1 = F$ (always towards y-axis)

$\vec{F}_2 = F$ (always towards point B)

$\vec{F}_3 = F$ (always along the tangent to path AB)

$\vec{F}_4 = F$ (always towards x-axis)

	Column-I		Column-II
A.	Work done by $\vec{F}_1$	p.	$\sqrt{2}FR$
B.	Work done by $\vec{F}_2$	q.	$\frac{1}{\sqrt{2}}FR$
C.	Work done by $\vec{F}_3$	r.	FR
D.	Work done by $\vec{F}_4$	s.	$\frac{\pi FR}{2}$

(*a*) A-(q); B-(r); C-(q); D-(r)

(*b*) A-(p); B-(q); C-(r); D-(p)

(*c*) A-(r); B-(p); C-(s); D-(r)

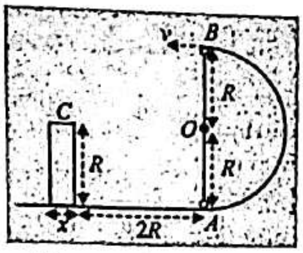
(*d*) A-(q); B-(r); C-(p); D-(r)

NUMERICAL TYPE QUESTIONS

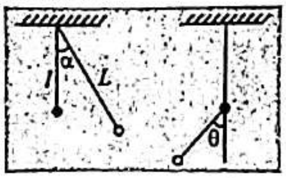
Answer should be rounded off upto two decimal places

24. A small ball is given some velocity at point A towards right so that it move on the semicircular track and does not leave contact up to the highest point. After leaving the highest point, it falls at the top of a building of height R and width ($x << 2R$). (All the surfaces are frictionless)

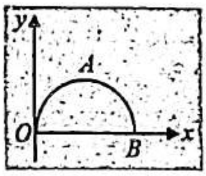
Find the velocity V_0 (in ms^{-1}) given to the ball at point A so that it may hit the top of the building ($R = 0.1$ m, $g = 10$ m/s^2).



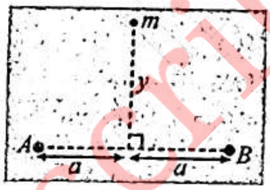
25. A simple pendulum consisting of a mass M attached to a string of length $L = 7\text{m}$ is released from rest at an angle $\alpha = 37°$. A pin is located at a distance $l = 4\text{m}$ below the pivot point. When the pendulum swings down, the string hits the pin as shown in figure. The maximum angle which the string makes with the vertical after hitting the pin is θ. Find the value of $\cos\theta$.



26. Given $\vec{F} = (xy^2)\hat{i} + (x^2y)\hat{j}N$. The work done by $\vec{F}$ when a particle is taken along the semicircular path OAB where the coordinates of B are (4, 0) is

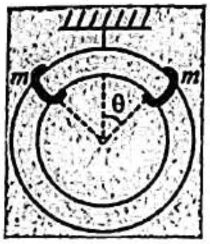


27. Potential energy of a particle of mass $m = 2$ kg, depends on distance y from line AB according to given relation $U = \dfrac{5}{\sqrt{y^2 + a^2}}$ J. The particle is projected from $y = \sqrt{3}$ m towards line AB (perpendicular to it) then what is the minimum velocity (in m/s) so that it cannot return to its initial point? [Given $a = 1$ in S.I unit]



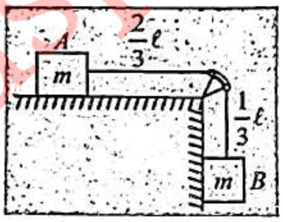
28. The potential energy (in SI units) of a particle of mass 2 kg in a conservative field is $U = 6x - 8y$. If the initial velocity of the particle is $\vec{u} = -1.5\hat{i} + 2\hat{j}$ m/s then find the total distance (in meter) travelled by the particle in first two and a half seconds.

29. A massless ring hangs from a thread and two beads of mass m slide on it without friction. The beads are released simultaneously from the top of the ring and slide down opposite sides. If the angle from vertical at which tension in the thread becomes zero for the first time is θ. Find $\sec\theta$.

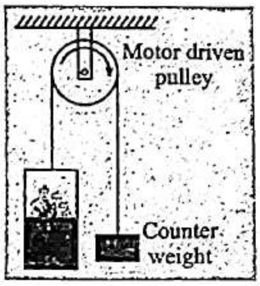


30. A particle is suspended from a fixed point by an inextensible string of length 5m. It is projected from its lowest position in the horizontal direction in a vertical plane with such velocity that the string slackens after the particle has reached a height 8m above the lowest position. The maximum height the particle can gain above this point in meters is ______ [$g = 10$ m/s^2]

31. Two objects A and B of mass m are connected with a uniform rope of length $l = 0.3$ m and mass m as shown in the figure. The rope can slide in a tube which forms a quarter of a circle, and which is fixed to the edge of the table. Initially 1/3 of the rope hangs vertically. Friction is negligible. If the system is released from rest, and B descends a distance of $l/3$, what is the speed (in m/s) of B? [Take $g = 10$ m/s^2]

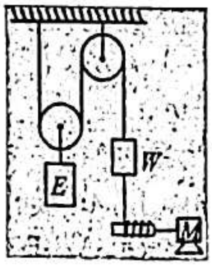


32. An elevator cab, whose total mass with its passengers is 600 kg ascends uniformly. The mass of the counterweight is 300 kg. The mechanical power of the motor is 10 kW and the efficiency is 60%. Find the speed (in m/s) of the lift. [Take $g = 10$ m/s^2]



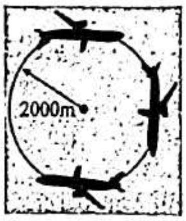
33. A block of mass 'M' = 20 kg rests on the top of a smooth horizontal fixed table top, resting inside a rail car, which is moving with a constant velocity $V_0 = 2$ m/s over a horizontal track. A person P riding in the car pushes the block with a horizontal force $F = 10$ N in the direction of velocity for a time 't' = 2 sec. Then at the end of this interval, the kinetic energy (in J) of the block as assessed by P is less than that computed by another person Q standing at rest on the ground by a value of ______ Joule.

34. The elevator E has a mass of 3000 kg when fully loaded and is connected as shown to a counterweight W of mass 1000 kg. Determine the power in kilowatts delivered by the motor when it has an upward velocity of 3 m/s and a deceleration of 0.5 m/s^2. [Take $g = 10$ m/s^2]



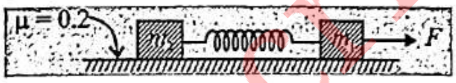
INTEGER TYPE QUESTIONS

35. An aircraft loops the vertical loop as shown with constant speed. At the top most point, the normal force exerted on the pilot by his seat is $\frac{1}{3}$ times the force exerted by the seat at the lowest point. What is the speed (in m/s) of the plane?

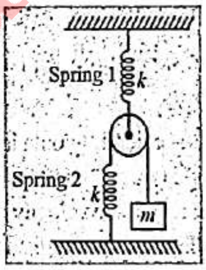


36. A uniform chain of length ℓ and mass m overhangs on a rough horizontal table with its 3/4 part on the table. The friction coefficient between the table and the chain is μ. The magnitude of work done by the friction during the period the chain slips off the table is $\frac{n\mu mgl}{64}$. Find n.

37. Two blocks of masses m_1 and m_2 are connected by a spring of stiffness $k = 200$ Nm^{-1}. The coefficient of friction between the blocks and the surface is μ. Find the minimum constant horizontal force F (in Newton) to be applied to m_1 in order to slide the mass m_2. (Initially spring is in its natural length). (Take $m_1 = 3$ kg, $m_2 = 5$ kg, $g = 10$ m/s^2, $\mu = 0.2$)

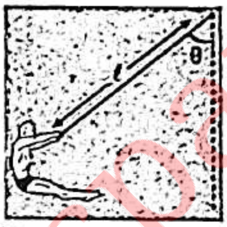


38. All springs, string and pulley shown in figure are light. Initially when both the springs were in their natural state, the system was released from rest. The maximum displacement of block m is $x \times \left(\frac{5mg}{k}\right)$. Calculate x.

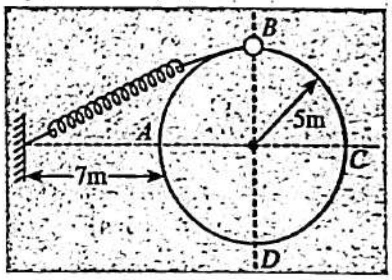


39. A particle of mass 'M' is moved rectilinearly under constant power P_0. At some instant after the start, its speed is v and at a later instant, the speed is $2v$. Neglecting friction, distance travelled (in m) by the particle as its speed increases from v to $2v$ is $7x$. Find x (Take $P_0 = 4$ watt, $M = 12$ kg, $v = 3$m/s).

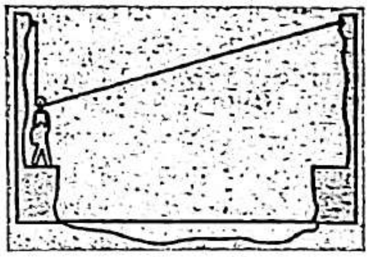
40. A circus trapeze consists of a bar suspended by two parallel ropes, each of length ℓ, allowing performers to swing in a vertical circular arc (figure). Suppose a performer with mass m holds the bar and steps off an elevated platform, starting from rest with the ropes at an angle θ with respect to the vertical. Assume the size of the performer's body is small compared to the length ℓ, air resistance is negligible. Determine the angle θ (in degree) for which the force exerted by the bar on the performer at the bottom of the swing is twice as large as the gravitational force exerted on the performer.



41. A collar B of mass 2kg is constrained to move along a vertical smooth and fixed circular track of radius 5m as shown in figure. The spring is in plane of the track and has a spring constant of 200 N/m. It is underformed when collar is at A. It starts from rest at B. What is the normal force (in N) exerted by the track on the collar when it passes through A? [$g = 10$ m/s^2]

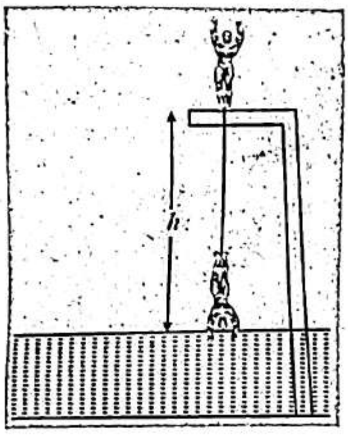


42. A soldier has to cross a 80 m wide river on a ropeway grabbing on a pulley. The length of the rope is 100 m, its ends are fixed at the same height on both banks of the river and the rope is tight at shown. The soldier starts from a platform under one of the fixed points with zero initial velocity. (Friction is absent every where.)



If the highest speed of the soldier during the crossing is v (in m/s). What is v^2? [Take g = 10 m/s^2]

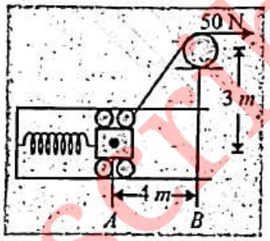
43. A man of height $h_0 = 2\text{m}$ is bungee jumping from a platform situated at a height $h = 25\text{m}$ above a lake. One end of an elastic rope is attached to his foot and the other end is fixed to the platform. He starts falling from rest in vertical position. The length and elastic properties of the rope are chosen so that his speed will have been reduced to zero at instant when his head reaches the surface of water. Ultimately the jumper is hanging from the rope with his head 8m above the water. Find the maximum acceleration achieved during the jump in m/s^2 (take $g = 10 \text{ m/s}^2$). Ignore mass of the rope and assume that jumper's center of mass is half way up his body.



44. An elastic string of force constant 100 N/m is being used to lower a block of mass 1kg with a constant speed of 4 m/s. The upper end of the string is suddenly stopped. What is the maximum tension in the string (in N)

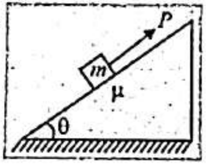
SINGE CORRECT TYPE QUESTIONS

45. A 20 kg block attached to a spring of spring constant 5 Nm^{-1} is released from rest at A. The spring at this instant is having an elongation of 1 m. The block is allowed to move in smooth horizontal slot with the help of a constant force 50 N in the rope as shown. The velocity of the block as it reaches B is (assume the rope to be light)



(*a*) 4 ms^{-1} (*b*) 2 ms^{-1} (*c*) 1 ms^{-1} (*d*) 3 ms^{-1}

46. A block of mass m is being pulled up a rough incline by an agent delivering constant power P. The coefficient of friction between the block and the incline is μ. The maximum speed of the block during the course of ascent is



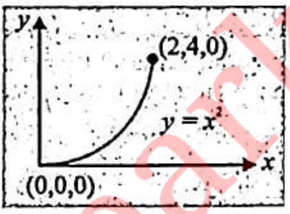
(*a*) $v = \dfrac{P}{mg\sin\theta + \mu mg\cos\theta}$

(*b*) $v = \dfrac{P}{mg\sin\theta - \mu mg\cos\theta}$

(*c*) $v = \dfrac{2P}{mg\sin\theta - \mu mg\cos\theta}$

(*d*) $v = \dfrac{3P}{mg\sin\theta - \mu mg\cos\theta}$

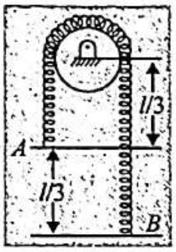
47. A force $\vec{F} = (3xy - 5z)\hat{j} + 4z\hat{k}$ is applied on a particle. The work done by the force when the particle moves from point (0, 0, 0) to point (2, 4, 0) as shown in figure is



(*a*) $\dfrac{280}{5}$ units (*b*) $\dfrac{140}{5}$ units

(*c*) $\dfrac{232}{5}$ units (*d*) $\dfrac{192}{5}$ units

48. Two ends A and B of a smooth chain of mass m and length l are situated as shown in figure. If an external agent pulls A till it comes to same level of B, work done by external agent is



(*a*) $\dfrac{mgl}{36}$ (*b*) $\dfrac{mgl}{15}$ (*c*) $\dfrac{mgl}{9}$ (*d*) $\dfrac{mgl}{18}$

49. A body is moving down an inclined plane of slope 37°. The coefficient of friction between the body and the plane varies as $\mu = 0.3x$, where x is the distance travelled down the plane by the body. The body will have maximum speed. $\left(\sin 37° = \dfrac{3}{5}\right)$

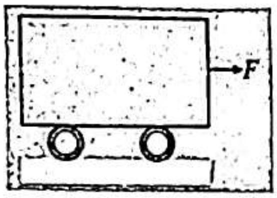
(*a*) At $x = 1.16$ m

(*b*) At $x = 2$

(*c*) At bottom most point of the plane

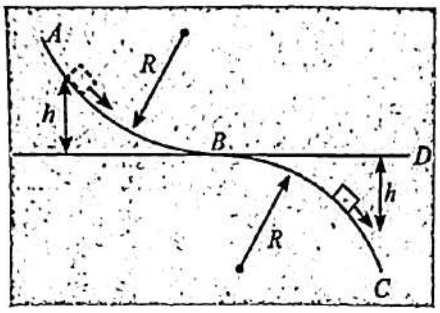
(*d*) At $x = 2.5$ m

50. A car of mass m is accelerating on a level smooth road under the action of a single force F. The power delivered to the car is constant and equal to P. If the velocity of the car at an instant is v, then after travelling how much distance it becomes double?



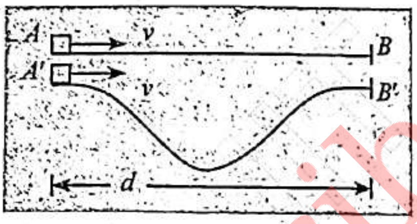
(a) $\frac{7mv^3}{3P}$ (b) $\frac{4mv^3}{3P}$ (c) $\frac{mv^3}{P}$ (d) $\frac{18mv^3}{7P}$

51. Figure shows a smooth track in a vertical plane, consisting of two circular arcs AB and BC of the same radius $R = 2$m. The common tangent to the arcs, BD, is horizontal. A small block placed on the track at a height h above BD leaves the track at exactly the same depth below. Determine h in cm.



(a) 10 cm (b) 20 cm (c) 40 cm (d) 80 cm

52. Two bodies of equal mass are projected horizontally with equal velocity v to move along the paths AB and $A'B'$ as shown in the figure. The path AB and $A'B'$ are frictionless and lie in a vertical plane. Let the time taken by body in making displacement d be t_1 and t_2 for path AB and $A'B'$ respectively. (Assume that body does not lose contact at any point). Then



(a) $t_1 = t_2$ (b) $t_1 < t_2$

(c) $t_1 > t_2$ (d) cannot be predicted

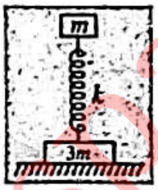
53. The potential energy function for the force between two atoms in a diatomic molecule is given by

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

where a and b are positive constants and x is the distance between the atoms. How much minimum energy must be supplied to separate the two atoms.

(a) $\frac{2a^2}{b}$ (b) $\frac{b}{2a^2}$ (c) $\frac{b^2}{4a}$ (d) $\frac{-b^2}{4a}$

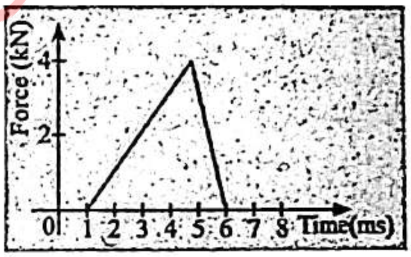
54. In the figure shown the spring constant is K. The mass of the upper disc is m and that of the lower disc is $3m$. The upper block is depressed down from its equilibrium position by a distance $\delta = 5mg/k$ and released at $t = 0$. Find the velocity of 'm' when normal reaction on $3m$ is mg.



(a) zero (b) $g[m/k]^{1/2}$

(c) $2g[m/k]^{1/2}$ (d) $4g[m/k]^{1/2}$

55. A point object of mass 2.0 kg moves along the x-axis with a velocity of + 4.0 m/s. A net horizontal force acting along the x-axis is applied to the object with the force-time profile shown. What is the total work that was done by the force?



(a) 20 J (b) 25 J (c) 65 J (d) 81 J

PYQ's (Past Year Questions)

WORK DONE BY CONSTANT FORCE, VARIABLE FORCE

1. Two bodies of mass 4g and 25g are moving with equal kinetic energies. The ratio of magnitude of their linear momentum is: **[27 Jan, 2024 (Shift-I)]**

(a) 3 : 5

(b) 5 : 4

(c) 2 : 5

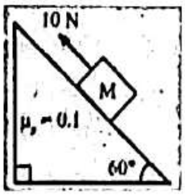
(d) 4 : 5

2. A block of mass 100kg slides over a distance of 10 m on a horizontal surface. If the co-efficient of friction between the surface is 0.4, then the work done against friction (in J) is: **[29 Jan, 2024 (Shift-I)]**

(a) 4200 (b) 3900 (c) 4000 (d) 4500

3. A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 60° by a force of 10 N parallel to the inclined surface as shown in figure. When the block is pushed up by 10 m along inclined surface, the work done against frictional force is:

[g = 10 m/s²] **[30 Jan, 2024 (Shift-II)]**



(*a*) $5\sqrt{3}$ J (*b*) 5 J (*c*) 5×10^3 J (*d*) 10 J

4. An artillery piece of mass M_1 fires a shell of mass M_2 horizontally. Instantaneously after the firing, the ratio of kinetic energy of the artillery and that of the shell is: **[31 Jan, 2024 (Shift-I)]**

(*a*) $M_1/(M_1 + M_2)$ (*b*) $\frac{M_2}{M_1}$

(*c*) $M_2/(M_1 + M_2)$ (*d*) $\frac{M_1}{M_2}$

5. A bullet of mass 50 g is fired with a speed 100 m/s on a plywood and emerges with 40 m/s. The percentage loss of kinetic energy is: **[06 April, 2024 (Shift-I)]**

(*a*) 32% (*b*) 44% (*c*) 16% (*d*) 84%

6. Four particles *A*, *B*, *C*, *D* of mass $m/2$, m, $2m$, $4m$, have same momentum, respectively. The particle with maximum kinetic energy is: **[06 April, 2024 (Shift-I)]**

(*a*) *D* (*b*) *C* (*c*) *A* (*d*) *B*

7. When kinetic energy of a body becomes 36 times of its original value, the percentage increase in the momentum of the body will be: **[06 April, 2024 (Shift-II)]**

(*a*) 500% (*b*) 600% (*c*) 6% (*d*) 60%

8. Three bodies *A*, *B* and *C* have equal kinetic energies and their masses are 400 g, 1.2 kg and 1.6 kg respectively. The ratio of their linear momenta is: **[08 April, 2024 (Shift-I)]**

(*a*) $1 : \sqrt{3} : 2$ (*b*) $1 : \sqrt{3} : \sqrt{2}$

(*c*) $\sqrt{2} : \sqrt{3} : 1$ (*d*) $\sqrt{3} : \sqrt{2} : 1$

9. A force $(3x^2 + 2x - 5)$ N displaces a body from $x = 2$ m to $x = 4$ m. Work done by this force is ………… J. **[09 April, 2024 (Shift-II)]**

10. Identify the correct statements from the following:

A. Work done by a person in lifting a bucket out of a well by means of a rope tied to the bucket is negative.

B. Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative.

C. Work done by friction on a body sliding down an inclined plane is positive.

D. Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity is zero.

E. Work done by the air resistance on an oscillating pendulum is negative.

Choose the correct answer from the options given below: **[29 Jan, 2023 (Shift-II)]**

(*a*) B and E only (*b*) A and C only

(*c*) B, D and E only (*d*) B and D only

11. A force $F = (5 + 3y^2)$ acts on a particle in the y direction, where F is newton and y is in meter. The work done by the force during a displacement from $y = 2m$ to $y = 5m$ is ________. **[01 Feb, 2023 (Shift-II)]**

12. Arrange the four graphs in descending order of total work done; where W_1, W_2, W_3, and W_4, are the work done corresponding to figure *A*, *B*, *C* and D respectively. **[26 June, 2022 (Shift-II)]**

(A)

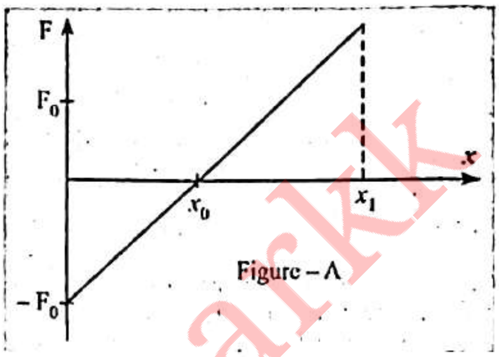


Figure – A

(B)

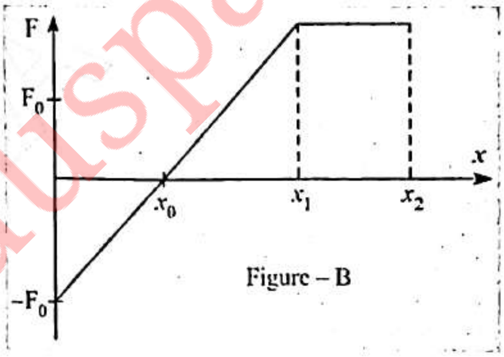


Figure – B

(C)

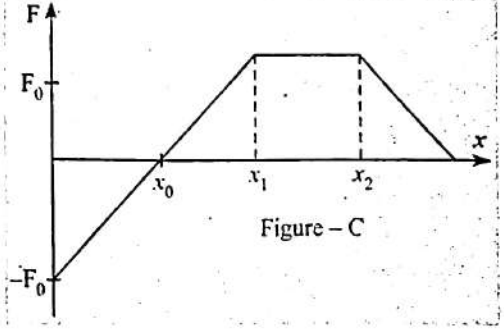


Figure – C

(D)

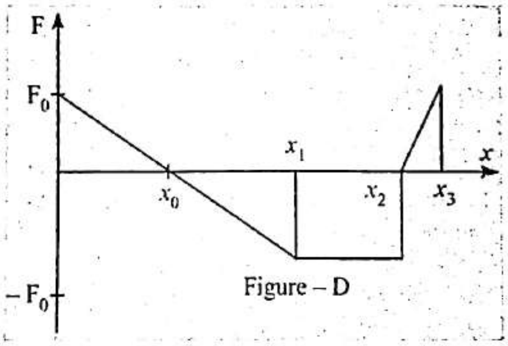


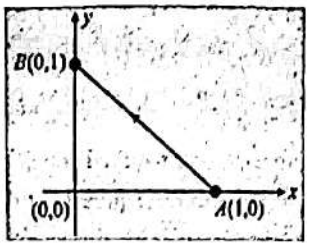
Figure – D

(*a*) $W_3 > W_2 > W_1 > W_4$ (*b*) $W_3 > W_2 > W_4 > W_1$

(*c*) $W_2 > W_3 > W_4 > W_1$ (*d*) $W_2 > W_3 > W_1 > W_4$

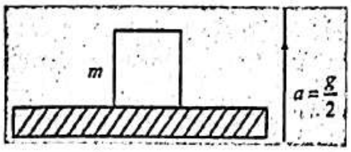
13. A force of $F = (5y + 20)\hat{j} N$ acts on a particle. The work done by this force when the particle is moved from $y = 0 m$ to $y = 10\ m$ is __________ J. [25 July, 2021 (Shift-II)]

14. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point $A(1,0)$ to $B(0,1)$ along the line segment is (All quantities are in SI units) [09 Jan, 2020 (Shift-I)]



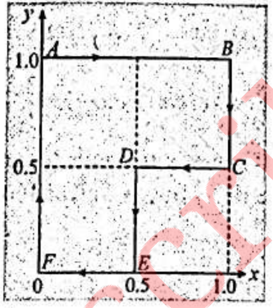
(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 2 (d) 1

15. A block of mass m is kept on a platform which starts from rest with constant acceleration $g/2$ upward, as shown in figure. Work done by normal reaction on block in time t is [10 Jan, 2019 (Shift-I)]



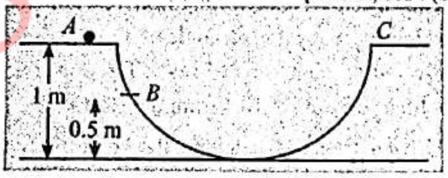
(a) $-\frac{mg^2t^2}{8}$ (b) $\frac{mg^2t^2}{8}$ (c) 0 (d) $\frac{3mg^2t^2}{8}$

16. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})$ N where x and y are in meter and $a = -1$ N/m^{-1}. The work done on the particle by this force $\vec{F}$ will be ____ Joule [JEE Adv, 2019]



WORK ENERGY THEOREM

17. A particle is placed at the point A of a frictionless track ABC as shown in figure. It is gently pushed toward right. The speed of the particle when it reaches the point B is: (Take $g = 10$ m/s^2). [30 Jan, 2024 (Shift-I)]



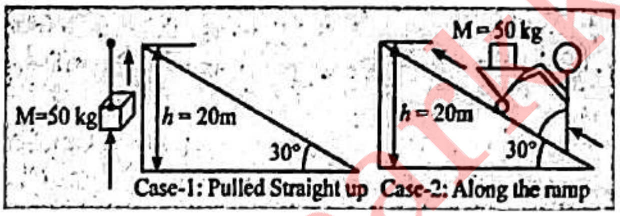
(a) 20 m/s (b) $\sqrt{10}$ m/s (c) $2\sqrt{10}$ m/s (d) 10 m/s

18. If a rubber ball falls from a height h and rebounds upto the height of $h/2$. The percentage loss of total energy of the initial system as well as velocity of ball before it strikes the ground, respectively, are: [04 April, 2024 (Shift-I)]

(a) 50%, $\sqrt{\frac{gh}{2}}$ (b) 50%, $\sqrt{gh}$

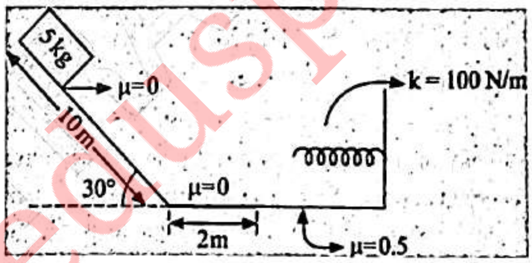
(c) 40%, $\sqrt{2gh}$ (d) 50%, $\sqrt{2gh}$

19. A body of mass 50 kg is lifted to a height of 20 m from the ground in the two different ways as shown in the figures. The ratio of work done against the gravity in both the respective cases, will be: [05 April, 2024 (Shift-I)]



(a) 1 : 1 (b) 2 : 1 (c) 3 : 2 (d) 1 : 2

20.



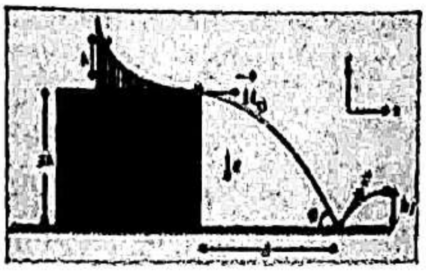
A block is simply released from the top of an inclined plane as shown in the figure above. The maximum compression in the spring when the block hits the spring is: [08 April, 2024 (Shift-II)]

(a) $\sqrt{6}$ m (b) 2 m (c) 1 m (d) $\sqrt{5}$ m

21. A particle of mass m moves on a straight line with its velocity increasing with distance according to the equation $v = \alpha\sqrt{x}$, where α is a constant. The total work done by all the forces applied on the particle during its displacement from $x = 0$ to $x = d$, will be: [09 April, 2024 (Shift-I)]

(a) $\frac{m}{2\alpha^2 d}$ (b) $\frac{md}{2\alpha^2}$ (c) $\frac{m\alpha^2 d}{2}$ (d) $2m\alpha^2 d$

22. A slide with a frictionless curved surface, which becomes horizontal at its lower end, is fixed on the terrace of a building of height $3h$ from the ground, as shown in the figure. A spherical ball of mass m is released on the slide from rest at a height h from the top of the terrace. The ball leaves the slide with a velocity $\vec{u}_0 = u_0\hat{x}$ and falls on the ground at a distance d from the building making an angle θ with the horizontal. It bounces off with a velocity $\vec{v}$ and reaches a maximum height h_1. The acceleration due to gravity is g and the coefficient of restitution of the ground is $\frac{1}{\sqrt{3}}$. Which of the following statements is are correct? [JEE Adv, 2023]



(a) $\vec{u}_0 = \sqrt{2gh}\,\hat{x}$ (b) $\vec{v} = \sqrt{2gh}\,(\hat{x} - \hat{z})$

(c) $\theta = 60^\circ$ (d) $\frac{d}{h_1} = 2\sqrt{3}$

23. A body of mass 0.5 kg travels on straight line path velocity $u = (3x^2 + 4)$ m/s. The net work done by the force during its displacement from $x = 0$ to $x = 2$m is:

[25 July, 2022 (Shift-I)]

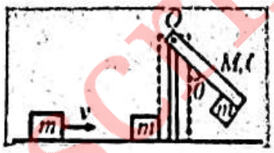
(a) 64 J (b) 60 J (c) 120 J (d) 128 J

24. A ball of mass 4 kg, moving with a velocity of 10 ms^{-1}, collides with a spring of length 8m and force constant 100 Nm^{-1}. The length of the compressed spring is x m. The value of x, to the nearest integer, is_____.

[18 March, 2021 (Shift-II)]

25. A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) _______. **[05 Sep, 2020 (Shift-II)]**

26. A block of mass $m = 1$ kg slides with velocity $v = 6$ m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass $M = 2$ kg and length $l = 1$ m, the value of θ is approximately (take $g = 10$ m/s^2) **[3 Sep, 2020 (Shift-I)]**



(a) 49° (b) 55° (c) 63° (d) 69°

27. A student skates up a ramp that makes an angle 30° with the horizontal. He/she starts (as shown in the figure) at the bottom of the ramp with speed v_0 and wants to turn around over a semicircular path xyz of radius R during which he/she reaches a maximum height h (at point y) from the ground as shown in the figure. Assume that the energy loss is negligible and the force required for this turn at the highest point is provided by his/her weight only, then (g is the acceleration due to gravity) **[JEE Adv, 2020]**

(a) $v_0^2 - 2gh = \frac{1}{2}gR$

(b) $v_0^2 - 2gh = \frac{\sqrt{3}}{2}gR$

(c) The centripetal force required at points x and z is zero

(d) The centripetal force required is maximum at points x and z

28. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds? **[09 Jan, 2019 (Shift-II)]**

(a) 850 J (b) 950 J

(c) 875 J (d) 900 J

POWER

29. A body of mass 2 kg begins to move under the action of a time dependent force given by $\vec{F} = (6t\hat{i} + 6t^2\hat{j})$ N. The power developed by the force at the time t is given by:

[31 Jan, 2024 (Shift-II)]

(a) $(6t^4 + 9t^5)$W (b) $(3t^3 + 6t^5)$W

(c) $(9t^5 + 6t^3)$W (d) $(9t^3 + 6t^5)$W

30. A body is moving unidirectionally under the influence of a constant power source. Its displacement in time t is proportional to: **[05 April, 2024 (Shift-II)]**

(a) t^2 (b) $t^{2/3}$ (c) $t^{3/2}$ (d) t

31. The ratio of powers of two motors is $\frac{3\sqrt{x}}{\sqrt{x}+1}$, that are capable of raising 300 kg water in 5 minutes and 50 kg water in 2 minutes respectively from a well of 100 m deep. The value of x will be **[13 April, 2023 (Shift-I)]**

(a) 2 (b) 4 (c) 2.4 (d) 16

32. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = \left(t\hat{i} + 3t^2\hat{j}\right)N$, where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. The power developed by above force, at the time $t = 2s$, will be _______ W.

[24 Jan, 2023 (Shift-II)]

33. A block of mass 5 kg starting from rest pulled up on a smooth incline plane making an angle of 30° with horizontal with an affective acceleration of 1 ms^{-2}. The power delivered by the pulling force at $t = 10$ s from the start is ____________ W. [Use g = 10 ms^{-2}] (calculate the nearest integer value)

[11 April, 2023 (Shift-II)]

34. If the maximum load carried by an elevator is 1400 kg (600 kg – Passenger + 800 kg – elevator), which is moving up with a uniform speed of $3\,ms^{-1}$ and the frictional force acting on it is 2000 N, then the maximum power used by the motor is _______ kW ($g = 10\,m/s^2$). [10 April, 2023 (Shift-II)]

35. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration (a) is varying with time t as $a = k^2rt^2$, where k is a constant.

The power delivered to the particle by the force acting on it is given as [28 June, 2022 (Shift-I)]

(*a*) Zero (*b*) $mk^2r^2t^2$

(*c*) mk^2r^2t (*d*) mk^2rt

36. Sand is being dropped from a stationary dropper at a rate of $0.5\,kgs^{-1}$ on a conveyor belt moving with a velocity of $5ms^{-1}$. The power needed to keep the belt moving with the same velocity will be: [27 July, 2022 (Shift-I)]

(*a*) 1.25 W (*b*) 2.5 W

(*c*) 6.25 W (*d*) 12.5 W

37. A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time 't' is proportional to: [18 March, 2021 (Shift-I)]

(*a*) $t^{1/2}$ (*b*) t (*c*) $t^{3/2}$ (*d*) $t^{2/3}$

38. A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to: [20 July, 2021 (Shift-II)]

(*a*) $t^{\frac{3}{4}}$ (*b*) $t^{\frac{3}{2}}$ (*c*) $t^{\frac{1}{2}}$ (*d*) $t^{\frac{1}{4}}$

39. An automobile of mass 'm' accelerates starting from origin and initially at rest, while the engine supplies constant power P. The position is given as a function of time by: [27 July, 2021 (Shift-II)]

(*a*) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$ (*b*) $\left(\frac{9P}{8m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

(*c*) $\left(\frac{9m}{8P}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$ (*d*) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{2}{3}}$

40. An elevator in a building can carry a maximum of 10 persons with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator must be at least ($g = 10\,m/s^2$) [07 Jan, 2020 (Shift-II)]

(*a*) 62360 W (*b*) 56300 W

(*c*) 48000 W (*d*) 66000 W

41. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746 W, $g = 10\,ms^{-2}$) [07 Jan., 2020 (Shift-I)]

(*a*) $1.5\,ms^{-1}$ (*b*) $1.9\,ms^{-1}$

(*c*) $2.0\,ms^{-1}$ (*d*) $1.7\,ms^{-1}$

IDEA OF POTENTIAL ENERGY EQUILIBRIUM

42. The potential energy function (in J) of a particle in a region of space is given as $U = (2x^2 + 3y^3 + 2z)$. [29 Jan., 2024 (Shift-I)]

Here x, y and z are in meter. The magnitude of x-component of force (in N) acting on the particle at point $P(1, 2, 3)$ m is:

(*a*) 2 (*b*) 6 (*c*) 4 (*d*) 8

43. Potential energy as a function of r is given by, $U = \frac{A}{r^{6}} - \frac{B}{r^{3}}$, where r is the interatomic distance, A and B are positive constant. The equilibrium distance between the two atoms will be: [24 June, 2022 (Shift-II)]

(*a*) $\left(\frac{A}{B}\right)^{\frac{1}{3}}$ (*b*) $\left(\frac{B}{A}\right)^{\frac{1}{3}}$ (*c*) $\left(\frac{2A}{B}\right)^{\frac{1}{3}}$ (*d*) $\left(\frac{B}{2A}\right)^{\frac{1}{3}}$

44. The potential energy of a particle of mass 4 kg in motion along the x-axis is given by $U = 4(1 - \cos 4x)$ J. The time period of the particle for small oscillation ($\sin\theta \simeq \theta$) is $\left(\frac{\pi}{K}\right)$ s. The value of K is _______. [28 July, 2022 (Shift-II)]

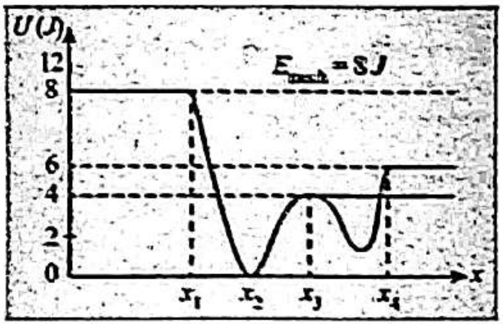
45. Given below is the plot of a potential energy function $U(x)$ for a system, in which a particle is in one dimensional motion, while a conservative force $F(x)$ acts on it. Suppose that $E_{mech} = 8$ J, the incorrect statement for this system is: [27 July, 2021 (Shift-II)]

(*a*) at $x = x_3$, K.E. = 4 J.

(*b*) at $x > x_4$, K.E. is constant throughout the region.

(*c*) at $x < x_1$, K.E. is smallest and the particle is moving at the slowest speed.

(*d*) at $x = x_2$, K.E. is greatest and the particle is moving at the fastest speed.



[where K.E. = kinetic energy]

46. The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as

$$U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$$

Where, α and β are positive constants. The equilibrium between two atoms will be $\left(\frac{2\alpha}{\beta}\right)^{\frac{a}{b}}$, where a = ______

[25 Feb, 2021 (Shift-I)]

VERTICAL CIRCULAR MOTION

47. A ball suspended by a thread swings in a vertical plane so that its magnitude of acceleration in the extreme position and lowest position are equal. The angle (θ) of thread deflection in the extreme position will be: **[27 Jan, 2024 (Shift-II)]**

(a) $\tan^{-1}(\sqrt{2})$ (b) $2\tan^{-1}\left(\frac{1}{2}\right)$

(c) $\tan^{-1}\left(\frac{1}{2}\right)$ (d) $2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

48. A stone of mass 900g is tied to a string and moved in a vertical circle of radius 1m making 10 rpm. The tension in the string, when the stone is at the lowest point is (if $\pi^2 = 9.8$ and $g = 9.8\ m/s^2$) **[29 Jan, 2024 (Shift-II)]**

(a) 97 N (b) 9.8 N

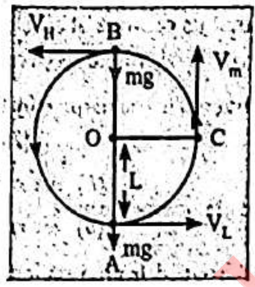
(c) 8.82 N (d) 17.8 N

49. The bob of a pendulum was released from a horizontal position. The length of the pendulum is 10m. If it dissipates 10% of its initial energy against air resistance, the speed with which the bob arrives at the lowest point is: [Use, g: $10\ ms^{-2}$] **[29 Jan, 2024 (Shift-II)]**

(a) $6\sqrt{5}\ ms^{-1}$ (b) $5\sqrt{6}\ ms^{-1}$

(c) $5\sqrt{5}\ ms^{-1}$ (d) $2\sqrt{5}\ ms^{-1}$

50. A bob of mass 'm' is suspended by a light string of length 'L'. It is imparted a minimum horizontal velocity at the lowest point A such that it just completes half circle reaching the top most position B. The ratio of kinetic energies $\frac{(K.E.)_A}{(K.E.)_B}$ is: **[29 Jan, 2024 (Shift-II)]**

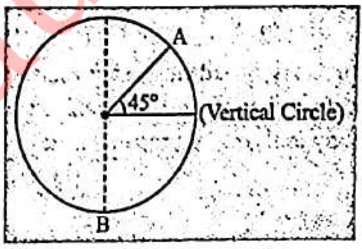


(a) 3 : 2 (b) 5 : 1 (c) 2 : 5 (d) 1 : 5

51. A simple pendulum of length 1 m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of $2 \times 10^2\ ms^{-1}$. The bullet gets embedded into the bob. The height to which the bob rises before swinging back is: (use $g = 10\ m/s^2$) **[01 Feb, 2024 (Shift-I)]**

(a) 0.30 m (b) 0.20 m (c) 0.35 m (d) 0.40 m

52. A body of m kg slides from rest along the curve of vertical circle from point A to B in frictionless path. The velocity of the body at B is: **[04 April, 2024 (Shift-II)]**



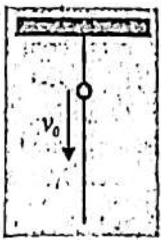
(given, $R = 14$ m, $g = 10\ m/s^2$ and $\sqrt{2} = 1.4$)

(a) 19.8 m/s (b) 21.9 m/s (c) 16.7 m/s (d) 10.6 m/s

MEDITATIVE BREATHING EXERCISE

- Set a timer for 10 minutes. Sit in a quiet space, close your eyes, and focus solely on your breathing. Each time your mind wanders, gently bring your focus back to your breath.
- Strengthens the brain's ability to return to a point of focus, even when distractions occur.

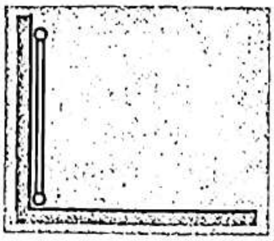
1. A light elastic cord of force constant k and relaxed length l is suspended from the ceiling. A bead of mass m threaded on the cord is held motionless close to the ceiling and released. If the bead slides down the cord with a constant speed v relative to the ceiling, find expression for thermal power dissipated during sliding of the bead.



(a) $mgv_0\left(\frac{mg}{kl}\right)$ (b) $mgv_0\left[\frac{mg+2kl}{2(mg+kl)}\right]$

(c) $mgv_0\left[\frac{mg+kl}{mg}\right]$ (d) $mgv_0\left[\frac{kl}{mg}\right]$

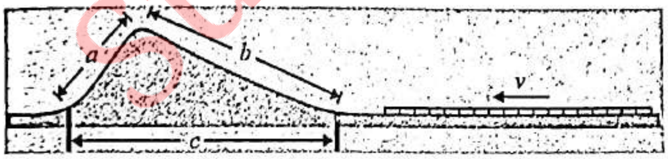
2. A dumbbell constructed by affixing small identical balls each of mass m at the ends of a light rod of length l stands vertically on a frictionless floor touching a frictionless wall as shown in the figure. If the lower ball is gently pushed away from the wall, the dumbbell begins to slide. Denoting acceleration due to gravity by g, find expression for speed of the lower ball, when the upper ball is leaving the wall.



(a) $\sqrt{\frac{4gl}{9}}$ (b) $\sqrt{\frac{8gl}{9}}$

(c) $\sqrt{\frac{4gl}{27}}$ (d) $\sqrt{\frac{8gl}{27}}$

3. A prototype train of length $l = 168$ m is running at a constant speed v with its engine off on horizontal portion of a track that has a hilly portion as shown in the figure. Dimensions of the hilly portion are $a = 60$ m, $b = 80$ m and $c = 100$ m respectively. What should be the minimum speed of the train to pass over the hill?

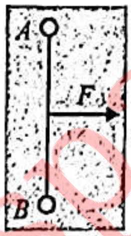


(a) 10 m/s (b) 15 m/s (c) 20 m/s (d) 25 m/s

4. A plank is being pushed on a frictionless horizontal floor with a constant velocity $u = 1.5$ m/s. When a block is gently placed on the plank, it slides for a while, acquires velocity of the plank and then moves together with the plank. If the agency pushing the plank has to do a work $W = 22.50$ J during the sliding of the block, the mass of the block.

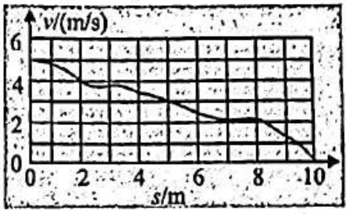
(a) 5 kg (b) 7.5 kg (c) 10 kg (d) 20 kg

5. Two small identical discs each of mass $m = 1.0$ kg placed on a frictionless horizontal floor are connected by a light inextensible cord of length $l = 2.0$ m. Now the midpoint of the cord is pulled perpendicular to the line joining centres of the discs by a constant force $F = 4.0$ N. What is velocity of approach of the discs, when they are about to collide?



(a) 1 m/s (b) 2 m/s (c) 3 m/s (d) 4 m/s

6. On a horizontal surface of non-uniform texture, friction varies from point to point. On this surface a small disc projected with speed 5 m/s covers 10 m along a straight path before it stops. Variation in speed v of the disc with distance s covered is shown in the figure. If the disc is projected with speed 4 m/s from the same initial point in the same direction, how much total distance will it cover?



(a) 2 m (b) 4 m

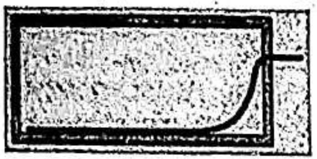
(c) 5 m (d) 8 m

7. A boy is holding one end of an inextensible cord supporting a block of mass m at its lower end. From this block, another block of mass M is suspended with the help of a light spring of stiffness k. The boy brings the assembly over a table and lowers it until the lower block touches the tabletop and then releases the cord. How far will the upper block move before coming to an instantaneous rest first time?

(a) $\frac{(M+m)g}{k}$ (b) $\frac{2(M+m)g}{k}$

(c) $\frac{(M-m)g}{k}$ (d) $\frac{2(M-m)g}{k}$

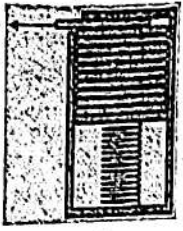
8. A light elastic cord of length $l_0 = 5.0$ m and stiffness $k = 50$ N/m is inside a fixed box with a small portion of length $x_0 = 20$ cm left out of a hole on a wall of the box as shown in the figure. If the cord is pulled out of the box, a frictional force $F = 25$ N acts on the cord. How much work has to be done in slowly pulling out the entire length of the cord?



(*a*) 120.00 J (*b*) 125.00 J

(*c*) 132.25 J (*d*) 135.25 J

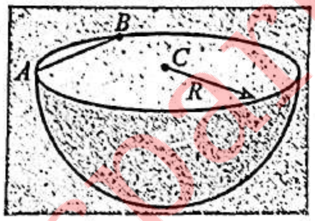
9. A special box used to store several plate-like articles contains a light spring loaded piston inside it. When the box is empty, the piston touches the top inner surface of the box. The plates can be inserted or pulled out through an opening at the top of the box as shown in the figure. Initially the box contains $n = 20$ identical square plates each of edge length $l = 5$ cm. Coefficient of friction between all the surfaces in contact is $\mu = 0.5$. All the plates are pulled out one by one in two different trials, in the first trial the opening was at the top and in the second trial the box was inverted. If difference of work done by the agent pulling the plates in both these trials is $\Delta W = 20$ J, calculate mass of a plate.



(*a*) 4×10^{-2} kg (*b*) 6×10^{-2} kg

(*c*) 8×10^{-2} kg (*d*) 10×10^{-2} kg

10. Consider a fixed hemispherical bowl of radius R with its opening horizontal. One end of a light rod of length $l = R$ is hinged at a point A on the brim of the bowl. A small bead of mass m is attached at the other end the rod. Initially the bead is held at a point B also on the brim and then released. Find the normal reaction exerted on the bead when the bead passes the lowest position. Acceleration of free fall is g.



(*a*) $\sqrt{3}mg$ (*b*) $\dfrac{mg}{\sqrt{3}}$ (*c*) $\dfrac{4mg}{\sqrt{3}}$ (*d*) $\dfrac{2mg}{\sqrt{3}}$

> *"It is during our darkest moments that we must focus to see the light."*
> **– Aristotle**

ANSWER KEY

CONCEPT APPLICATION

1. (*c*) 2. (i) Zero (ii) 500J, 3. [150 J] 4. [8 J] 5. (*a*) 6. (*a*) zero (*b*) zero (*c*) $-\mu mgvt$ (*d*) $\mu mgvt$

7. [135 J] 8. [0 J] 9. [25 J] 10. [$W = -3.5$ J] 11. [3] 12. (*a*) $\frac{1}{2}kx^2$ (*b*) Zero (*c*) $-\frac{1}{2}kx^2$

13. (*a*) 32 J, (*b*) 5 m/s 14. (*c*) 15. (*c*) 16. [0.5 J] 17. (*a*) 18. (*a*) 19. $\frac{mg}{k}$ 20. (*a*) 21. (*b*)

22. (*a*) 23. $\left\{\frac{M+2(m-m')}{M+m+m'}ag\right\}$ 24. $F_x = \frac{-\partial U}{\partial x} = -2ax + by^2, F_y = 2bxy$

25. $x = 0$, stable equilibrium, $x = \pm\sqrt{\frac{a}{2b}}$ unstable equilibrium 26. (*b*) 27. (*b*) 28. [5 rad/s] 29. [90°]

SCHOOL LEVEL PROBLEMS

1. (*d*) 2. (*d*) 3. (*c*) 4. (*d*) 5. (*c*) 6. (*d*) 7. (*b*) 8. (*a*) 9. (*a*) 10. (*a*)
11. (*b*) 12. (*a*) 24. (i)-(*d*), (ii)-(*a*), (iii)-(*c*), (iv)-(*a*), (v)-(*a*) 25. (i)-(*d*), (ii)-(*d*), (iii)-(*b*), (iv)-(*c*), (v)-(*d*)

PRARAMBH (TOPICWISE)

1. (*c*) 2. (*a*) 3. (*b*) 4. (*b*) 5. (*b*) 6. (*a*) 7. (*c*) 8. (*d*) 9. (*c*) 10. (*b*)
11. (*b*) 12. (*b*) 13. (*c*) 14. (*a*) 15. (*c*) 16. (*a*) 17. (*c*) 18. (*b*) 19. (*b*) 20. (*b*)
21. (*a*) 22. (*d*) 23. (*c*) 24. (*b*) 25. (*b*) 26. (*d*) 27. (*b*) 28. (*d*) 29. (*a*) 30. (*b*)
31. (*a*) 32. (*c*) 33. (*c*) 34. (*d*) 35. (*d*) 36. (*b*) 37. (*d*) 38. (*a*) 39. (*a*) 40. (*b*)
41. (*c*) 42. (*b*) 43. (*d*) 44. (*b*) 45. (*a*) 46. (*b*) 47. (*a*) 48. (*b*) 49. (*c*) 50. (*a*)
51. (*c*) 52. (*b*) 53. (*b*) 54. (*a*) 55. (*c*) 56. (*c*) 57. (*c*) 58. (*d*) 59. (*a*) 60. (*b*)
61. (*d*) 62. (*b*) 63. (*b*)

PRABAL (JEE MAIN LEVEL)

1. (*a*) 2. (*c*) 3. (*d*) 4. (*a*) 5. (*b*) 6. (*a*) 7. (*d*) 8. (*c*) 9. (*b*) 10. (*c*)
11. (*b*) 12. (*a*) 13. (*c*) 14. (*d*) 15. (*b*) 16. (*a*) 17. (*a*) 18. (*a*) 19. (*a*) 20. (*b,d*)
21. (*b*) 22. (*a*) 23. (*b*) 24. (*b*) 25. (*c*) 26. (*c*) 27. [0010] 28. [0007] 29. [0002] 30. [0005]
31. [0053] 32. [0001] 33. [0375] 34. [0007] 35. [0000] 36. [0005]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (*c*) 2. (*a, d*) 3. (*a, b, c, d*) 4. (*a, b, c, d*) 5. (*a, b, c, d*) 6. (*b, c, d*) 7. (*a, d*) 8. (*b, c, d*) 9. (*b, d*) 10. (*a, d*)
11. (*b, c*) 12. (*b, c*) 13. (*a*) 14. (*d*) 15. (*b*) 16. (*d*) 17. (*b*) 18. (*c*) 19. (*d*) 20. (*c*)
21. (*d*) 22. (*b*) 23. (*c*) 24. [2.45] 25. [0.53] 26. [0.00] 27. [1.58] 28. [21.87] 29. [1.50] 30. [0.96]
31. [1.00] 32. [2.00] 33. [80.00] 34. [19.50] 35. [200] 36. [18] 37. [11] 38. [2] 39. [27] 40. [60]
41. [1480] 42. [240] 43. [40] 44. [50] 45. (*b*) 46. (*a*) 47. (*d*) 48. (*a*) 49. (*d*) 50. (*a*)
51. (*c*) 52. (*c*) 53. (*c*) 54. (*d*) 55. (*c*)

PYQ's (PAST YEAR QUESTIONS)

1. (*c*) 2. (*c*) 3. (*b*) 4. (*b*) 5. (*d*) 6. (*c*) 7. (*a*) 8. (*a*) 9. (58) 10. (*a*)
11. [132] 12. (*a*) 13. [450] 14. (*d*) 15. (*d*) 16. [0.75] 17. (*b*) 18. (*d*) 19. (*a*) 20. (*b*)
21. (*c*) 22. (*a, c, d*) 23. (*b*) 24. [2] 25. [18.00] 26. (*c*) 27. (*a, d*) 28. (*d*) 29. (*d*) 30. (*c*)
31. (*d*) 32. [100] 33. [300] 34. [48] 35. (*c*) 36. (*d*) 37. (*c*) 38. (*b*) 39. (*a*) 40. (*d*)
41. (*b*) 42. (*c*) 43. (*c*) 44. [2] 45. (*c*) 46. [1] 47. (*b*) 48. (*b*) 49. (*a*) 50. (*b*)
51. (*b*) 52. (*b*)

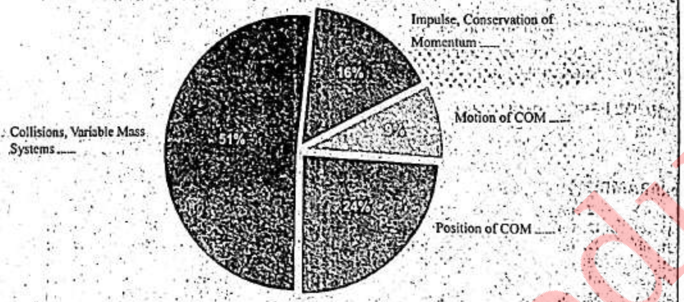
PW CHALLENGERS

1. (*b*) 2. (*d*) 3. (*c*) 4. (*c*) 5. (*d*) 6. (*c*) 7. (*b*) 8. (*c*) 9. (*d*) 10. (*a*)

CHAPTER 7

Centre of Mass & System of Particles

Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

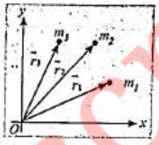
CENTRE OF MASS

In a system of extended bodies there is one special point that has some interesting and simple properties no matter how complicated the system is. This point is called the **center of mass (COM)**.

For a system of n particles whose position vectors are $\vec{r}_1, \vec{r}_2, ..., \vec{r}_n$ as shown in the figure, the position vector for the center of mass $\vec{r}_{cm}$ is defined as

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n}$$

$$\text{or} \quad \vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M}$$



where $M = \Sigma m_i$ is the total mass of the system.

The components of the above equation may be written as

$$x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{M}; \; y_{cm} = \frac{\sum_{i=1}^{n} m_i y_i}{M}; \; z_{cm} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$$

Note: The location of the centre of mass is independent of the origin used to locate it.

The centre of mass of the system of particles depends only on the masses of the particles and the positions of the particles relative to one another.

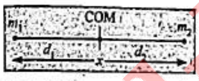
Position of COM of Two Particles System

In case of two bodies, the ratio of distance of centre of mass from the bodies is in the inverse ratio of their masses.

Let us choose origin at the location of center of mass in the figure shown, then

$$x_1 = -d_1$$
$$x_2 = d_2$$
$$x_{cm} = 0$$

$$\Rightarrow 0 = \frac{-m_1 d_1 + m_2 d_2}{m_1 + m_2} \text{ or } m_1 d_1 = m_2 d_2$$



In figure, $x = d_1 + d_2$.

Solving Equation 1 and 2, $m_1 d_1 = m_2 d_2$

$$m_1 d_1 = m_2 \times (x - d_1)$$

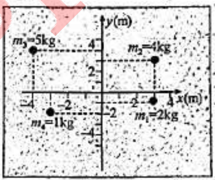
$$\Rightarrow d_1 = \frac{m_2 x}{m_1 + m_2} \text{ and } d_2 = \frac{m_1 x}{m_1 + m_2}$$

Here, d_1, d_2 are the distances of COM from m_1, m_2 respective

Thus, COM is located nearer to heavier body.

Train Your Brain

Example 1: Find the center of mass of the four point masses as shown in figure.



Sol. The total mass $M = 12$ kg. From the component equations, we have

$$x_{cm} = \frac{(2\text{kg})(3\text{m}) + (4\text{kg})(3\text{m}) + (5\text{kg})(-4\text{m}) + (1\text{kg})(-3\text{m})}{12\text{kg}}$$

$$= -\frac{5}{12}\text{ m}$$

$$y_{cm} = \frac{(2\text{kg})(-1\text{m}) + (4\text{kg})(3\text{m}) + (5\text{kg})(4\text{m}) + (1\text{kg})(-2\text{m})}{12\text{kg}}$$

$$= \frac{28}{12}\text{ m}$$

The position vector of the centre of mass is

$$\vec{r}_{cm} = (-0.42\hat{i} + 2.3\hat{j})\text{ m}$$

Example 2: When 'n' number of particles of masses $m, 2m, 3m, ..., nm$ are at distances $x_1 = 1, x_2 = 2, x_3 = 3, ...$ $x_n = n$ units respectively from origin on the x-axis, then find the distance of centre of mass of the system from origin.

(a) $\frac{2(n+1)}{2}$ (b) $\left(\frac{2n+1}{3}\right)$

(c) $\left(\frac{2n-1}{3}\right)$ (d) $\frac{n(n+1)}{2}$

Sol. (*b*) Using formula of C.O.M, we have.

$$x_{cm}=\frac{\sum m_i x_i}{\sum m_i}$$

$$=\frac{m(1)+2m(2)+3m(3)+\ldots\ldots\ldots+nm(n)}{m+2m+3m+\ldots\ldots+nm}$$

$$=\frac{m(1^2+2^2+\ldots\ldots+n^2)}{m(1+2+\ldots\ldots+n)}$$

$$\left\{\begin{array}{l} \because 1^2+2^2+\ldots+n^2=\dfrac{n(n+1)(2n+1)}{6} \\ \because 1+2+\ldots\ldots+n\ =\dfrac{n(n+1)}{2} \end{array}\right\}$$

$$=\frac{n(n+1)(2n+1)\times 2}{6\times n(n+1)}=\frac{(2n+1)}{3}$$

Concept Application

1. Two blocks of masses 10 kg and 30 kg are placed on X-axis. The first mass is moved on the axis by a distance of 2 cm right. By what distance should the second mass be moved to keep the position of centre of mass unchanged?

   (*a*) $\frac{2}{3}$ cm, left (*b*) $\frac{3}{2}$ cm, left

   (*c*) $\frac{2}{3}$ cm, right (*d*) $\frac{3}{2}$ cm, right

2. When '*n*' number of particles of masses $m, 2m, 3m, \ldots nm$ are at distances $x_1=1, x_2=4, x_3=9, \ldots x_n=n^2$ units respectively from origin on the X-axis, then find the distance of their centre of mass from origin.

   (*a*) $\frac{n(n-1)}{2}$ (*b*) $\frac{n(n+1)}{2}$

   (*c*) $\frac{2n(n-1)}{3}$ (*d*) $\frac{2n(n+1)}{3}$

3. If the distance between the centers of the atoms of potassium and bromine in **KBr (potassium-bromide)** molecule is 0.282×10^{-9}m, find the distance of centre of mass of this two particle system from potassium (mass of bromine = 80 u, and of potassium = 39 u)

   (*a*) 0.189 nm (*b*) 0.471 nm

   (*c*) 0.894 nm (*d*) 0.671 nm

CENTRE OF MASS OF CONTINUOUS MASS SYSTEMS

For continuous distribution of mass, the centre of mass is defined as

$$\vec{r}_{cm}=\frac{1}{M}\int \vec{r}\,dm$$

where $\vec{r}$ is the position vector of a small mass element dm.

The components of this equation are

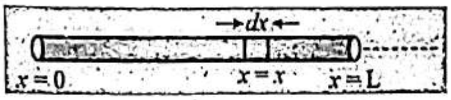
$$x_{cm} = \frac{1}{M}\int x dm,\ y_{cm} = \frac{1}{M}\int y dm \text{ and } z_{cm} = \frac{1}{M}\int z dm$$

Center of Mass of a Uniform Rod

Suppose a rod of mass M and length L is lying along the x-axis with its one end at $x = 0$ and the other end at $x = L$. Mass per unit length of the rod $= \frac{M}{L}$

Hence, dm, (the mass of the element dx situated at coordinate $x = \frac{M}{L}dx$

The coordinates of the element dx are $(x, 0, 0)$. Therefore, x-coordinate of COM of the rod will be



$$x_{COM} = \frac{\int_0^L x\, dm}{\int dm} = \frac{\int_0^L (x)\left(\frac{M}{L}dx\right)}{M}$$

$$= \frac{1}{L}\int_0^L x\, dx = \frac{L}{2}$$

The y-coordinate of COM is

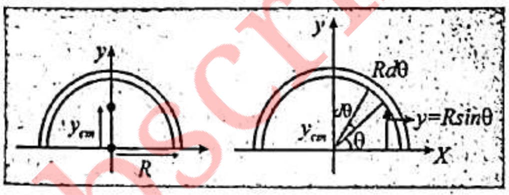
$$y_{COM} = \frac{\int y dm}{\int dm} = 0$$

Similarly, $z_{COM} = 0$

i.e., the coordinates of COM of the rod are $\left(\frac{L}{2}, 0, 0\right)$, i.e. it lies at the center of the rod.

Center of Mass of a Semicircular Ring

Figure shows the object (semicircular ring). By observation we can say that the x-coordinate of the center of mass of the ring is zero as the half ring is symmetrical about y-axis on both sides of the origin. We are only required to find the y-coordinate of the center of mass.



To find y_{cm} we use

$$y_{cm} = \frac{1}{M}\int dm\, y \quad \text{... (i)}$$

Here for dm we consider an elemental arc of the ring at an angle θ from the x-direction of angular width $d\theta$. If radius of the ring is R then its y coordinate will be $R\sin\theta$, here dm is given as

$$dm = \frac{M}{\pi R} \times R\, d\theta \quad \text{... (ii)}$$

So from equation (*i*) and (*ii*) we have

$$y_{cm} = \frac{1}{M}\int_0^\pi \frac{M}{\pi R} R d\theta\,(R\sin\theta) = \frac{R}{\pi}\int_0^\pi \sin\theta\, d\theta$$

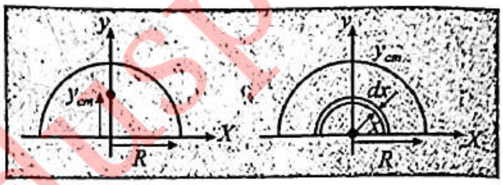
$$y_{cm} = \frac{R}{\pi}[-\cos\theta]_0^\pi$$

$$y_{cm} = \frac{2R}{\pi} \quad \text{... (iii)}$$

Center of Mass of a Semicircular Disc

Figure shows the half disc of mass M and radius R. Here, we are only required to find the y-coordinate of the center of mass of this disc as center of mass will be located on its half vertical diameter. Here to find y_{cm}, we consider a small elemental semicircular ring of mass dm of radius x on the disc (disc can be considered to be made up such thin rings of increasing radii) which will be integrated from 0 to R. Here dm is given as

$$dm = \frac{2M}{\pi R^2}(\pi x)dx$$



Now the y-coordinate of the element is $\frac{2x}{\pi}$, as in previous section, we have derived that the center of mass of a semi circular ring lies at $\frac{2R}{\pi}$.

Here y_{cm} is given as

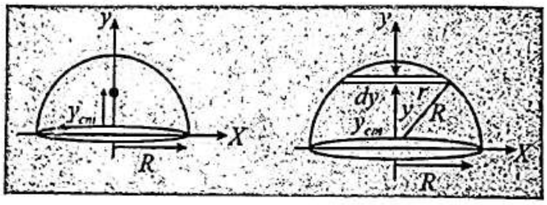
$$y_{cm} = \frac{1}{M}\int dm y$$

$$y_{cm} = \frac{1}{M}\int_0^R dm\,\frac{2x}{\pi} = \frac{1}{M}\int_0^R \frac{4M}{\pi R^2}x^2\,dx = \frac{4}{\pi R^2}\left[\frac{x^3}{3}\right]_0^R$$

$$\Rightarrow y_{cm} = \left(\frac{4R}{3\pi}\right)$$

Center of Mass of a Solid Hemisphere

The hemisphere is of mass M and radius R. To find its center of mass (only y-coordinate), we consider an element disc of width dy, mass dm at a distance y from the center of the hemisphere. The radius of this elemental disc will be given as, $r = \sqrt{R^2 - y^2}$



The mass dm of this disc can be given as

$$dm = \frac{3M}{2\pi R^3} \times \pi r^2\, dy = \frac{3M}{2R^3}(R^2 - y^2)\, dy$$

y_{cm} of the hemisphere is given as

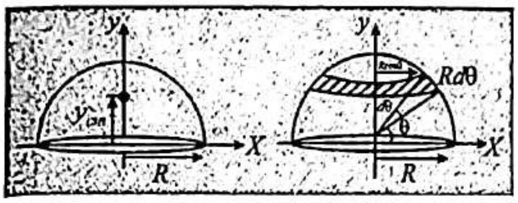
$$y_{cm} = \frac{1}{M}\int_0^R dm\, y = \frac{1}{M}\int_0^R \left[\frac{3M}{2R^3}(R^2 - y^2)\, dy\right] y$$

$$= \int_0^R \left[\frac{3M}{2R^3}(R^2 - y^2)\, dy\right] y \Rightarrow y_{cm} = \frac{3R}{8}$$

Center of Mass of a Hollow Hemisphere

A hollow hemisphere of mass M and radius R. Now we consider an elemental circular strip of angular width $d\theta$ at an angular distance θ from the base of the hemisphere. This strip will have an area.

$$dS = 2\pi R \cos\theta\, R d\theta$$



Its mass dm is given as

$$dm = \frac{M}{2\pi R^2}\, 2\pi R \cos\theta\, R d\theta$$

Here y - coordinate of this strip of mass dm can be taken as $R \sin\theta$. Now we can obtain the center of mass of the system as.

$$y_{cm} = \frac{1}{M}\int_0^{\frac{\pi}{2}} dm\, R\sin\theta$$

$$= \frac{1}{M}\int_0^{\frac{\pi}{2}} \left(\frac{M}{2\pi R^2} 2\pi R^2 \cos\theta\, d\theta\right) R\sin\theta$$

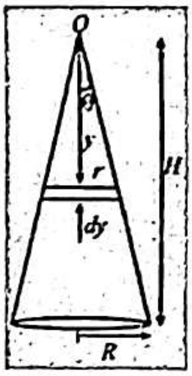
$$= R\int_0^{\frac{\pi}{2}} \sin\theta\, \cos\theta\, d\theta \Rightarrow y_{cm} = \frac{R}{2}$$

Center of mass of a Solid Cone

A solid cone has mass M, height H and base radius R. Obviously the center of mass of this cone will lie somewhere on its axis, at a height less than $H/2$. To locate the center of mass we consider an elemental disc of width dy and radius r, at a distance y from the apex of the cone. Let the mass of this disc be dm, which can be given as

$$dm = \frac{3M}{\pi R^2 H} \times \pi r^2\, dy$$

here y_{cm} can be given as

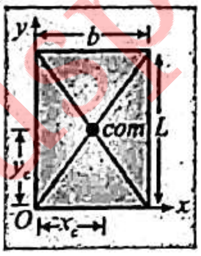


$$y_{cm} = \frac{1}{M}\int_0^H y\, dm \qquad = \frac{1}{M}\int_0^H \left(\frac{3M}{\pi R^2 H}\pi\left(\frac{Ry}{H}\right)^2 dy\right) y$$

$$= \frac{3}{H^3}\int_0^H y^3\, dy = \frac{3H}{4} \text{ (from vertex)}$$

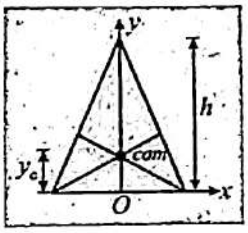
Center of Mass of Some Other Common Systems

- Rectangular plate (By symmetry)



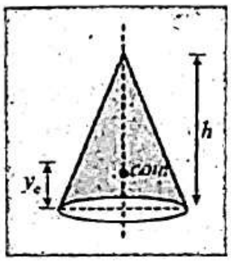
$$x_c = \frac{b}{2};\; y_c = \frac{L}{2}$$

- A triangular plate (By qualitative argument)



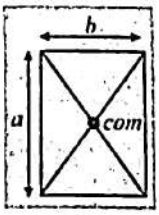
at the centroid: $y_c = \frac{h}{3}$

- A circular cone (hollow)

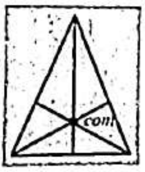


$$y_c = \frac{h}{3}$$

- Center of mass of rectangle at $\left(\frac{a}{2},\frac{b}{2}\right)$



- Center of mass of an equilateral triangle lies at point when medians meet.



Notes:

(i) Center of mass of a symmetrical body having uniform density lies at its geometrical center.

For example, center of mass of sphere (hollow or solid) lies at its center, center of mass of a ring or a disc at the center of the ring and center of mass of a uniform rod at its mid point.

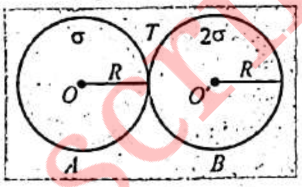
(ii) If a line or a plane divides a body into two symmetrical parts then, center of mass lies on that line or that plane.

For example, the center of a rectangular plate will lie on the diagonals. The point where diagonals meet will be its center of mass.

(iii) For locating the center of mass of a composite body, each extended component of the body can be replaced by a particle placed at its center of mass.

Train Your Brain

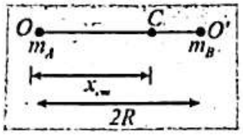
Example 3: Two circular disc having radius R and mass density σ and 2σ respectively are placed as shown in figure. Then find out the position of COM of the system (from point 'O').



Sol. Mass of disc A: $m_A = \sigma\pi R^2 = m_1$

Mass of disc B: $m_B = 2\sigma\pi R^2 = m_2$

Due to symmetry the COM of disc A lie at point O and COM of disc B lie at point O'.



We can replace two discs by two particles placed at their center.

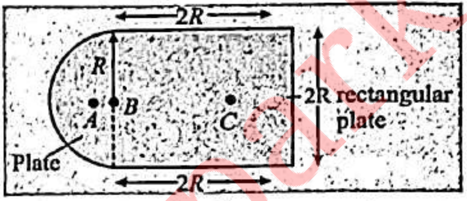
Let us choose x-axis along OO' with origin at O. Then,

$x_1 = 0, x_2 = 2R$

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{0 + 2\sigma\pi R^2 \times 2R}{\sigma\pi R^2 + 2\sigma\pi R^2} = \frac{4R}{3}$$

So the centre of mass lie in the disc B at distance $\frac{4R}{3}$ from O.

Example 4: Find out the position of centre of mass of an object shown in figure below having uniform density.



Sol. We divide the above problem in two parts

(i) First find out position of centre of mass of both semicircular plate and rectangular plate separately.

(ii) Then find the position of centre of mass of given structure.

Centre of mass of semicircular disc lie at $\frac{4R}{3\pi}$

$$\Rightarrow AB = \frac{4R}{3\pi}$$

Centre of mass of rectangular plate lie at the centre of plate at point C

$$\Rightarrow BC = R$$

Let us choose x-axis along AC with origin at point B.

$$x_1 = -AB = -\frac{4R}{3\pi}$$

$$x_2 = BC = R$$

$$m_1 = m_{SC} = \frac{\sigma\pi R^2}{2}$$

$$m_2 = m_R = \sigma 4R^2$$

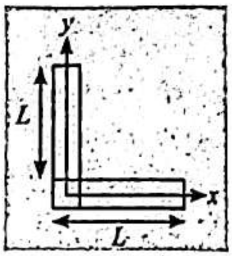
$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{\frac{-\sigma\pi R^2}{2} \times \frac{4R}{3\pi} + \sigma 4R^2 \times R}{\frac{\sigma\pi R^2}{2} + \sigma 4R^2}$$

$$= \frac{20}{24 + 3\pi}R$$

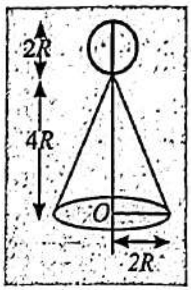
Concept Application

4. The x and y coordinates of centre of mass of two thin uniform rods of same length but made up of different materials & kept (as shown) can be. (if the meeting point is the origin of co-ordinates)



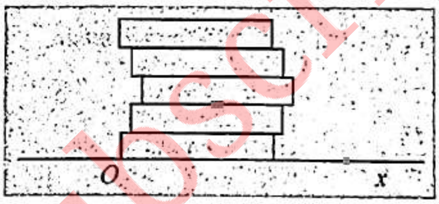
(a) (L/2, L/2) (b) (2L/3, L/2)
(c) (L/3, L/3) (d) (L/3, L/6)

5. A man has constructed a toy as shown in figure. If density of the material of the sphere is 12 times that of the cone, compute the position of the centre of mass. [Centre of mass of a cone of height h is at height of $h/4$ from its base.]



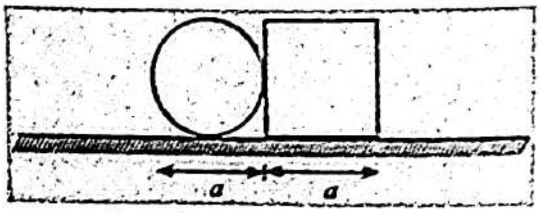
(a) $4R$ from O (b) $3R$ from O
(c) R from O (d) $2R$ from O

6. Five homogeneous bricks, each of length L, are arranged as shown in figure. Each brick is displaced with respect to the one in contact by $L/5$. Find the x-coordinate of the centre of mass relative to the origin O shown.



(a) $\frac{32L}{50}$ (b) $\frac{30L}{50}$ (c) $\frac{33L}{50}$ (d) $\frac{25L}{50}$

7. A square plate of edge 'a' and a circular disc of same diameter are placed touching each other at the midpoint of an edge of the plate as shown in figure. If mass per unit area for the two plates are same then find the distance of centre of mass of the system from the centre of the disc.



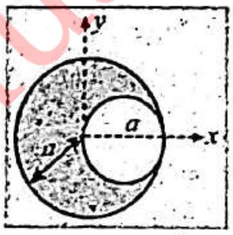
(a) $\frac{4a}{(4+\pi)}$ (b) $\frac{4a}{3+\pi}$
(c) $\frac{3a}{4+\pi}$ (d) $\frac{(4-\pi)a}{2(4+\pi)}$

CENTER OF MASS OF TRUNCATED BODIES

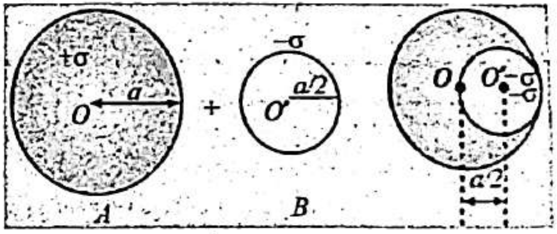
If some mass m is removed from a rigid body then the position of centre of mass of the remaining portion is obtained by assuming that in a remaining part $+m$ & $-m$ mass is there at the location of removed portion. Further steps are explained by following example.

Train Your Brain

Example 5: Find the position of centre of mass of the disc of radius a from which a cavity of radius $a/2$ is dug out. The mass density of the disc is σ.



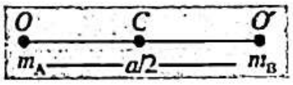
Sol. The body can be visualized as a superposition of a disc of radius a having mass density $+\sigma$ and another disc of $a/2$ radius having $-\sigma$ mass density placed at the location of cavity. So now the problem is



The centre of mass of both disc A & B lie in their respective centre such as O & O'.

Now

$\Rightarrow$ C.O.M. of the lamina $x_{com} = \frac{m_B a/2}{m_A + m_B}$



$m_A = \sigma(\pi a^2)$, $m_B = -\sigma(\pi)(a/2)^2$

$= -\sigma\pi\frac{a^2}{4}$

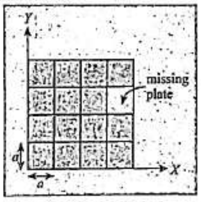
$$\Rightarrow x_{com} = \frac{-\left(\sigma\pi\frac{a^2}{4}\right)\times\frac{a}{2}}{\sigma\pi a^2 - \sigma\pi\frac{a^2}{4}} = \frac{-4a}{24} = \frac{-a}{6}$$

x-axis divides the body into two symmetrical parts, therefore C.O.M lies on x-axis

$\Rightarrow y_{cm} = 0$

C.O.M lie on leftward side from point O at distance of $a/6$.

Example 6: In the figure shown below, fifteen identical square plates each of side a and mass m are arranged side by side with one plate missing at the position indicated. Find the location of center of mass.



Sol. The missing plate can be visualized to be a superposition of plates of mass m and mass $-m$. The system now consists of one large plate of mass $16m$ and another plate of mass $-m$.

$m_1 = 16m, x_1 = 2a, y_1 = 2a$

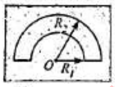
$m_2 = -m, x_1 = 3.5a, y_2 = 2.5a$

$$x_{cm} = \frac{16m\times2a - m\times3.5a}{16m-m}, y_{cm} = \frac{16m\times2a - m\times2.5a}{16m-m}$$

$$x_{cm} = \frac{57a}{30} \qquad y_{cm} = \frac{59a}{30}$$

Concept Application

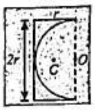
8. Find the distance of centre of mass of an annular half disc from point O shown in figure



(a) $\frac{4}{3\pi}\left(\frac{R_2^3-R_1^3}{R_2^2-R_1^2}\right)$ (b) $\frac{8}{3\pi}\left(\frac{R_2^3-R_1^3}{R_2^2-R_1^2}\right)$

(c) $\frac{4}{3\pi}\left(\frac{R_2^2-R_1^2}{R_2^3-R_1^3}\right)$ (d) $\frac{8}{3\pi}\left(\frac{R_2^2-R_1^2}{R_2^3-R_1^3}\right)$

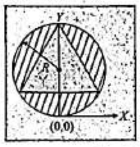
9. A semicircular portion of radius 'r' is cut from a uniform rectangular plate as shown in figure. The distance of centre of mass 'C' of remaining plate, from point 'O' is:



(a) $\frac{2r}{(3r-\pi)}$ (b) $\frac{3r}{2(4-\pi)}$

(c) $\frac{2r}{(4+\pi)}$ (d) $\frac{2r}{3(4-\pi)}$

10. From a uniform disc of radius R, an equilateral triangle of side $\sqrt{3}R$ is cut, as shown in the figure. The new position of centre of mass is



(a) $(0, 0)$ (b) $(0, R)$

(c) $\left(0, \frac{\sqrt{3}R}{2}\right)$ (d) None of these

MOTION OF THE CENTRE OF MASS

Now we can discuss the physical importance of the centre of mass concept. Consider the motion of a group of particles $m_1, m_2, \ldots\ldots, m_n$ and whose total mass is M. From the definition of the centre of mass, we have

$$M\vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots\ldots\ldots\ldots\ldots\ldots + m_n\vec{r}_n$$

Differentiating this equation w.r.t. time, we obtain

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots\ldots + m_n\vec{v}_n$$

where $\vec{v}_1 = \frac{d\vec{r}_1}{dt}$ is the ve telocity of the first particle, etc., and $\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt}$ is the velocity of the centre of mass.

Differentiating the above equation w.r.t. time, we obtain

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots\ldots\ldots + m_n\vec{a}_n$$

where $\vec{a}_1$ is the acceleration of the first particle, etc., and $\vec{a}_{cm}$ is the acceleration of the centre of mass. Now, from Newton's second law the force $\vec{F}_1$ acting on the first particle is given by $\vec{F}_1 = m_1\vec{a}_1$. Likewise, $\vec{F}_2 = m_2\vec{a}_2$, etc. We can then write the above equation as $M\vec{a}_{cm} = \vec{F}_1 + \vec{F}_2 + \ldots\ldots\ldots + \vec{F}_n = \vec{F}_{net}$

Now consider the net force on a system of particles.

There are two kinds of forces:

(i) **Internal forces,** resulting from the forces between the particles within the system.

(ii) **External forces,** arising from the forces between the particles in the system and objects outside the system.

For example, consider a system of two blocks joined together with a spring. If the system is allowed to fall freely under gravity, then the gravitational force acting on each block is the external force and the spring force acting on each block is internal.

The net force on the system is vector sum of internal and external force acting on the system. However, from Newton's third law, internal forces will occur in equal and opposite pairs so they contribute nothing to the sum.

$$\sum\vec{F}_{int} = 0$$

Thus the Newton's second law may be written as

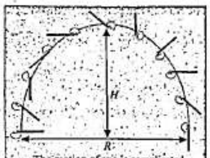
$$\vec{F}_{net} = \sum\left(\vec{F}_{ext} + \vec{F}_{int}\right) = \sum\vec{F}_{ext}$$

$$\text{or} \sum\vec{F}_{ext} = M\vec{a}_{cm}$$

This states that the centre of mass of a system of particles moves as though all the mass of the system is concentrated at the centre of mass and all the external forces were applied at that point.

The acceleration of center of mass only depends only on the external force acting on the system; irrespective of internal forces acting between the particles of the system.

For example an axe thrown in air at an angle θ with the horizontal will perform a complicated motion of rotation as well as parabolic motion under the effect of gravitation but the acceleration of its center of mass will equal to g in downward direction and hence



The motion of axe is complicated but the COM is moving in a parabolic motion.

$$H_{com} = \frac{u^2\sin^2\theta}{2g}; R_{com} = \frac{u^2\sin^2\theta}{g}\ T = \frac{2u\sin\theta}{g}$$

One important consequence of this is that if $\sum\vec{F}_{external} = \vec{0}$ then $\vec{v}_{cm}$ = constant.

Notes:

(i) If centre of mass is initially at rest and $\sum \vec{F}_{external} = 0$ on centre of mass, then displacement of centre of Mass will be zero.

Hence shift in position of centre of mass is zero.

(ii) If $\Sigma\vec{F}ext$ is zero on COM then $\vec{v}_{com}$ = constant but kinetic energy of system may or may not be constant because kinetic energy of the system can also change due to internal forces.

Train Your Brain

Example 7: A dog of mass 10 kg stands on a stationary boat of mass 40 kg so that he is 20 m from the shore. He then walks 8 m on the boat towards the shore and halts. How far is he from the shore now? Assume that there is no friction between the boat and water.

Sol. In the horizontal direction, there is no force on the system (dog + boat). Therefore, the centre of mass of the system does not move in the horizontal direction. Since the dog moves towards the shore, the boat moves away from the shore to keep centre of mass stationary. Let d be the distance by which the boat moves backwards and let x be the initial distance of the boat from the shore. The initial x-coordinate of the

centre of mass $= \dfrac{10 \times 20 + 40x}{10 + 40} = x_i$

The final x-coordinate of the centre of mass

$$x_f = \frac{10(20 - 8 + d) + 40(x + d)}{10 + 40}$$

Equating the two, we get

$$d = \frac{8}{5}\text{ m}$$

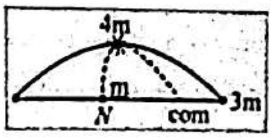
$\therefore$ The dog is $20 - 8 + \dfrac{8}{5} = 13.6$ m from the shore.

Example 8: A projectile is fired at a speed of 100 m/s at an angle of 37° above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio 1 : 3, the smaller coming to rest. Find the distance from the launching point to the point where the heavier piece lands.

Sol. Internal force do not effect the motion of the centre of mass, the centre of mass hits the ground at the position where the original projectile would have landed. The range of the original projectile is,

$$x_{COM} = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{2 \times 10^4 \times \frac{3}{5} \times \frac{4}{5}}{10}\text{ m}$$

$$= 960\text{ m}$$



The centre of mass will hit the ground at this position. As the smaller block comes to rest after breaking, it falls down vertically and hits the ground at half of the range, i.e., at $x = 480$ m. If the heavier block hits the ground at x_2, then

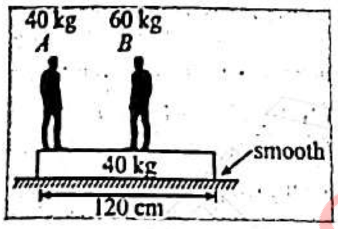
$$x_{COM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$960 = \frac{(m)(480) + (3m)(x_2)}{(m + 3m)}$$

$$x_2 = 1120 \text{ m}$$

Concept Application

11. Two men 'A' and 'B' are standing on a plank. 'B' is at the middle of the plank and 'A' is at the left end of the plank. Bottom surface of the plank is smooth. System is initially at rest and masses are as shown in figure. 'A' and 'B' start moving such that the position of 'B' remains fixed with respect to ground and 'A' meets 'B'. Then the point where A meets B is located at:



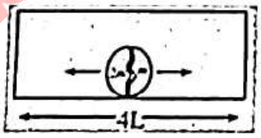
(*a*) the middle of the plank

(*b*) 30 cm from the left end of the plank

(*c*) the right end of the plank

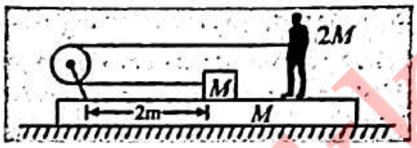
(*d*) None of these

12. A bomb of mass $3m$ is kept inside a closed box of mass $3m$ and length $4L$ at it's centre. It explodes in two parts of mass m & $2m$. The two parts move in opposite direction and stick to the opposite side of the walls of box. Box is kept on a smooth horizontal surface. What is the distance moved by the box during this time interval.



(*a*) 0 (*b*) $\frac{L}{3}$ (*c*) $\frac{L}{6}$ (*d*) $\frac{L}{12}$

13. A block of mass M is tied to one end of a massless rope. The other end of the rope is in the hands of a man, of mass $2M$. The block and the man are resting on a rough wedge of mass M. The whole system is resting on a smooth horizontal surface. The man starts walking towards right while holding the rope in his hands. Pulley is massless and frictionless. Find the displacement of the wedge when the block meets the pulley. Assume wedge is sufficiently long so that man does not fall down.



(*a*) 1/2 m towards right

(*b*) 1/2 m towards left

(*c*) The wedge does not move at all

(*d*) 1 m towards left

LINEAR MOMENTUM AND ITS CONSERVATION

When the sum of the forces on an object is zero, the equation

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

tells us that the time derivative of momentum is zero.

i.e., $\frac{d\vec{p}}{dt} = 0$

$\Rightarrow \vec{p} =$ constant

Consequently, one can state a conservation law for momentum:

When the net force on a particle is zero, its momentum is constant.

The real utility of the momentum conservation concept comes about when it is applied to a collection of particles. For a system of particles, the total momentum is simply the vector sum of the momentum of each of the particles in the system. i.e.,

$$\vec{P} = \sum_{i=1}^{N} \vec{p}_i = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n = M\vec{V}_{cm}$$

Differentiating with respect to time, $\frac{d\vec{P}}{dt} = M\vec{a}_{cm} = \vec{F}_{\text{ext}}$

Now, if the summation of external forces is zero, then

$$\frac{d\vec{P}}{dt} = 0$$

Thus, $\vec{P} = \text{constant}\ \left(\vec{V}_{cm} = \text{constant}\right)$

In the absence of a net external force, the momentum of a system is conserved and velocity of center mass is constant.

The conservation of momentum law can be used to relate the initial motion of particles within a system to the motion of those same particles some time later. The law emphasizes the equality of momentum before and after something happens within the system.

Thus, the conservation law is usually written as

$$\vec{P}_{\text{initial}} = \vec{P}_{\text{final}} \qquad \sum_{i=1}^{N} \vec{p}_i = \sum_{f=1}^{N} \vec{p}_f$$

Center of Mass Frame:

A reference frame attached to center of mass of the system of particles is called center of mass frame. In this frame center of mass is always at the origin. Therefore in this frame velocity of center of mass and momentum of the system will always be zero.

It is special frame and presents simple interpretation and solution to several complex problems

Relationship between kinetic energy of a system of particles in ground frame and center of mass frame:

Let $\vec{v}_i$ and $\vec{v}_i'$ be the velocity of i^{th} particle in ground frame and center of mass frame, then

$\vec{v}_i' = \vec{v}_i - \vec{v}_{cm}$ or $\vec{v}_i = \vec{v}_i' + \vec{v}_{cm}$

$$\text{K.E} = \sum_{i=1}^{n} \frac{1}{2} m_i \vec{v}_i \cdot \vec{v}_i = \sum_{i=1}^{n} \frac{1}{2} m_i v'^2 + \frac{1}{2}\left(\sum m_i\right)\vec{V}_{cm}^2 + \sum m_i \vec{v}_i' \cdot \vec{V}_{cm}$$

But $\sum m_i \vec{v}_i' = MV_{cm}' = 0$

$$\frac{1}{2}\sum m_i v_i^2 = \frac{1}{2}\sum m_i v_i'^2 + \frac{1}{2} MV_{cm}^2$$

$$\Rightarrow \text{K.E} = K.E_{\text{rel'cm}} + \frac{1}{2} MV_{cm}^2$$

The first term on the right hand side is the kinetic energy relative to center of mass and second term is the kinetic energy due to translation of the center of mass

Since $V_{cm}^2 \geq 0$, K.E. is minimum in the center of mass frames

For a system of two particles of mass m_1 and m_2 which we encounter very frequently, kinetic energy relative to center of mass is given by

$$K_{\text{rel'cm}} = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} v_{rel}^2 = \frac{1}{2}\mu v_{rel}^2 \text{ where } \mu = \frac{m_1 m_2}{m_1 + m_2}$$

Here μ is called the reduced mass of the two particle system and v_{rel} is the magnitude of velocity one particle with respect to other.

$v_{rel} = |\vec{v}_1 - \vec{v}_2| = |\vec{v}_2 - \vec{v}_1|$

For an isolated system value $V_{cm,f} = V_{cm,i}$, hence, change in K.E is given by

$$\Delta K = \frac{1}{2}\mu\left(v_{\text{rel},f}^2 - v_{\text{rel},i}^2\right)$$

Train Your Brain

Example 9: A particle of mass $m_1 = 4$ kg moves at $5\hat{i}$ m/s, while $m_2 = 2$ kg moves at $2\hat{i}$ m/s. Find

(*a*) the kinetic energy of center of mass

(*b*) the kinetic energy of the system with respect to center of mass.

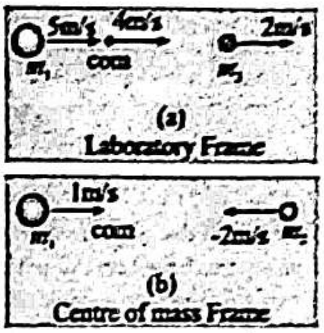
Sol. The velocity of the centre of mass is obtained as

$$v_{cm} = \frac{(4\text{kg})(5\text{m/s}) + (2\text{kg})(2\text{m/s})}{6\text{kg}} = 4 \text{ m/s}$$

The following figure shows the velocities relative to the centre of mass

$v_1' = v_1 - v_{cm} = +1$ m/s

$v_2' = v_2 - v_{cm} = -2$ m/s



Thus,

(*a*) $K_{cm} = \frac{1}{2}(m_1 + m_2)v_{cm}^2 = 48$ J

(*b*) $K' = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 = 6$ J

or alternatively

$$K' = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} v_{rel}^2 = \frac{1}{2} \times \frac{4 \times 2}{6}(5-2)^2 = 6\text{J}$$

Example 10: The last stage of a rocket is travelling at a speed of 7600 m/s. This last stage is made up of two parts that are clamped together, namely, a rocket case with a mass of 290.0 kg and a payload capsule with a mass of 150.0 kg. When the clamp is released, a compressed spring causes the two parts to separate with a relative speed of 910.0 m/s. What are the speeds of the two parts after they have separated? Assume that all velocities are along the same line.

Sol. We assume no external forces act on the system composed of the two parts.

So, total momentum of the system is conserved.

Now,

$(m_c + m_p)\,V = m_c v_c + m_p v_p$ ... (*i*)

Where,

m_c = mass of rocket case.

m_p = mass of payload capsule.

v_p = velocity fo payload capsule.

v_c = velocity of rocket case.

v = speed of rocket.

$\because\ v_p = v_c + v_{rel}$ ... (*ii*)

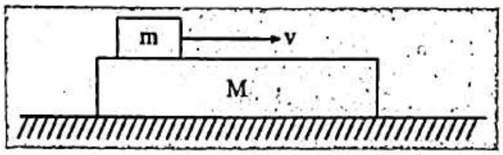
So, $v_c = \dfrac{(m_c + m_p)v - m_p v_{rel}}{m_c + m_p}$

$= \dfrac{(290 + 150)7600 - 150(910)}{290 + 150} = 7290$ m/s

Now, from equation (*ii*), we have

$v_p = v_c + v_{rel} = 7290 + 910 = 8200$ m/s

Example 11: A long plank of mass M, with a block of mass m placed on it, rests on a smooth horizontal surface. The block m is set in motion in the horizontal direction with velocity v. Due to friction between the plank and the block, the block slows down and moves in one piece with the plank. Find the work performed by frictional forces.



Sol. We choose the block and the plank as our system. Since friction is absent between the plank and the horizontal surface, the linear momentum in the horizontal direction will be conserved (No external force in the horizontal direction). But since there is friction between the plank and block (an internal non-conservative force), mechanical energy of the system will not be conserved. Hence

$P_i = P_f$ ... (*i*)

$\Delta KE + \Delta PE = W_{friction}$ ... (*ii*)

Let the final velocity of the block and plank be v′

From eqn (*i*), we get

$mv = (m + M)\,v'$

$\Rightarrow v' = \dfrac{m}{M+m}v$ ... (*iii*)

Substituting the value of v' in eqn (*ii*), we obtain

$$W_{friction} = \frac{1}{2}(m+M)v'^2 - \frac{1}{2}mv^2$$

$$= -\frac{1}{2}\frac{mM}{M+m}v^2$$

Alternatively, Since $v_{cm,f} = v_{cm,i}$

$$\Delta K = \frac{1}{2}\frac{mM}{M+m}\left[v_{rel,f}^2 - u_{rel}^2\right]$$

$u_{rel} = v - 0 = v$

$v_{rel} = v' - v' = 0$

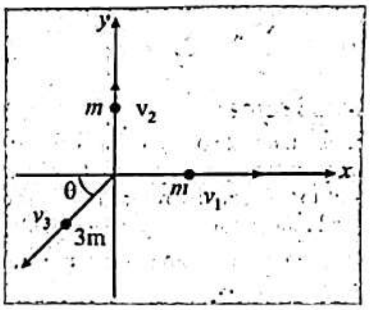
$$\Rightarrow \Delta K = -\frac{1}{2}\frac{mM}{Mm}v^2 = W_{friction}$$

Example 12: A body of mass 1kg initially at rest explodes and breaks into three fragments of masses in the ratio 1:1:3. The two pieces of equal mass fly off perpendicular to each other with a speed of 30 m/s. What is the velocity of the heavier fragment?

Sol. Internal forces can not change the momentum of the system

Let $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ be the velocities of the three fragments.

Let x and y axis point in the direction of $\vec{v}_1$ and $\vec{v}_2$ and let $\vec{v}_3$ make an angle θ with -ve X-axis.



Conserving momentum in x and y directions, we have

$mv_1 - 3mv_3\cos\theta = 0$ ... (*i*)

$mv_2 - 3mv_3\sin\theta = 0$ ... (*ii*)

Here, $v_1 = v_2 = 30$m/s

Solving eqn (*i*) and eqn (*ii*), we find

$\cos\theta = \sin\theta \Rightarrow \theta = 45°$

and $v_3 = \dfrac{\sqrt{2}v_1}{3} = \dfrac{\sqrt{2}}{3}\times 30 = 10\sqrt{2}\,m/s$

Therefore, the third fragment flies with a speed of $10\sqrt{2}$ m/s at an angle of 135° with $\vec{v}_2$

Example 13: A 75 kg man stands at the rear end of a platform of mass 25 kg and length 4m, which moves initially at 4 m/s over a frictionless surface. At $t = 0$, he walks at 2 m/s relative to the platform and then stops at the front end. During the period of walking, find the displacement of

(*a*) the platform, (*b*) the man

(*c*) the center of mass

Sol. Initially the man, the platform, and the c.o.m. have the same velocity, 4m/s. When he begins to walk forward, his increase in momentum must be compensated by a decrease in the platform's momentum. Let us say that the velocity of the platform relative to the ground while he is walking i.e.

$\vec{v}_{PG} = v_p\hat{i}$

The velocity of man relative to the ground is then

$\vec{v}_{MG} = \vec{v}_{MP} + \vec{v}_{PG} = (2 + v_P)\,\hat{i}$:

From the conservation of momentum, we have

$(75 + 25)\times 4 = 75(2 + v_p) + 25\,v_P$

Thus, the velocity of the platform is $v_P = 2.5$ m/s, and the velocity of the man is $v_m = 4.5$ m/s.

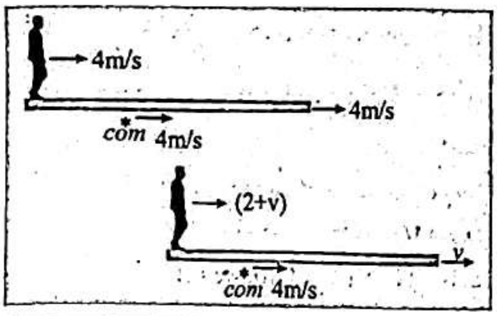
Since the velocity of man relative to platform is 2m/s, therefore, it takes $t = \dfrac{4m}{2m/s} = 2$ s for him to walk from the rear to the front.

Displacement of platform is $x_p = v_p t = (2.5)(2) = 5$m

Displacement of the man is $x_m = v_m t = (4.5)(2) = 9$m

Displacement of the center of mass

$x_{cm} = v_{cm}t = (4)(2) = 8$ m



Concept Application

14. A stationary body explodes into two fragments of masses m_1 and m_2. If momentum of one fragment is p, the minimum energy of explosion is

(a) $\dfrac{p^2}{2(m_1+m_2)}$ (b) $\dfrac{p^2}{2\sqrt{m_1 m_2}}$

(c) $\dfrac{p^2(m_1+m_2)}{2m_1 m_2}$ (d) $\dfrac{p^2}{2(m_1-m_2)}$

15. A flat car of mass M is at rest on a frictionless floor with a child of mass m standing at its edge. If child jumps off from the car towards right with an initial velocity u, with respect to the car, find the velocity of the car after his jump.

(a) $\dfrac{mu}{m+M}$ (b) $\dfrac{mu}{m-M}$

(c) $\dfrac{2mu}{m-M}$ (d) $\dfrac{2mu}{m+M}$

16. A gun of mass M, fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height 'h'. The recoil velocity of the gun is

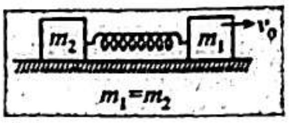
(a) $\left(\dfrac{2m^2gh}{M(m+M)}\right)^{\frac{1}{2}}$ (b) $\left(\dfrac{2m^2gh}{M(m-M)}\right)^{\frac{1}{2}}$

(c) $\left(\dfrac{2m^2gh}{2M(m-M)}\right)^{\frac{1}{2}}$ (d) $\left(\dfrac{2m^2gh}{2M(m+M)}\right)^{\frac{1}{2}}$

SPRING BLOCK SYSTEMS

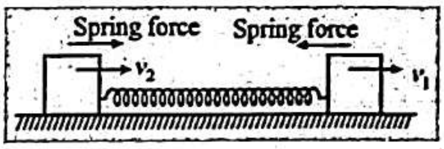
Analysis of Motion of Spring Block System in Ground Frame

- ❖ Consider a two blocks system connected with a spring placed on a smooth surface. An impulse acts on frame block so that it acquires a velocity v_0. As there is no external force in horizontal direction and non-conservative forces are absent, both momentum and mechanical energy are conserved. A ground observer see the sequence of events as follows.



For simplicity, let us assume $m_1 = m_2 = m$

(a) Spring begins to stretch, now spring force retards m_1 but accelerates m_2. Stretch in spring continues till velocity v_1 and v_2 are equal i.e., v_{rel} is zero. At the state of maximum extension both blocks have same velocity.



(b) Forward translation of system continues with velocity

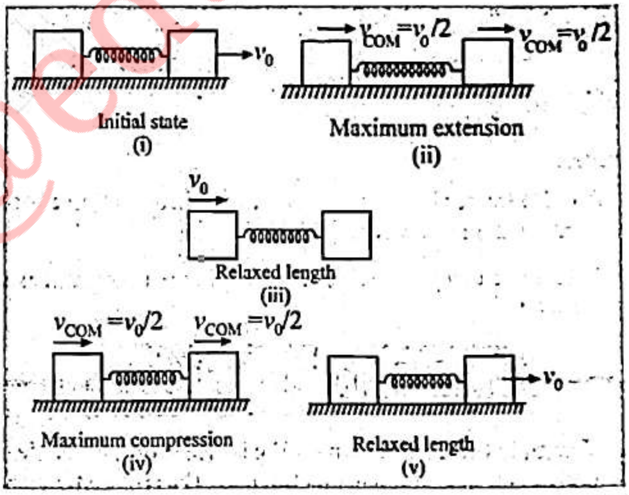
$$v_{cm} = \frac{m_2 v_0}{m_1+m_2} = \frac{v_0}{2}$$

In the absence of any external force $\vec{v}_{CM}$ = const. At the instant of maximum stretch of spring both the blocks have same velocity which is equal to velocity of centre of mass.

In COM frame blocks will be stationary at this moment. (Fig. ii)

(c) While translating forward spring begins to return to its natural length. When spring regains its natural length, (Fig. ii)

(d) At the instant of maximum compression velocity of both blocks is same in ground frame. (Fig. iv)

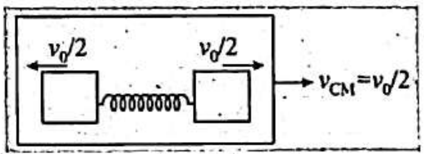


In ground reference frame system moves forward with spring getting compressed and relaxed, then extended alternatively.

Analysis of Motion of Spring Block System in COM Frame

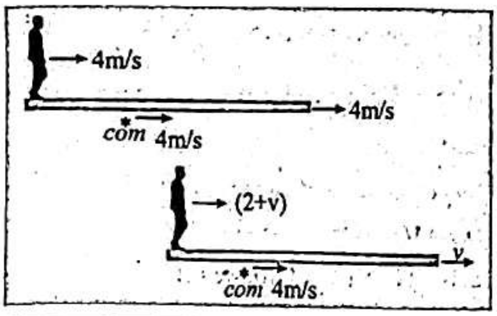
Now lets see what happens in COM frame.

(a) Initial state



Displacement of the center of mass

$x_{cm} = v_{cm}t = (4)(2) = 8$ m



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SPRING BLOCK SYSTEMS

Analysis of Motion of Spring Block System in Ground Frame

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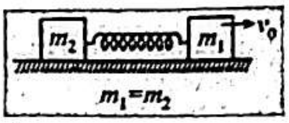
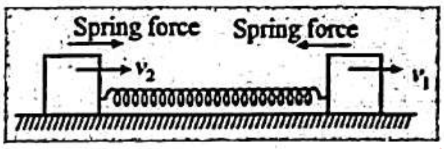


For simplicity, let us assume $m_1 = m_2 = m$

(a) Spring begins to stretch, now spring force retards m_1 but accelerates m_2. Stretch in spring continues till velocity v_1 and v_2 are equal i.e., v_{rel} is zero. At the state of maximum extension both blocks have same velocity.



(b) Forward translation of system continues with velocity

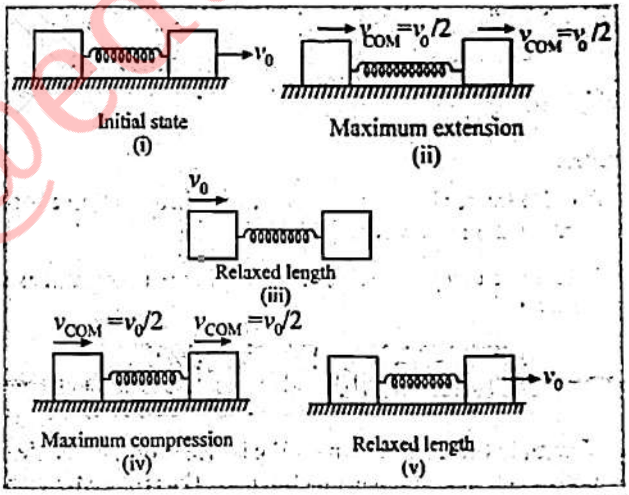
$$v_{cm} = \frac{m_2 v_0}{m_1+m_2} = \frac{v_0}{2}$$

In the absence of any external force $\vec{v}_{CM}$ = const. At the instant of maximum stretch of spring both the blocks have same velocity which is equal to velocity of centre of mass.

In COM frame blocks will be stationary at this moment. (Fig. ii)

(c) While translating forward spring begins to return to its natural length. When spring regains its natural length, (Fig. ii)

(d) At the instant of maximum compression velocity of both blocks is same in ground frame. (Fig. iv)



Initial state (i)

Maximum extension (ii)

Relaxed length (iii)

Maximum compression (iv)

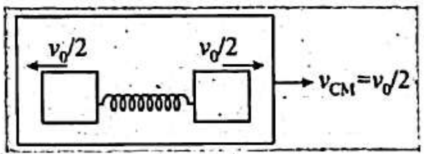
Relaxed length (v)

In ground reference frame system moves forward with spring getting compressed and relaxed, then extended alternatively.

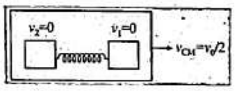
Analysis of Motion of Spring Block System in COM Frame

Now lets see what happens in COM frame.

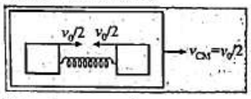
(a) Initial state



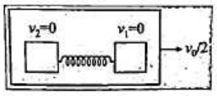
(*b*) At maximum extension



(*c*) At relaxed length of spring



(*d*) At maximum compression



In COM frame the observer sees only oscillatory motion of blocks.

At the maximum extension (or maximum compression) of spring blocks are at rest in COM frame.

or $-\left[\frac{1}{2}kx_{max}^2-0\right]=\Delta KE_{system/CM}$

or $-\frac{1}{2}kx_{max}^2=\left[\frac{1}{2}m(v_0/2)^2+\frac{1}{2}m(v_0/2)^2\right]$

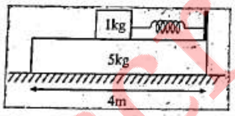
or $x_{max}=\sqrt{\frac{m}{2k}}v_0$

Notes:

- COM frame is also known as zero-momentum frame because momentum is always zero in this frame.
- To solve complex problems, COM frame is sometimes very useful.

Train Your Brain

Example 14: A plank of mass 5kg is placed on a frictionless horizontal plane. Another block of mass 1kg is placed over the plank. A massless spring of natural length 2 m is fixed to the plank by its one end. The other end of spring is compressed by the block by half of spring's natural length. The system is now released from the rest. What is the velocity of the plank when block leaves the plank? All the surfaces are smooth. (The stiffness constant of spring is 100 Nm^{-1})



Sol. Let the velocities of the block and the plank, when the block leaves the spring, be u and v, respectively.

By conservation of energy, $\frac{1}{2}kx^2=\frac{1}{2}mu^2+\frac{1}{2}Mv^2$

(where, M = mass of the plank, m = mass of the block)

$\Rightarrow 100=u^2+5v^2$

By conservation of momentum,

$mu+Mv=0\Rightarrow u=-5v$

Solving Eqs. (i) and (ii), we get

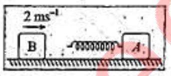
$30v^2=100$

$\Rightarrow v=\sqrt{\frac{10}{3}}\text{ ms}^{-1}$

From this moment until block falls, both plank and block keep their velocity constant.

Thus, when block falls, velocity of plank $=\sqrt{10/3}\text{ ms}^{-1}$

Example 15: Two blocks A and B of equal mass m = 1 kg are lying on a smooth horizontal surface as shown in figure. A spring of force constant k = 200 Nm^{-1} is fixed at one end of block A. Block B collides with block A with velocity v_0 = 2 ms^{-1}. Find the maximum compression of the spring.



Sol. At maximum compression (x_m), velocity of both the blocks is same, say v

Applying conservation of linear momentum, we have

$(m_A+m_B)v=m_Bv_0$

or $(1+1)v=(1)v_0$

or $v=\frac{v_0}{2}=\frac{2}{2}=1\text{ ms}^{-1}$

Using conservation of mechanical energy, we have

$\frac{1}{2}m_Bv_0^2=\frac{1}{2}(m_A+m_B)v^2+\frac{1}{2}kx_m^2$

Substituting the given values in above equation, we get

$\frac{1}{2}\times(1)\times(2)^2=\frac{1}{2}\times(1+1)\times(1)^2+\frac{1}{2}\times(200)\times x_m^2$

or $x_m=0.1\text{ m}=10\text{ cm}$

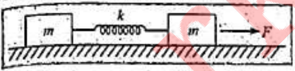
Alternatively,

$\Delta K=\frac{1}{2}\frac{1\times1}{2}[0^2-2^2]=-1\text{J}\ [v_{cm}=\text{constant}]$

$\Delta U=\frac{1}{2}k(x_m^2-0)=-\Delta K=1$

$x_m=\sqrt{1/100}=0.1m$

Example 16: Two blocks each of mass m, connected by a weightless spring of stiffness k and length L_0 lie on a horizontal surface. A constant force F starts acting on one of the bars (see fig.). Find the maximum distance between the bars during the subsequent motion of the system.



Sol. Consider the system of two blocks and the spring. Force F is external to the system. Therefore neither mechanical energy nor linear momentum will be conserved. Distance between the two blocks will be maximum and minimum when they have the same velocity. Let it be v.

Let d be the distance moved by the right block and be elongation of the spring when the separation between the blocks is maximum. Consequently, distance moved by the center of mass will be $\left(d-\frac{x_o}{2}\right)$.

The acceleration of the center of mass is given by

$a_{cm}=\frac{F}{2m}$

From eqns of constant acceleration, we have

$v^2=2\left(\frac{F}{2m}\right)(d-x_o/2)$ ... (*i*)

and from work-energy theorem, we have

$\Delta KE+\Delta PE$ = work done by F

$\Rightarrow\frac{1}{2}mv^2+\frac{1}{2}mv^2+\frac{1}{2}kx_o^2=Fd$ ... (*ii*)

Solving eqn (*i*) and eqn (*ii*), we find $x_0=F/k$

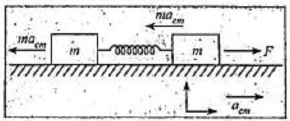
So, the maximum separation is

$\left(L_o+\frac{F}{k}\right)$

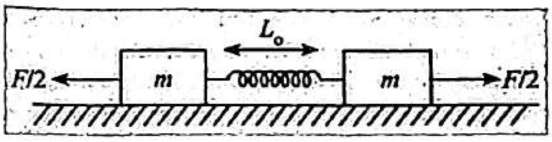
Alternate method(Center of mass frame)

Solving problem in the center of mass frame is always handy as the system is symmetrically placed about the center of mass.

However, a frame fixed to the center of mass in this problem is a non-inertial frame as the acceleration of center of mass is $F/2m$ towards right. Therefore, pseudo forces will act on the blocks towards left.



Setting $a_{cm}=\frac{F}{2m}$, we find that the problem is equivalent to



As the center of mass is at rest in this frame, the blocks move in the opposite direction and come to instantaneous rest at the same time. The extension of the spring is maximum at this instant. Let the two blocks move through distance x_1 and x_2. The work done on the system by the forces is , therefore

$$W = (F/2)x_1 + (F/2)x_2 = (F/2)(x_1 + x_2)$$

This increases the potential energy of the system. Hence

$$(F/2)(x_1 + x_2) = \frac{1}{2}k(x_1 + x_2)^2$$

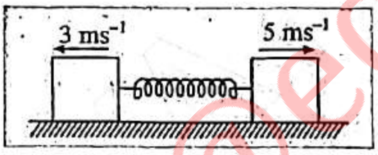
$$(x_1 + x_2) = F/k$$

The maximum distance between the blocks is, therefore

$$L_0 + x_1 + x_2 = L_0 + F/k$$

Concept Application

17. Two identical blocks each of mass 1 kg are joined together with a compressed spring. When the system is released from rest the two blocks appear to be moving with unequal speeds in the opposite direction as shown in figure below. Choose the correct statements (s):



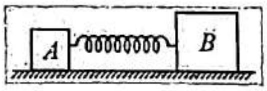
(*a*) Total momentum of the system is conserved.

(*b*) Whatever may be the speed of the blocks the centre of mass will remain stationary

(*c*) The centre of mass of the system is moving with a velocity of 2 ms^{-1} at the given instant

(*d*) The centre of mass of the system is moving with a velocity of 1 ms^{-1} at the given instant

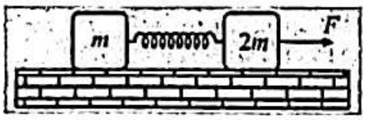
18. Two blocks *A* and *B* of mass *m* and 2m are connected by a massless spring of force constant *k* and placed on a smooth horizontal plane. The spring is stretched by an amount *x* and then released. The relative velocity of the blocks when the spring comes to its natural length is



(*a*) $\left(\sqrt{\frac{3k}{2m}}\right)x$ (*b*) $\left(\sqrt{\frac{2k}{3m}}\right)x$

(*c*) $\sqrt{\frac{2kx}{m}}$ (*d*) $\sqrt{\frac{3km}{2x}}$

19. Two blocks of masses m and $2m$ are kept on a smooth horizontal surface. They are connected by an ideal spring of force constant k. Initially, the spring is unstretched. A constant force is applied to the heavier block in the direction as shown in figure. Suppose at time t, displacement of smaller block is x, then displacement of the heavier block at this moment would be



(a) $\frac{x}{2}$ (b) $\frac{Ft^2}{6m}+\frac{x}{3}$

(c) $\frac{x}{3}$ (d) $\frac{Ft^2}{4m}-\frac{x}{2}$

IMPULSE

In the previous chapter, we have learnt the concept of work which was an integral of force with respect to displacement. Now we are going to learn another concept, called impulse.

Impulse is defined as the integral of force with respect to time.

$$\bar{I}=\int_{t_i}^{t_f}\bar{F}dt$$

Since force is a vector and time is a scalar, the result of the integral in above equation is a vector. If the force is constant (both in magnitude and direction), it may be removed from the integral so that the integral is reduced to

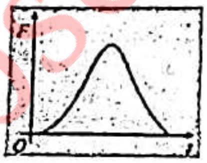
$$\bar{I}=\bar{F}\int_{t_i}^{t_f}dt=\bar{F}(t_f-t_i)=\bar{F}\Delta t$$

Graphically, the impulse is the area between the force curve and the $F=0$ axis, as shown in figure.

The SI unit of impulse is Ns.

If more than one force are acting on a particle, then the net impulse is given by the time integral of the net force.

$$\bar{I}_{net}=\int_{t_i}^{t_f}\bar{F}_{net}dt$$



IMPULSE – MOMENTUM THEOREM

In the previous chapter, we have learnt that work done by a force brings about change in kinetic energy of a particle. Let us see what physical quantity changes due to impulse of a force.

According to Newton's second law, the net force acting on a particle is equal to the product of mass and acceleration.

$$\bar{F}_{net}=m\bar{a}$$

Since $\bar{a}=\frac{d\bar{v}}{dt}$, therefore

$$\bar{F}_{net}=m\frac{d\bar{v}}{dt}$$

Substituting the value of net force, we get

$$\bar{I}_{net}=\int_{t_i}^{t_f}\left(m\frac{d\bar{v}}{dt}\right)dt$$

$$\text{or } \bar{I}_{net}=\int_{v_i}^{v_f}m\,d\bar{v}$$

Notice when we change the variable of integration from t to v, we must also change the time limits of the integral to the corresponding limits of v.

For constant mass,

$$\bar{I}_{net}=m\int_{v_i}^{v_f}d\bar{v}$$

$$\text{or } \bar{I}_{net}=m(\bar{v}_f-\bar{v}_i)=m\bar{v}_f-m\bar{v}_i$$

$$\text{Thus } \bar{I}_{net}=\bar{p}_f-\bar{p}_i$$

$$\text{or } \bar{I}_{net}=\Delta\bar{p}$$

The above equation shows that the net impulse of forces acting on a particle is equal to the change in momentum of the particle. This is called the **Impulse-Momentum Theorem.**

Train Your Brain

Example 17: Find the impulse due to the force $\bar{F}=a\hat{i}+bt\hat{j}$, where $a=2$ N and $b=4$ N/s, if this force acts from $t_i=0$ to $t_f=0.3$ s.

Sol. $\bar{I}=\int_{t_i}^{t_f}\bar{F}dt=\int_0^{0.3}(a\hat{i}+bt\hat{j})dt$

or $\bar{I}=a\hat{i}\int_0^{0.3}dt+b\hat{j}\int_0^{0.3}tdt$

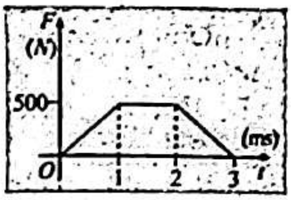
or $\bar{I}=at\hat{i}\Big|_0^{0.3}+\frac{bt^2}{2}\hat{j}\Big|_0^{0.3}$

or $\bar{I}=(2)(0.3)\hat{i}+\frac{(4)(0.3)^2\hat{j}}{2}$

thus $\bar{I}=(0.6\hat{i}+0.18\hat{j})$ Ns

Example 18: Figure shows the variation of force acting on a body with time. Calculate the impulse of this force.

Sol. Impulse of a force is the area under the graph.



Thus,

$$I = \left[\frac{1}{2}(500)(1) + (500)(2-1) + \frac{1}{2}(500)(3-2)\right] \times 10^{-3}$$

or $I = [250 + 500 + 250] \times 10^{-3}$

or $I = 1$ Ns

Example 19: A 2 kg bock is moving at a speed of 6 m/s. How large a force F is needed to stop the block in a time of 0.5 ms?

Sol. Impulse on block = Change in momentum of block

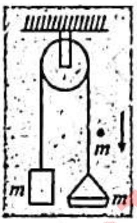
$Ft = mv_f - mv_i$

$F(5 \times 10^{-4}) = 2(0) - (2)(6)$

or $F = -2.4 \times 10^4$ N

The negative sign indicates that the force opposes the motion.

Example 20: A block of mass m and a pan of same mass are connected by a string going over a smooth light pulley as shown in figure. Initially the system is at rest until a particle of mass m falls on the pan and sticks to it. If the particle strikes the pan with a speed v find the speed with which the system moves just after the collision.



Sol. Mass m strikes the pan with speed v and stick to it. Let speed of system just after collision is v_1. For pan and mass m.

Impulse = change in momentum

$-\int T\,dt = 2mv_1 - mv$

For block of mass m

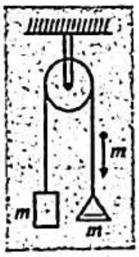
impulse $= mv_1$

$\int T\,dt = mv_1$

$-mv = 2mv_1 - mv$

$mv = 3mv_1$

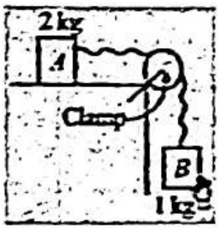
$v_1 = v/3$



Concept Application

20. A ball falling with velocity $\vec{v}_i = (-0.65\hat{i} - 0.35\hat{j})$ m/s is subjected to a net impulse $\vec{I} = (0.6\hat{i} + 0.18\hat{j})$ Ns. If the ball has a mass of 275 g, calculate its velocity immediately following the impulse.
    (a) $\vec{v}_f = (1.53\hat{i} - 0.305\hat{j})$ m/s
    (b) $\vec{v}_f = (0.305\hat{i} - 1.53\hat{j})$ m/s
    (c) $\vec{v}_f = (1.53\hat{i} + 0.305\hat{j})$ m/s
    (d) $\vec{v}_f = (0.305\hat{i} + 1.53\hat{j})$ m/s

21. Two blocks A and B are joined by means of slacked string passing over a massless pulley as shown in diagram. The system is released from rest and it becomes taut when B falls a distance 0.5 m, then
    (a) Find the common velocity (m/s) of two blocks just after string become taut.
    (b) Find the magnitude of impulse (Nm/s) on the pulley by the clamp during the small interval while string becomes taut.



COLLISIONS

Collision is a short event between objects that come in contact with each other. The interaction between two or more objects is called collision if there exists three identifiable stages to this interaction: before, during and after. In the before and after stage the interaction forces are zero or approaches zero asymptotically. Between these two states the interaction forces are large and often the dominating forces governing the objects motion. If we ignore external forces the total momentum of system will be conserved. The magnitude of the interacting force is often unknown. Therefore, the conservation of momentum statement is useful for relating the initial velocities before the interaction to the final velocities after the interaction without requiring a detailed knowledge of the interaction forces.

In a collision the effect of external non impulsive forces such as force of gravity, force due to spring are not taken into account. Due to short duration of time of collision, impulse due to them will be negligible.

Types of Collision

Collisions may be either elastic or inelastic. Linear momentum is conserved in both cases. A perfectly elastic collision is defined as one in which the total kinetic energy of the particles is also conserved.

$$\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$$

In an inelastic collision, the total kinetic energy of the particles changes. Some of the kinetic energy is stored as potential energy associated with a change in internal structure or state, and is not immediately recovered. Some of the energy may be used to raise the system (e.g. an atom) to a state with higher energy. Or, it may be converted into thermal energy of vibrating atoms and molecules or into light, sound or some other form of energy.

In a completely inelastic collision, the two bodies couple or stick together.

Line of Impact

The line passing through the common normal to the surfaces in contact during impact is called line of impact. The impulse during collision acts along this line on both the bodies.

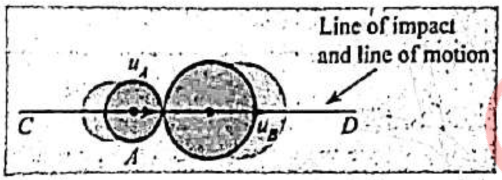
Direction of Line of impact can be determined by:

(*a*) Geometry of colliding objects like spheres, discs, wedge etc.

(*b*) Direction of change of momentum.

If one particle is stationary before the collision then the line of impact will be along its motion after collision.

Head-on collision: In head-on elastic collision the velocities of the particles are along the same line before and after the collision. In the figure shown below two balls *A* and *B* are approaching each other such that their centres are moving along line of impact *CD*.

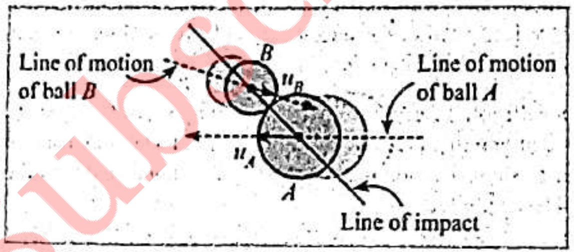

Line of impact
and line of motion
u_A
u_B
C
D
A

Head on Collision

Oblique Collision:

If the velocities of the particles are along different lines before and after the collision, then collision is called oblique collision

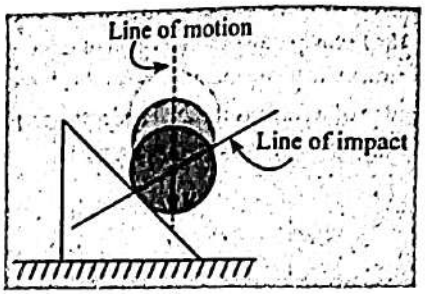
(i) Two balls A and B are approaching each other such that their centre are moving along dotted lines as shown in figure. Since line of impact is not along the initial velocities of the bodies, their direction of motion changes.


Line of motion
of ball B
B
u_B
Line of motion
of ball A
u_A
A
Line of impact

Oblique Collision

(ii) Ball falling on a stationary wedge.

Velocity of the body is not line of impact.


Line of motion
Line of impact

Oblique Collision

Coefficient of Restitution (e)

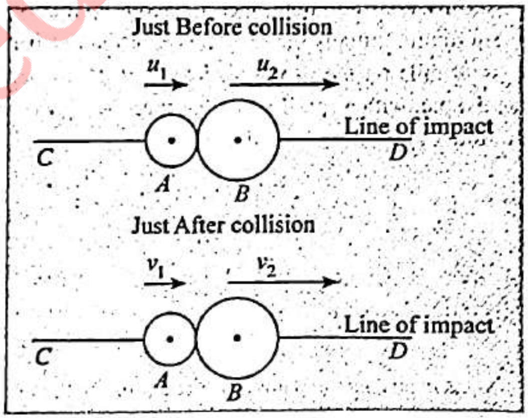
The coefficient of restitution is defined as the ratio of the impulses of recovery and deformation of either body.

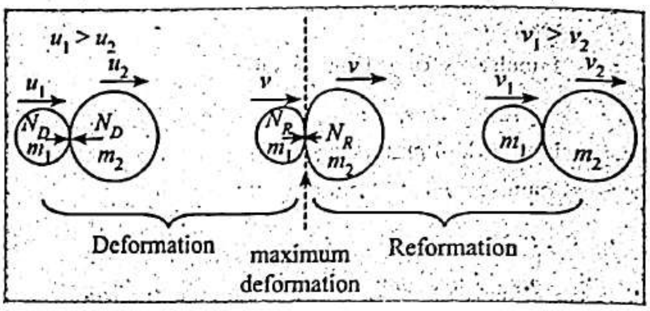
$$e=\frac{\text{Impulse of reformation}}{\text{Impulse of deformation}}=\frac{\int F_r\,dt}{\int F_d\,dt}$$

If the colliding bodies are not in translatory motion then the coefficient of restitution is

$$e=\frac{\text{velocity of separation of points of contact along line of impact}}{\text{velocity of approach of points of contact along line of impact}}$$

Consider two smooth balls *A* and *B* approaching each other such that their centres are moving along line *CD* in absence of external impulsive force. Let the velocities of *A* and *B* just before collision be u_1 and u_2 respectively. and the velocities of A and B just after collision be v_1 and v_2 respectively.


Just Before collision
u_1
u_2
Line of impact
C
D
A
B
Just After collision
v_1
v_2
Line of impact
C
D
A
B


$u_1 > u_2$
u_1
u_2
N_D
m_1
N_D
m_2
v
v
N_R
m_1
N_R
m_2
$v_1 > v_2$
v_1
v_2
m_1
m_2
Deformation
maximum
deformation
Reformation

During deformation v_A is greater than v_B and two bodies deform until $v_A = v_B = v$. During reformation v_A is smaller than v_B and they reform (deformation decreases). Note that during deformation kinetic energy decreases while during reformation kinetic energy increases while total momentum remains constant.

Applying momentum conservation

⇒ $m_1u_1 + m_2u_2 = (m_1 + m_2)v = m_1v_1 + m_2v_2$

The common velocity during maximum deformation is given by

$$\Rightarrow \quad v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \quad \dots (i)$$

Impulse of Deformation:

J_D = change in momentum of any one body during deformation.

$= m_2(v - u_2)$ (for m_2)

$= m_1(-v + u_1)$ (for m_1)

Impulse of Reformation:

J_R = change in momentum of any one body during Reformation.

$= m_2(v_2 - v)$ (for m_2)

$= m_1(v - v_1)$ (for m_1)

Therefore,

$$e = \frac{\text{Impulse of Reformation } (\vec{J}_R)}{\text{Impulse of Deformation } (\vec{J}_D)} = \frac{v_2 - v}{v - u_2}$$

$$v_2 - v = v_2 - \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{m_1}{m_1 + m_2}(v_2 - v_1)$$

$$v - u_2 = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} - u_2 = \frac{m_1}{m_1 + m_2}(u_1 - u_2)$$

Substituting values of $v_2 - v$ and $v - u_2$

we get $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

$$= \frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

If the object under consideration in collision are not in translatory then $e = \dfrac{\text{Velocity of separation in point of contact along line of impact}}{\text{Velocity of approach of point of contact along line of impact}}$

Note: e is independent of shape and mass of object but depends on the material. The coefficient of restitution is constant for two particular objects.

(*a*) For elastic collision

$e = 1$ ⇒ Impulse of Reformation = Impulse of Deformation

⇒ Final kinetic energy = Initial kinetic energy

(*b*) For perfectly inelastic collision

$e = 0$ ⇒ Impulse of Reformation = 0

⇒ Velocity of separation = 0

⇒ Loss in kinetic energy is maximum

(*c*) For Inelastic collision

$0 < e < 1$ ⇒ Impulse of Reformation < Impulse of Deformation

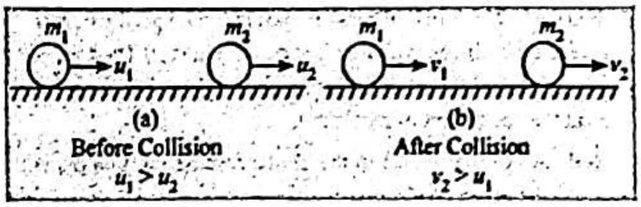
⇒ Velocity of separation < Velocity of approach

Final kinetic energy is loss that initial kinetic energy

Note: In case of collisions between macroscopic bodies e is always less than unity.

∴ $0 \le e \le 1$

COLLISION IN ONE DIMENSION (HEAD ON)



$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow (u_1 - u_2)e = (v_2 - v_1)$$

By momentum conservation,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$v_2 = v_1 + e(u_1 - u_2)$

and $v_1 = \dfrac{m_1u_1 + m_2u_2 - m_2e(u_1 - u_2)}{m_1 + m_2}$

$$v_2 = \frac{m_1u_1 + m_2u_2 + m_1e(u_1 - u_2)}{m_1 + m_2}$$

Loss in K.E.

$$= \frac{1}{2}\mu\left[u_{rel}^2 - v_{rel}^2\right]$$

$$= \frac{1}{2}\mu\left[(1 - e^2)(u_1 - u_2)^2\right]$$

$$= \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}[1 - e^2](u_1 - u_2)^2$$

Special Case:

(1) $e = 0 \Rightarrow v_1 = v_2$

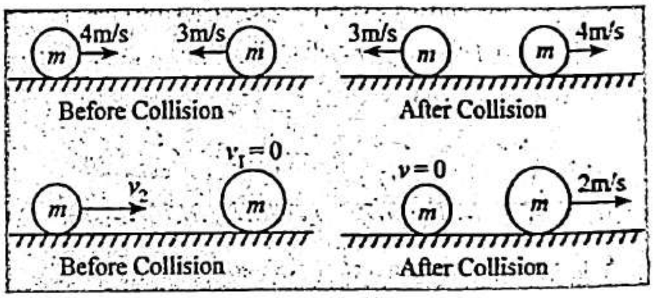
⇒ for perfectly inelastic collision, both the bodies, move with same vel. after collision.

(2) $e = 1$

and $m_1 = m_2 = m$,

we get $v_1 = u_2$ and $v_2 = u_1$

i.e., when two particles of equal mass collide elastically and the collision is head on, they exchange their velocities., e.g.



(3) $m_2 >> m_1, e = 1$

$v_2 = u_2$

$v_1 = u_2 - e(u_1 - u_2) = 2u_2 - u_1$

(4) $m_1 >> m_2, e = 1$

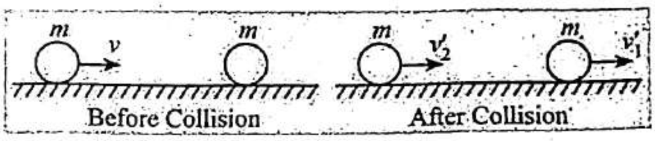
$v_1 = u_1$

$v_2 = u_1 - e(u_1 - u_2) = 2u_1 - u_2$

Train Your Brain

Example 21: A ball of mass m moving at a speed v makes a head on collision with an identical ball at rest. The kinetic energy of the balls after the collision is 3/4th of the original. Find the coefficient of restitution.

Sol. As we have seen in the above discussion, that under the given conditions:



$$v_1' = \left(\frac{1+e}{2}\right)v \text{ and } v_2' = \left(\frac{1-e}{2}\right)v$$

Given that $K_f = \frac{3}{4}K_i$

$$\text{or } \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 = \frac{3}{4}\left(\frac{1}{2}mv^2\right)$$

Substituting the value, we get

$$\left(\frac{1+e}{2}\right)^2 + \left(\frac{1-e}{2}\right)^2 = \frac{3}{4}$$

or $(1+e)^2 + (1-e)^2 = 3$

or $\quad 2 + 2e^2 = 3$

or $e^2 = \frac{1}{2}$ or $e = \frac{1}{\sqrt{2}}$

Example 22: Two balls of masses 2 kg and 4 kg are moving towards each other with velocities 4 m/s and 2 m/s respectively on a frictionless surface. After colliding the 2 kg balls returns back with velocity 2m/s. Then find:

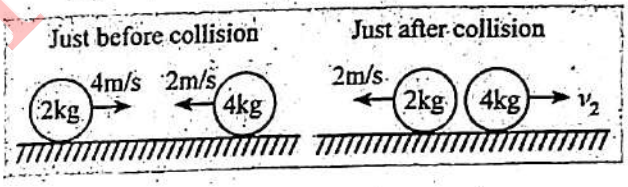
(*a*) velocity of 4 kg ball after collision

(*b*) coefficient of restitution e.

(*c*) Impulse of deformation J_D.

(*d*) Maximum potential energy of deformation.

(*e*) Impulse of reformation J_R.



Sol. (*a*) By momentum conservation,

$$2(4) - 4(2) = 2(-2) + 4(v_2)$$

$$\Rightarrow v_2 = 1 \text{ m/s}$$

(*b*) $$e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = \frac{1-(-2)}{4-(-2)} = \frac{3}{6}$$

$= 0.5$

(c) At maximum deformed state, by conservation of momentum, common velocity is v = 0.

$J_D = m_1(v-u_1) = m_2(v-u_2)$

$= 2(0-4) = -8$ N-s

$= 4(0-2) = -8$ N-s

or $= 4(0-2) = -8$ N-s

(d) Potential energy at maximum deformed state U = loss in kinetic energy during deformation.

or $U = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1+m_2)v^2$

$= \left(\frac{1}{2}2(4)^2 + \frac{1}{2}4(2)^2\right) - \frac{1}{2}(2+4)(0)^2$

or $U = 24$ Joule

(e) $J_R = m_1(v_1 - v) = m_2(v - v_2)$

$= 2(-2-0) = -4$ N-s

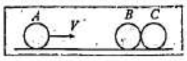
or $= 4(0-1) = -4$ N-s

or $e = \frac{J_R}{J_D}$

$\Rightarrow J_R = eJ_D$

$= (0.5)(-8) = -4$ N-s

Example 23: As shown in figure *A*, *B* and *C* are identical balls *B* and *C* are at rest and, the ball *A* moving with velocity *v* collides elastically with ball B, then after collision-



(a) All the three balls move with velocity v/3

(b) *A* comes to rest and (*B* + *C*) moves with velocity v/2

(c) *A* moves with velocity v/2 and (*B* + *C*) moves with velocity v/2

(d) *A* and *B* come to rest and *C* moves with velocity *v*

Sol. (d) There will be two elastic collisions and in each collision velocities will get exchanged.

Example 24: A sphere of mass *m* moving with velocity *u* hits another stationary sphere of same mass. If *e* is the coefficient of restitution, what is the ratio of velocities of two spheres after the collision?

Sol. By definition of coefficient of restitution

$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{u - 0}$

i.e. $v_2 - v_1 = eu$ ... (i)

And by conservation of momentum

$mu = mv_1 + mv_2$

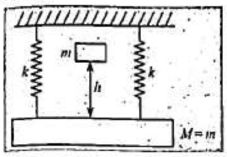
or $v_2 + v_1 = u$ ... (ii)

Solving equations (i) and (ii) for v_1 and v_2, we get

$v_1 = u(1-e)/2$ and $v_2 = u(1+e)/2$

so $\frac{v_1}{v_2} = \left[\frac{1-e}{1+e}\right]$

Example 25: A plate of mass *M* is suspended from a ceiling with the help of two springs. The spring constant for each of the springs is k. A body of mass m fell from a height h and strikes the plate. The collision is completely inelastic. Find the maximum displacement of the plate from its original position if $M = m$?



Sol. The elongation of the spring before the collision is given by

$mg = 2kx_i \Rightarrow x_i = \frac{mg}{2k}$

Note: In problems involving collision, it is possible to make a clear distinction between times that are before, during and after the collision. Accordingly, one must break problems into three parts and analyze each part separately. The following questions should help you solve problems involving collisions.

(a) What happens before the collision?

(b) What happens during collision?

(c) What happens after the collision?

Before collision:

Loss in potential energy = gain in kinetic energy

Therefore, velocity of the body m just before collision is $\sqrt{2gh}$ in the downward direction

During collision: During collision, linear momentum on the system is conserved. Note that there are external forces acting on the system of body m and plate, namely *mg*, *Mg* and *kx*. But their effect is negligible as time of collision is very small.

Hence, $m\sqrt{2gh} = (m+M)u = 2mv$

$\Rightarrow v = \sqrt{gh/2}$

in which *v* is the velocity of the system immediately after the collision.

After collision: After the collision mechanical energy of the system will be conserved since friction is absent. (In the system we have body *m*, plate *M* and two springs). At the point of maximum displacement, the instantaneous velocity of the plate will be zero.

⇒ Let the displacement of the plate be S, then

$x_f = x_i + s = \frac{mg}{2k} + s$

⇒ Conserving mechanical energy, we have

$\Delta KE + \Delta PE = 0$

$$\left[0 - \frac{1}{2}2m\left(\sqrt{gh/2}\right)^2\right] + 2 \times \frac{1}{2}k\left(x_f^2 - x_i^2\right) - 2mgs = 0$$

$$\Rightarrow -mgh/2 + k\left[\left(\frac{mg}{2k} + s\right)^2 - \left(\frac{mg}{2k}\right)^2\right] - 2mgs = 0$$

$$\Rightarrow -mgh/2 + ks^2 + mgs - 2mgs = 0$$

$$\Rightarrow ks^2 - mgs - mgh/2 = 0$$

$$s = \frac{-mg + \sqrt{m^2g^2 + 2mgkh}}{2k}$$

Example 26: A large block of wood of mass *M* is suspended from a ceiling by two cords of length *L*. A bullet of mass *m* is fired into the block, comes quickly to rest in the block. As a result of this, the block swings through an angle θ_0. Find the velocity of the bullet (such a device is called ballistic pendulum).

Sol. During collision, momentum of the system is conserved. Hence

$mv = (M+m)V$ ... (i)

where *V* is the velocity of system immediately after the collision.

After the collision, mechanical energy will be conserved. If the block swings by an angle θ_0, the center of mass of the system rises by

$h = L - L\cos\theta_0 = L(1-\cos\theta_0)$

$\Rightarrow \frac{1}{2}(M+m)V^2 = (M+m)gL(1-\cos\theta_0)$ ... (ii)

$V = \sqrt{2gL(1-\cos\theta_0)}$ ... (iii)

Substituting this value in eqn (i), we get

$v = \frac{M+m}{m}\sqrt{2gL(1-\cos\theta_0)}$

Example 27: A moving particle of mass *m* makes a head on elastic collision with a particle of mass 2*m* which is initially at rest. Find fractional decrease in kinetic energy of the colliding particle

Sol. Let *u* be the initial velocity of body of mass *m* and v_1 and v_2 be the velocities of bodies of mass *m* and 2m respectively after the collision. As in collision momentum is always conserved

$mu = mv_1 + 2mv_2$

i.e. $v_1 + 2v_2 = u$ or $(u - v_1) = 2v_2$ ... (i)

And as the collision in elastic

$\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2$

i.e. $v_1^2 + 2v_2^2 = u^2$ or $\left(u^2 - v_1^2\right) = 2v_2^2$ ... (ii)

Dividing equation (*ii*) by (*i*), we get

$$u + v_1 = v_2$$

Substituting this value of v_2 in equation (*i*),
$u - v_1 = 2(u + v_1)$, i.e., $v_1 = -(u/3)$

So $K_f = \frac{1}{2}mv_1^2 = \frac{1}{2}m\left(-\frac{u}{3}\right)^2 = \frac{1}{2}mu^2 \times \frac{1}{9} = \frac{1}{9}K_i$

$$\left[\text{as } K_i = \frac{1}{2}mu^2\right]$$

$$\therefore \quad \frac{K_i - K_f}{K_i} = \frac{8}{9}$$

Concept Application

22. A smooth small spherical ball of mass m, moving with velocity u collides head on with another small spherical ball of mass $3m$, which was initially at rest. Two-third of the initial kinetic energy of the system is lost. The coefficient of restitution between the spheres is

(*a*) $\frac{1}{3}$ (*b*) $\frac{1}{\sqrt{3}}$ (*c*) $\frac{1}{2}$ (*d*) zero

23. Two elastic bodies P and Q having equal masses are moving along the same line with velocities of 16 m/s and 10m/s respectively. Their velocities after the elastic collision will be in m/s -

(*a*) 0 and 25 (*b*) 5 and 20
(*c*) 10 and 16 (*d*) 20 and 5

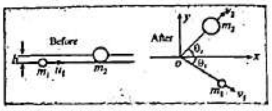
24. Two solid balls of rubber A and B whose masses are 200gm and 400gm respectively, are moving in mutually opposite directions. If the velocity of A is 0.3 m/s and both the balls come to rest after collision, then the velocity of ball B is-

(*a*) 0.15 ms^{-1} (*b*) –0.15 ms^{-1}
(*c*) 1.5 ms^{-1} (*d*) none of these

COLLISION IN TWO DIMENSIONS

In two or three dimensions (except for a completely inelastic collision) the conservation laws alone cannot tell us the motion of particles after a collision, if we know the motion before the collision. For example, for a two-dimensional elastic collision, which is simplest case, we have four unknowns, namely the two components of velocity for each of two particles after collision, but we have only three known relations between them, one for the conservation of kinetic energy and a conservation of momentum relation for each of the two dimensions. Hence we need more information than just the initial conditions. When we do not know the actual forces of interaction, as is often the case, the additional information must be obtained from experiment. It is simplest to specify the angle of recoil of one of the colliding particles.

A typical situation is shown in figure. The distance b between the initial line of motion and a line parallel to it through the center of the target particle is called the impact parameter. This is a measure of the directness of the collision. $b = 0$, corresponding to a head-on collision. The direction of motion of the incident particle m_1 after collision makes an angle θ_1 with the initial direction, and the target projectile m_2, initially at rest, moves in a direction after collision making an angle θ_2 with the initial direction of the incident projectile.



Applying the conservation of momentum, which is a vector relation, we obtain two scalar equations; for the x-component of motion we have

$$m_1u_1 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2$$

and for the y-component

$$0 = m_1v_1\sin\theta_1 - m_2v_2\sin\theta_2$$

If the collision is an elastic one we have,

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

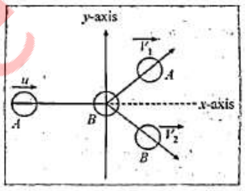
We have four unknowns v_1, v_2, θ_1 and θ_2 and only three equations relating them.

We can determine the motion after collision only if we specify a value for one of these quantities such as θ_1.

Train Your Brain

Example 28: A ball A moving with a speed of 9 m/s strikes identical stationary ball B such that after the collision, the direction of each ball makes an angle of 30° with the original line of motion.

(*a*) Find the speed of the balls after the collision
(*b*) Is kinetic energy conserved in the collision?



Sol. Choose x-axis along the direction of $\vec{u}$ and y-axis perpendicular to its direction.

Applying conservation of linear momentum along X and Y-axis, we have

$$mu = mv_1\cos30° + mv_2\cos30° \quad \text{... (i)}$$

and $$0 = mv_1\sin30° - mv_2\sin30° \quad \text{... (ii)}$$

After solving eqn (*i*) and eqn (*ii*) we find

$$v_1 = v_2 = 3\sqrt{3} \text{ m/s.}$$

(*b*) Let K_1 and K_2 be the kinetic energy before and after the collision. Then

$$K_1 = \frac{1}{2}mu^2 + 0 = \frac{81}{2}m$$

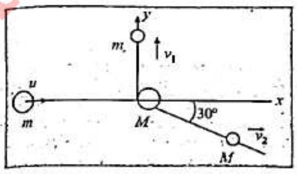
and $$K_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}m\left(3\sqrt{3}\right)^2 \times 2$$

$$= 27m$$

Since $K_2 < K_1$, kinetic energy of the system is not conserved

Example 29: A particle of mass m having collided with a stationary particle of mass M deviated by an angle $\pi/2$ whereas particle M recoiled at an angle $\theta = 30°$ to the direction of initial motion of particle m. How much and in what way has the kinetic energy changed after the collision if $\frac{M}{m} = 5$?

Sol.



From conservation of momentum in X and Y direction, we have

$$mu = Mv_2\cos30° \quad \text{... (i)}$$

$$0 = mv_1 - Mv_2\sin30° \quad \text{... (ii)}$$

Solving eqn (*i*) and eqn (*ii*) for v_1 and v_2, we get

$$v_1 = u/\sqrt{3}, v_2 = \frac{m}{M}\frac{2u}{\sqrt{3}}$$

Let K_1 and K_2 be the initial and final kinetic energy of the particles. Then

$$K_1 = \frac{1}{2}mu^2$$

and $$K_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$$

$$= \frac{1}{2}m\left[\frac{1}{3} + \frac{4m}{3M}\right]u^2$$

The percentage loss in K.E. is

$$\frac{K_1 - K_2}{K_1} \times 100 = \frac{2}{5} \times 100 = 40\%$$

Example 30: A gas molecule having a speed of 300 m/s collides elastically with another molecule of the same mass which is initially at rest. After the collision the first molecule moves at an angle of 30° to its initial direction. Find the speed of each molecule after collision and the angle made with the incident direction by the recoiling target molecule.

Sol. This application corresponds exactly to the situation just discussed, with $m_1 = m_2$, the equations become

$u_1 = v_1\cos\theta_1 + v_2\cos\theta_2$ ...(*i*)

$v_1\sin\theta_1 = v_2\sin\theta_2$ ...(*ii*)

and $u_1^2 = v_1^2 + v_2^2$ ...(*iii*)

Squaring and adding the first two equations, we get

$u_1^2 + v_1^2 - 2u_1v_1\cos\theta_1 = v_2^2$

Combining this with the third equation, we obtain

$2v_1^2 = 2u_1v_1\cos\theta_1$

or (since $u_1 \neq 0$) $v_1 = u_1\cos\theta_1 = 260$ m/s

From the third equation, $v_2 = 150$ m/s

and finally from the second equation,

$\theta_2 = 60°$.

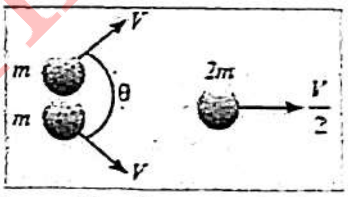
The molecules move apart at right angles

$(\theta_1 + \theta_2 = 90°)$.

Note: The student should be able to show that in an elastic collision between particles of equal mass, one of which is initially at rest, the recoiling particles always move off at right angles to one another.

Concept Application

25. After perfectly inelastic collision between two identical balls moving with same speed in different directions, the speed of the combined mass becomes half the initial speed. Find the angle between the direction of motion of two before collision.



(*a*) π/6 (*b*) π/3 (*c*) 2π/3 (*d*) π/2

26. Three identical particles with velocities $v_0\hat{i}$, $3v_0\hat{j}$ and $5v_0\hat{k}$ collide successively with each other in such a way that they form a single particle. The velocity vector of resultant particle is

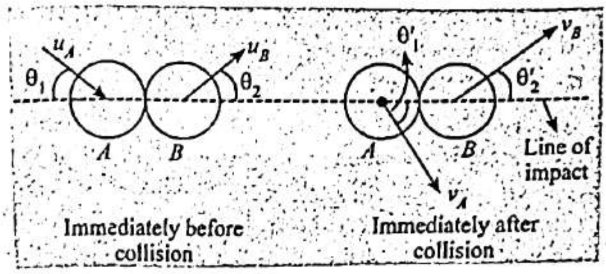
(*a*) $\frac{v_0}{3}(\hat{i} + \hat{j} + \hat{k})$ (*b*) $\frac{v_0}{3}(\hat{i} - \hat{j} + \hat{k})$

(*c*) $\frac{v_0}{3}(\hat{i} - 3\hat{j} + \hat{k})$ (*d*) $\frac{v_0}{3}(\hat{i} + 3\hat{j} + 5\hat{k})$

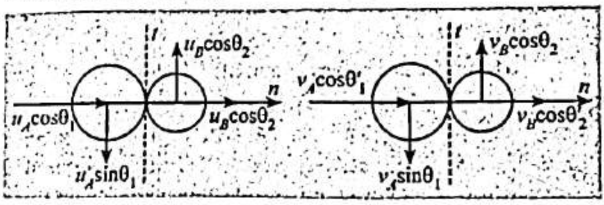
COEFFICIENT OF RESTITUTION AND LINE OF IMPACT IN OBLIQUE COLLISION

In a oblique collision velocities of the two colliding bodies is not along the line of impact. Therefore due to impact, direction of motion of bodies changes.

Consider two balls A and B moving with velocities u_A and u_B collide obliquely. After collision let the velocities of two bodies be v_A and v_B respective as shown in figure below.


u_A
θ_1
u_B
θ_2
A
B
θ'_1
v_B
θ'_2
A
B
Line of impact
v_A
Immediately before collision
Immediately after collision

To analyse the situation, let us consider the components of velocities along the common tangent (tan-axis) and line of impact n-axis (common normal).


t
$u_B\cos\theta_2$
n
$u_A\cos\theta_1$
$u_B\cos\theta_2$
$u_A\sin\theta_1$
$v_A\cos\theta'_1$
t
$v_B\cos\theta_2$
n
$v_B\cos\theta'_2$
$v_A\sin\theta_1$

$u_{At} = u_A \sin\theta_1$ $\quad$ $v_{At} = v_A \cos\theta'_1$

$u_{An} = u_A \cos\theta_1$ $\quad$ $v_{An} = v_A \sin\theta'_1$

$u_{Bt} = u_B \cos\theta_2$ $\quad$ $v_{Bt} = v_B \cos\theta'_2$

$u_{Bn} = u_B \cos\theta_2$ $\quad$ $v_{Bn} = v_B \cos\theta'_2$

Component along t-axis: If the surfaces of two colliding bodies are smooth, there is no impulsive force along the t-axis. Therefore, components of velocities of each body considered separately don't change

$v_{At} = u_{At}$

$v_{Bt} = u_{Bt}$

Components along n-axis: A pair of equal and opposite impulses act along common normal direction. Hence, linear momentum of individual particles do change along common normal direction, but total momentum is conserved. For components along n-axis, the collision can be treated same as head-on collision.

Conserving total momentum along n-axis

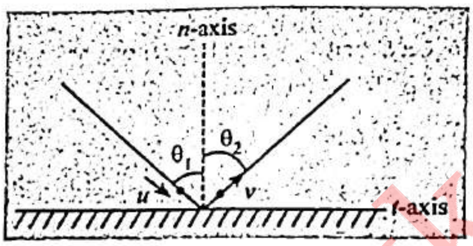
$m_A u_{An} + m_B u_{Bn} = m_A v_{An} + m_B v_{Bn}$

Concept of coefficient of restitution can be applied for n-component of velocities

Relative speed of separation = e (relative speed of approach)

$$e = \frac{v_{Bn} - v_{An}}{u_{An} - u_{Bn}}$$

Oblique Collision of a Particle with a Fixed Smooth Surface: Consider a ball moving with velocity u at an angle θ, with normal to the surface and rebounds with velocity v at angle θ_2 with the normal.


n-axis
θ_1
θ_2
u
v
t-axis

Since the surface is smooth, there is no impulse along the surface. Therefore,

$u_t = v_t \Rightarrow u\sin\theta_1 = v\sin\theta_2$

$v_n = eu_n \Rightarrow eu\cos\theta_2 = v\cos\theta_2$

$\Rightarrow \tan\theta_2 = \dfrac{\tan\theta_1}{e}$

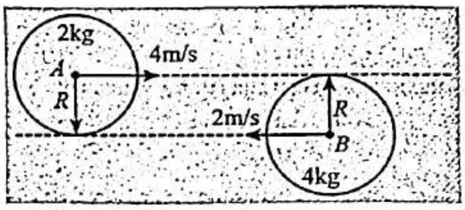
$v = u\sqrt{\sin^2\theta_1 + e^2\cos^2\theta_1}$

For elastic collision $e = 1$

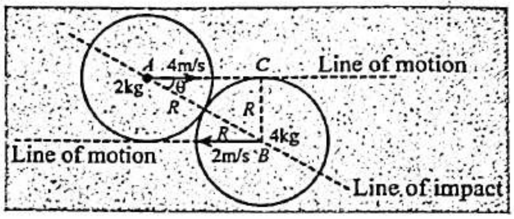
$v = u$ and $\theta_2 = \theta_1$

Train Your Brain

Example 31: Two spheres are moving towards each other. Both have same radius but their masses are 2 kg and 4 kg. If the velocities are 4 m/s and 2 m/s respectively and coefficient of restitution is $e = 1/3$, find the final velocities anlong line impact.


2kg
A
R
4m/s
2m/s
R
B
4kg

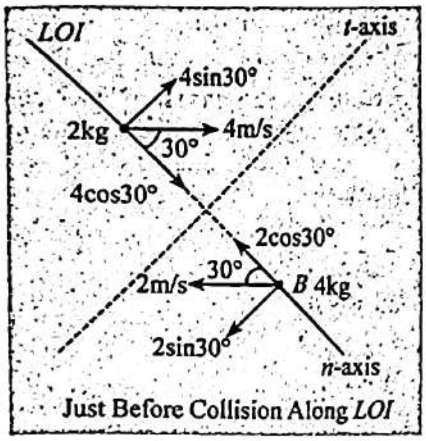
Sol.


A
4m/s
2kg
θ
R
C
Line of motion
R
R
4kg
Line of motion
2m/s
B
Line of impact

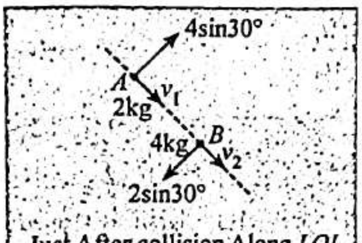
In ΔABC , $\sin\theta = \dfrac{BC}{AB} = \dfrac{R}{2R} = \dfrac{1}{2}$

or $\theta = 30°$

(*a*) By conservation of momentum along line of impact.



Just Before Collision Along *LOI*



Just After collision Along *LOI*

Let v_1 and v_2 be the final velocity of *A* and *B* respectively then, by conservation of momentum along line of impact,

$2(4\cos 30°) - 4(2\cos 30°) = 2(v_1) + 4(v_2)$

or $0 = v_1 + 2v_2$ ... (*i*)

By coefficient of restitution,

$$e = \frac{\text{velocity of separation along LOI}}{\text{veloctiy of approach along LOI}}$$

or $\frac{1}{3} = \frac{v_2 - v_1}{4\cos 30° + 2\cos 30°}$

or $v_2 - v_1 = \sqrt{3}$ ... (*ii*)

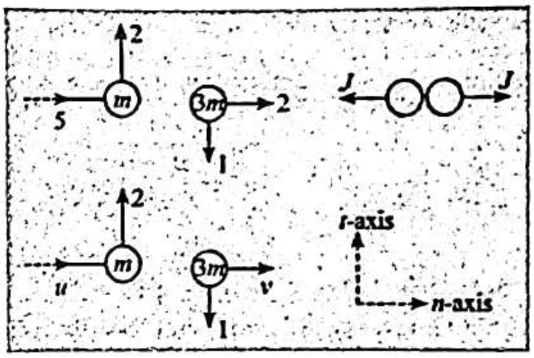
from the above two equations,

$v_1 = \frac{2}{\sqrt{3}}$ m/s and $v_2 = \frac{1}{\sqrt{3}}$ m/s

Example 32: Two smooth spheres, *A* and *B*, having equal radii, lie on a horizontal table. *A* is of mass *m* and *B* is of mass 3m. The spheres are projected towards each other with velocity vector $5\hat{i} + 2\hat{j}$ and $2\hat{i} - \hat{j}$ respectively and when they collide the line joining their centres is parallel to the vector $\hat{i}$.

If the coefficient of restitution between A and B is $\frac{1}{3}$, find the velocities after impact.

Sol. The line of impact, is parallel to the vector , the velocity components of *A* and *B* perpendicular to are unchanged by the impact.



Applying conservation of linear momentum and the law of restitution, we have in *x* direction

$5m + (3m)(2) = mu + 3mv$ ... (*i*)

and $\frac{1}{3}(5 - 2) = v - u$ ... (*ii*)

Solving these equations, we have $u = 2$ and $v = 3$

The velocities of *A* and *B* after impact are therefore,

$2\hat{i} + 2\hat{j}$ and $3\hat{i} - \hat{j}$ respectively

Concept Application

27. A ball after falling vertically through a distance h collides with a fixed inclined plane of inclination θ. If it moves horizontally just after the impact, find the coefficient of restitution?

28. A small sphere of mass 1kg is moving with a velocity $(6\hat{i} + \hat{j})\,\text{ms}^{-1}$. It hits a fixed smooth wall and rebound with velocity $(-4\hat{i} + \hat{j})\,\text{ms}^{-1}$. the coefficient of restitution between the sphere and the wall is

29. A ball moving translationally collides elastically with another stationary ball of same mass. At the moment of impact, the angle between the straight line passing through the centres of balls and direction of initial velocity of the striking ball is equal to $\alpha = 45°$. Assuming the balls to be smooth, find the fraction η of the kinetic energy of the striking ball that turned into potential energy at the moment of maximum deformation.

VARIABLE MASS SYSTEM

So far we have dealt with the dynamics of a system whose mass is constant. We now discuss the dynamics of a system, such as a rocket, whose mass varies. One may try to apply the following equation in such a case.

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt} = M\frac{d\vec{v}}{dt} + \vec{v}\frac{dM}{dt}$$

However, this equation is correct in only a few very special cases. This approach is not correct when mass actually enters or leaves the system.

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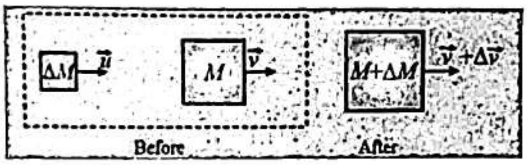
Let us examine the motion of a body of mass M, moving with velocity v. Another small body of mass ΔM approaches with velocity $\bar{u}$ along the same line. We assume $\bar{u} > \bar{v}$ and that after the collision the two bodies stick together and move at velocity $(\bar{v}+\Delta\bar{v})$, as shown in the figure.



By defining the system comprising of both the bodies we can use $\bar{F}_{\text{ext}} = \frac{d\bar{P}}{dt}$, where $\bar{P}$ is the total momentum of the system of constant mass, $M+\Delta M$. The change in momentum of the system in time Δt is

$$\Delta\bar{P} = (M+\Delta M)(\bar{v}+\Delta\bar{v}) - M\bar{v} - \Delta M\bar{u}$$
$$= M\Delta\bar{v} - (\bar{u}-\bar{v})\Delta M$$

Now $(\bar{u}-\bar{v}) = \bar{v}_{\text{rel}}$ is the velocity of ΔM relative to M before the collision.

We divide both sides of this equation by Δt and take the limit as $\Delta t \to 0$

$$\bar{F}_{\text{ext}} = \frac{d\bar{P}}{dt} = M\frac{d\bar{v}}{dt} - \bar{v}_{\text{rel}}\frac{dM}{dt}$$

It is convenient to rewrite it in the form

$$M\frac{d\bar{v}}{dt} = \bar{F}_{\text{ext}} + \bar{v}_{\text{rel}}\frac{dM}{dt}$$

The last term in the above equation $\left(\bar{v}_{rel}\frac{dM}{dt}\right)$, is the rate at which momentum is being transferred into (or out of) the system by the mass that system has collected (or ejected). It can be interpreted as the force exerted on the system by the mass that leaves or joins it. This force is referred to as the reaction force.

ROCKET PROPULSION

Consider a rocket of mass M with fuel of mass Δm. Their common velocity is v relative to some inertial frame. When the rocket engines are fired, the gases are expelled backward with an exhaust velocity $\bar{v}_{ex} = -v_{ex}\hat{i}$ relative to the rocket. This is a fixed quantity determined by the design of the engine and the type of fuel. If the rocket's velocity changes to $\bar{v}+\Delta\bar{v}$ relative to the inertial frame, then the velocity of gas with respect to the frame will be

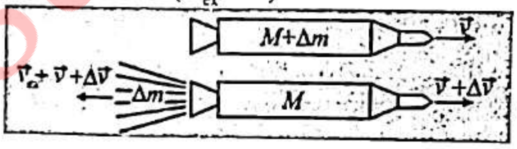
$$\bar{v}_{\text{gas}} = \bar{v}_{\text{ex}} + \bar{v} + \Delta\bar{v} = (-v_{\text{ex}} + v + \Delta v)\ \hat{i}$$

Applying the law of conservation of momentum, we get

$$(M+\Delta m)\, v = M(v+\Delta v) + \Delta m(-v_{\text{ex}} + v + \Delta v)$$

After some cancellation we find

$$0 = M\,\Delta v + \Delta m(-v_{\text{ex}} + \Delta v)$$



If both Δv and Δm are small quantities relative to v and M, respectively, their product, $\Delta v\Delta m$, is negligible in comparison with the other terms, and we are left with

$$\Delta v = v_{ex}\frac{\Delta m}{M}$$

Since an increase in the mass of the expelled gases corresponds exactly to the loss in mass of the rocket system, we have $\Delta m = -\Delta M$. In the limit as $\Delta M \to 0$ the above equation becomes

$$dv = -v_{ex}\frac{dM}{M}$$

On integrating both sides,

$$\int_{v_i}^{v_f} dv = -\int_{M_i}^{M_f} v_{ex}\frac{dM}{M}$$

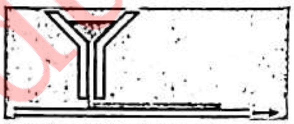
We find

$$v_f - v_i = v_{\text{ex}}\ln\frac{M_i}{M_f}$$

Train Your Brain

Example 33: A hopper releases grain at a rate dm/dt onto a conveyor belt that moves at a constant speed v. What is the power of the motor driving the belt?

Sol.



Let the system has some arbitrary length of belt whose mass we can call M. This mass of the system increases at the same rate as that the grain falls,

so $dM/dt = dm/dt$.

Since the grain falls vertically, $\bar{u} = 0$ and $\bar{v}_{rel} = \bar{u} - \bar{v} = -\bar{v}$, since the speed is constant,

$dv/dt = 0$. Thus from equation for the system of variable mass

$$\bar{F}_{\text{ext}} - \bar{v}\frac{dm}{dt} = 0 \Rightarrow \bar{F}_{\text{ext}} = \bar{v}\frac{dm}{dt}$$

where $\bar{F}_{\text{ext}}$ is the force needed to maintain constant speed because the mass is increasing. The power required $(P = \bar{F}\cdot\bar{v})$ is

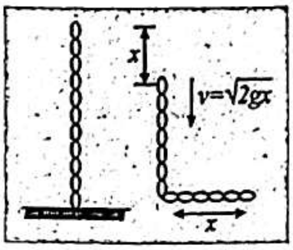
$$P = v^2\frac{dm}{dt}$$

It is interesting to compare this with the rate at which the kinetic energy of the grain increases.

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}v^2\frac{dm}{dt}$$

This is only half the power input. The other half is dissipated as heat when the grain lands on the belt and slips relative to it.

Example 34: A uniform chain of mass m and length l hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the table on the chain, when half of its length has fallen on the table. The fallen part does not form heap.



Sol. Let N be the normal reaction acting on the chain.

It consists of two parts

$$N = N_{wt} + N_{thrust}$$

$$N_{wt} = \frac{m}{l}xg$$

$$N_{thrust} = v_{rel}\frac{dm}{dt}$$

If v be the instantaneous velocity of the particle falling on the floor, then

$$v_{rel} = v - 0 = v$$

$$\frac{dm}{dt} = \frac{dm}{dx}\frac{dx}{dt} \text{ or } \frac{dm}{dt} = v\left(\frac{m}{l}\right)$$

Thus $N_{th} = v^2\left(\frac{m}{l}\right)$, Since $v = \sqrt{2gx}$ $\therefore$ $N_{th} = \frac{2mg}{l}x$

Total reaction is

$$N = \frac{mgx}{l} + \frac{2mgx}{l},\ N = 3mg\left(\frac{x}{l}\right)$$

$\{\because$ for $x = l/2\}$

$$N = \frac{3}{2}mg$$

Concept Application

30. A wagon filled with sand has a hole so that sand leaks through the bottom at a constant rate λ. An external force $\overline{F}$ acts on the wagon in the direction of motion. Assuming instantaneous velocity of the wagon to be $\bar{v}$ and initial mass of system to be m_0, the force equation governing the motion of the wagon is:

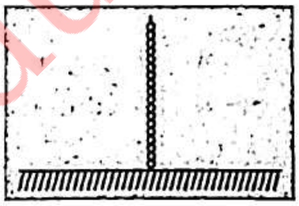
    (a) $\overline{F} = m_0\frac{d\bar{v}}{dt} + \lambda\bar{v}$

    (b) $\overline{F} = m_0\frac{d\bar{v}}{dt} - \lambda\bar{v}$

    (c) $\overline{F} = (m_0 - \lambda t)\frac{d\bar{v}}{dt}$

    (d) $\overline{F} = (m_0 - \lambda t)\frac{d\bar{v}}{dt} + \lambda\bar{v}$

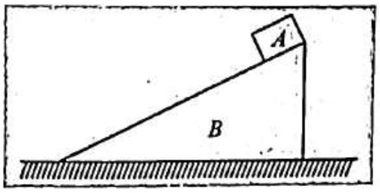
31. A uniform rope of mass m per unit length, hangs vertically from a support so that the lower end just touches the table top shown in figure. If it is released, at the time a length y of the rope has fallen, the force on the table is equivalent to the weight of a length ny of the rope. Find the value of n.



> *"The secret of success is constancy to purpose."*
>
> **– Benjamin Disraeli**

Aarambh (Solved Examples)

1. A block A (mass = $4M$) is placed on the top of a wedge B of base length l (mass = $20M$) as shown in figure. When the system is released from rest. Find the distance moved by the wedge B till the block A reaches at lowest end of wedge. Assume all surfaces are frictionless.



Sol. Initial position of center of mass

$$= \frac{X_B M_B + X_A M_A}{M_B + M_B} = \frac{X_B \cdot (20M) + \ell.(4M)}{24M} = \frac{5X_B + \ell}{6}$$

Final position of center of mass

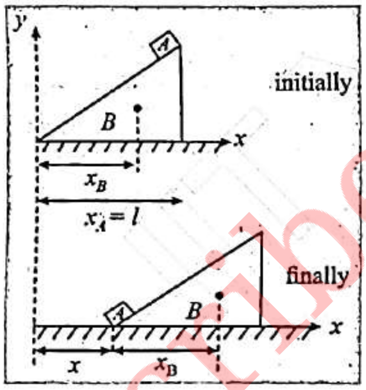
$$= \frac{(X_B + x)20M + 4Mx}{24M} = \frac{5(X_B + x) + x}{6}$$

since there is no horizontal force on system center of mass initially = center of mass finally.

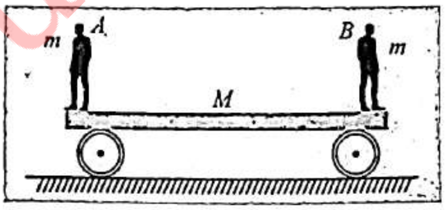
$$5X_B + l = 5X_B + 5x + x$$

$$\ell = 6x$$

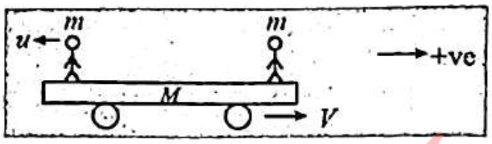
$$x = \frac{\ell}{6}$$



2. Two persons A and B, each of mass m are standing at the two ends of rail-road car of mass M. The person A jumps to the left with a horizontal speed u with respect to the car. Thereafter, the person B jumps to the right, again with the same horizontal speed u with respect to the car. Find the velocity of the car after both the persons have jumped off.



Sol. When person A jumps, let car achieve velocity V in forward direction. So, velocity of 'A' w.r.t. ground = $(u - V)$ in backward direction.

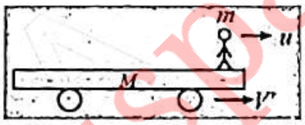


Applying momentum conservation.

$$(M+m)V - m(u-V) = 0, (M+m)V + mV = mu$$

$$V = \frac{m}{M+2m}u$$

When person B jumps, let velocity of car becomes V' in forward direction.



So velocity of 'B' w.r.t ground = $u + V'$

Applying momentum conservation:

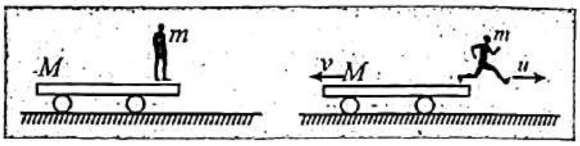
$$(m+M)V = MV' + m(u+V')$$

$$\frac{(m+M)mu}{M+2m} = (m+M)V' + mu$$

$$V' = \frac{-m^2u}{(M+m)(M+2m)}$$

3. A flat car of mass M is at rest on a frictionless floor with a child of mass m standing at its edge. If child jumps off from the car towards right with an initial velocity u, with respect to the car, find the velocity of the car after its jump.

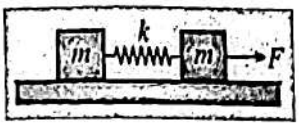
Sol. Let car attains a velocity v, and the net velocity of the child with respect to earth will be $u - v$, as u is its velocity with respect to car.



Initially, the system was at rest, thus according to momentum conservation, momentum after jump must be zero, as $m(u - v) = Mv$

$$v = \frac{mu}{m+M}$$

Paragraph for Question 4 to 5: Two blocks of equal mass m are connected by an unstretched spring and the system is kept at rest on a frictionless horizontal surface. A constant force F is applied on the first block pulling it away from the other as shown in figure.



4. The displacement of the centre of mass at time t is

(a) $\frac{Ft^2}{2m}$ (b) $\frac{Ft^2}{3m}$ (c) $\frac{Ft^2}{4m}$ (d) $\frac{Ft^2}{m}$

Sol. The acceleration of the centre of mass is

$$a_{COM} = \frac{F}{2m}$$

The displacement of the centre of mass at time t will be

$$x = \frac{1}{2}a_{com}t^2 = \frac{Ft^2}{4m}$$

Therefore, option (c) is the correct answer.

5. If the extension of the spring is x_0 at time t, then the displacement of the right block at this instant is:

(a) $\frac{1}{2}\left(\frac{Ft^2}{2m}+x_0\right)$ (b) $-\frac{1}{2}\left(\frac{Ft^2}{2m}+x_0\right)$

(c) $\frac{1}{2}\left(\frac{Ft^2}{2m}-x_0\right)$ (d) $\left(\frac{Ft^2}{2m}+x_0\right)$

Sol. Suppose the displacement of the first block is x_1 and that of the second is x_2. Then,

$$x = \frac{mx_1+mx_2}{2m} \quad \text{or,} \quad \frac{Ft^2}{4m} = \frac{x_1+x_2}{2}$$

$$\text{or} \quad x_1 + x_2 = \frac{Ft^2}{2m} \qquad \dots (i)$$

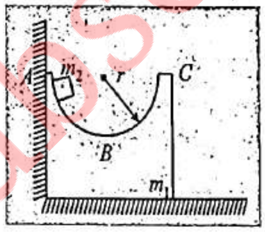
Further, the extension of the spring is $x_1 - x_2$. Therefore,

$$x_1 - x_2 = x_0 \qquad \dots (ii)$$

From Eqs. (i) and (ii), $x_1 = \frac{1}{2}\left(\frac{Ft^2}{2m}+x_0\right)$

Therefore, option (a) is the correct answer.

6. A wedge of mass m_1 with its upper surface hemispherical in shape, as shown in Figure, rests on a smooth horizontal, surface near the wall. A small block of mass m_2 slides without friction on the hemispherical surface of the wedge. What is the maximum velocity attained by the wedge?



Sol. As long as the block moves from A to B, the reaction on the wedge presses it to the wall. When the block reaches the lowermost position, its velocity from energy conservation is

$$v = \sqrt{2gr}$$

When the block moves along the right half of the wedge, during its upward journey as well as downward journey the reaction of the block on the wedge is towards right as shown in figure. Therefore during the entire motion of the block form B to C and C to B, the wedge is accelerated towards right. Thus to find the maximum velocity attained by the wedge at the instant when the block passes separated from the wall.

$$P_i = P_f$$

$$m_2\sqrt{2gr} = m_1v_1 + m_2v_2 \qquad \dots (i)$$

From energy conservation, $E_i = E_f$

$$m_2gr = \frac{m_1v_1^2}{2} + \frac{m_2v_2^2}{2} \qquad \dots (ii)$$

On solving eqns. (i) and (ii) simultaneously, we obtain two solutions

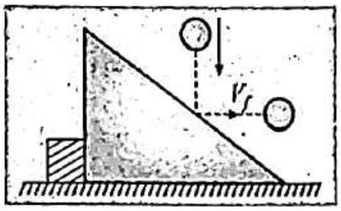
$$v_1 = 0, v_2 = \sqrt{2gr}$$

$$\text{and } v_1 = \frac{2m_2}{m_1+m_2}\left(\sqrt{2gr}\right), \; v_2 = \frac{m_2-m_1}{m_1+m_2}\left(\sqrt{2gr}\right)$$

The first solution corresponds to the instant when the block reaches for the first time at point B. At this instant the block moves with velocity v_2 and the wedge is at rest. The second solution corresponds to the instant when the block has the maximum velocity

$$(v_1)_{max} = \frac{2m_2\sqrt{2gr}}{m_1+m_2}$$

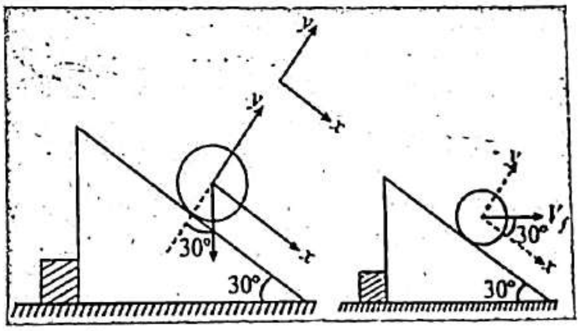
7. As shown in the figure a body of mass m moving vertically with speed 3 m/s hits a smooth fixed inclined plane and rebounds with a velocity v_f in the horizontal direction. If angle of inclination is 30°, the velocity v_f will be?



(a) 3 m/s (b) $\sqrt{3}$ m/s

(c) $1/\sqrt{3}$ m/s (d) This is not possible

Sol. The momentum along the inclined plane (x-axis) remains conserved. i.e. $\sum p_x = 0$



$$\Rightarrow \ MV_i \sin 30° = M\,V_f \cos 30°$$

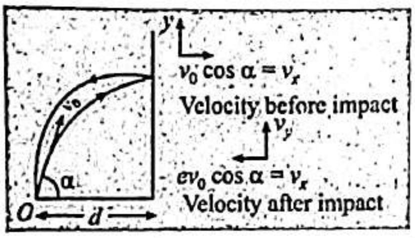
$$\Rightarrow V_f = V_i \frac{\sin 30°}{\cos 30°} = V_i \tan 30°$$

$$\Rightarrow V_f = V_i \frac{1}{(\sqrt{3})} = 3 \cdot \frac{1}{\sqrt{3}} = \sqrt{3}\ \text{m/s}$$

8. An inelastic ball is projected with velocity $v_0 = \sqrt{gh}$, at an angle α to the horizontal, towards a wall distant d from the point of projection. After collision the ball returns to the point of projection. What is the coefficient of restitution?

Sol. Time taken to reach the wall

$$t_1 = \frac{d}{v_0 \cos\alpha}$$



Time taken to return to the point of projection after impact

$$t_2 = \frac{d}{ev_0 \cos\alpha}$$

Note that x-component of velocity after impact is $ev_0 \cos\alpha$.

$$\text{Total time of flight } (T) = t_1 + t_2 = \frac{d}{v_0\cos\alpha} + \frac{d}{ev_0\cos\alpha}$$

$$T = \frac{d}{\sqrt{gh}\cos\alpha}\left[\frac{1+e}{e}\right].$$

There is no change in the vertical component of the velocity after impact, therefore total time of flight remains unchanged.

$$T = \frac{2v_0\sin\alpha}{g} = \frac{2\sqrt{gh}\sin\alpha}{g} \qquad \because v_0 = \sqrt{gh}$$

$$\Rightarrow \frac{d}{\sqrt{gh}\cos\alpha}\left(\frac{1+e}{e}\right) = \frac{2\sqrt{gh}\sin\alpha}{g} \text{ or } \left(\frac{1+e}{e}\right) = \frac{h\sin 2\alpha}{d}$$

$$\text{Or } \ e = \frac{d}{(h\sin 2\alpha - d)}$$

9. A smooth ring is kept on a smooth horizontal surface. From a point P of the ring a particle is projected at an angle α to the radius vector at P. If e is the coefficient of restitution between 'the ring and the particle, show that the particle will return to the point of projection after two reflections if

$$\cot^2\alpha = \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3}$$

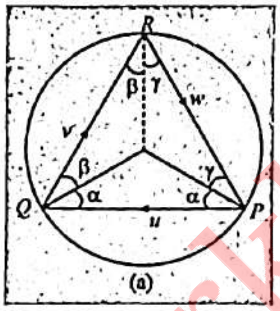
Sol. Let u be the velocity of projection at P. We can find the velocity of rebound at point Q from Fig. (b).

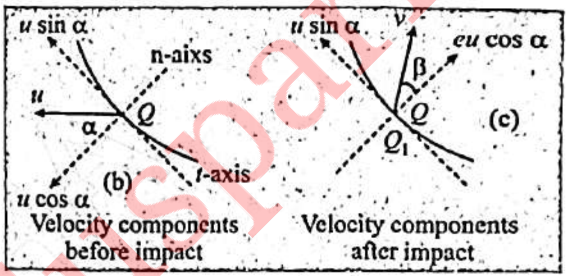
$$\tan\beta = \frac{u\sin\alpha}{eu\cos\alpha} = \frac{\tan\alpha}{e}$$

$$\text{and } v = \sqrt{u^2\sin^2\alpha + eu^2\cos^2\alpha}$$

$$\text{Similarly at } R \ \tan\gamma = \frac{v\sin\beta}{ev\cos\beta} = \frac{\tan\beta}{e} = \frac{\tan\alpha}{e^2}$$

$$\text{and } w = \sqrt{v^2\sin^2\alpha + ev^2\sin^2\alpha}$$





Velocity components before impact

Velocity components after impact

Since the particle returns to the point of projection,

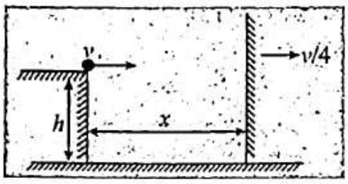
$$\alpha + \beta + \gamma = \pi/2 \qquad \text{or } \tan(\alpha+\beta+\gamma) = \infty$$

$$\text{or } \ 1 - [\tan\alpha\tan\beta + \tan\beta\tan\gamma + \tan\gamma\tan\alpha] = 0$$

$$\text{or } \ 1 = \tan^2\alpha\left[\frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3}\right], \cot^2\alpha = \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3}$$

where '–ve' sign signifies Backward direction

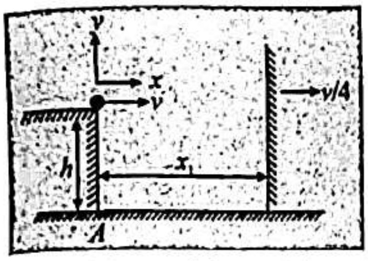
10. A particle is thrown from a height h horizontally with a speed v towards a vertical wall moving away with a speed $v/4$. The particle returns to the point of projection after suffering two elastic collisions, one with the wall and another with the ground. Find the total time of flight and initial separation x (as shown in diagram) between the particle and the wall.



Sol. While colliding with the wall, x component of particles velocity gets changed while y-component remains same. We have

$$\because e = \frac{v_{sep.}}{v_{app.}} = 1 \quad \text{(elastic collision)}$$

$$\Rightarrow \frac{v_1 + \frac{v}{4}}{v - \frac{v}{4}} = 1 \Rightarrow v_1 = \frac{v}{2}$$



It should be noted that total time of flight will be 'T', and t_1 is time of ball to the ground and first collision must be occurred with wall and second with ground. Whereas if wall was moving towards the ball then, first collision must occur at ground and second with wall.

$\therefore$ Total time of flight $(T) = 2\times\sqrt{\dfrac{2h}{g}}$

Let the separation between point A and wall is x_1 when ball hits the wall

$$\frac{x_1}{v}+\frac{x_1}{v/2}=T=2\sqrt{\frac{2h}{g}}$$

$\therefore$ time taken by ball to cover this distance

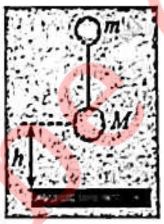
$$t_1=\frac{x_1}{v}=\frac{2}{3}\sqrt{\frac{2h}{g}} \Rightarrow x_1=\frac{2}{3}v\sqrt{\frac{2h}{g}}$$

$\therefore$ Initial separation $x=x_1-\dfrac{v}{4}(t_1)=x_1-\dfrac{x_1}{4}=\dfrac{3x_1}{4}$

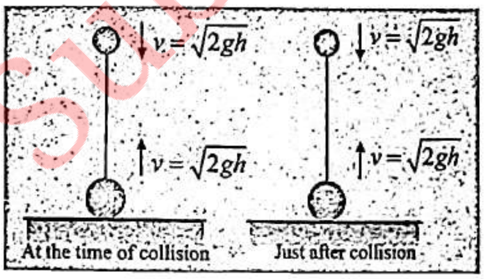
$$x=\frac{3}{4}\times\left(\frac{2}{3}v\sqrt{\frac{2h}{g}}\right)\Rightarrow\frac{v}{2}\sqrt{\frac{2h}{g}}$$

i.e., $x=\dfrac{v}{2}\sqrt{\dfrac{2h}{g}}$

11. A small ball of mass m is connected by an inextensible massless string of length l with an another ball of mass $M = 4\ m$. They are released with zero tension in the string from a height h as shown in figure. Find the time when the string becomes taut for the first time after the mass M collides with the ground. Take all collisions to be elastic.



Sol. Just after collision velocities of both the balls will be $\sqrt{2gh}$ in opposite directions. Relative acceleration between the two balls is zero and relative velocity of approach is $2\sqrt{2gh}$. Hence they will collide after a time



$$t=\frac{l}{2\sqrt{2gh}} \qquad \dots(i)$$

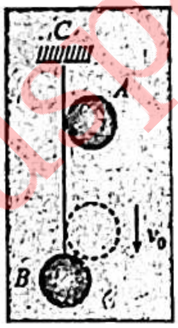
At the time of collision of two balls relative velocity of approach = $2\sqrt{2gh}$

Hence relative velocity of separation will also be $2\sqrt{2gh}$ (collision is elastic). Hence the string becomes tight after the same time

$$t=\frac{l}{2\sqrt{2gh}}$$

Hence the total time will be $2t$ or $\dfrac{l}{\sqrt{2gh}}$

12. Ball B is hanging from an inextensible cord BC. An identical ball A is released from rest when it is just touching the cord and acquires a velocity v_0 before striking ball B. Assuming perfectly elastic impact ($e = 1$) and no friction, determine the velocity of each ball immediately after impact.

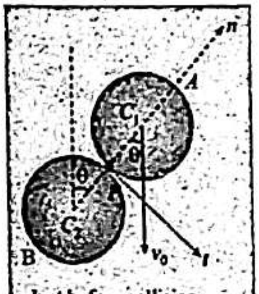


Sol. Ball A is free to move in a plane (after collision). So its velocity can be resolved in two mutually perpendicular directions. Let us resolve it along common tangent and common normal directions. Let v_t and v_n be the corresponding components of ball 'A' just after collision in these two directions. Ball B is attached to a vertical string. So, just after collision its velocity will be horizontal. Let it be v.

We have $\theta = 30°$

(i) Velocity component along common tangent direction remains unchanged. Hence,

$$u_t=v_t=v_0\sin\theta=\frac{v_0}{2} \qquad \dots(i)$$



Just before collision

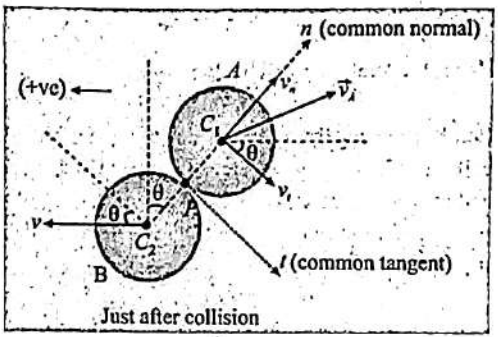
(ii) Linear momentum along horizontal will remain unchanged. Hence,

$$mv - mv_t\cos 30^\circ - mv_n \sin 30^\circ = 0$$

or $v - \frac{\sqrt{3}v_t}{2} - \frac{v_n}{2} = 0$ ... (*ii*)

(iii) $e = \frac{v_{sep.}}{v_{app.}} = 1$ ($\because$ elastic collision)

i.e $\frac{v_n + v\sin\theta}{v_0\cos\theta} = 1$



Just after collision

$\therefore\ v_n + V\sin\theta = v_0\cos\theta$

or $v_n + \frac{v}{2} = \frac{\sqrt{3}v_0}{2}\left(\text{as } \theta = 30^\circ\right)$. ... (*iii*)

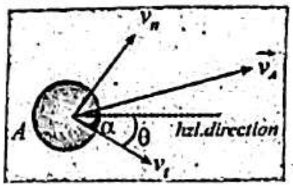
Solving Eqs. (*i*), (*ii*) and (*iii*), we get

$v = 0.693v_0,\ v_n = 0.52v_0$ and $v_t = 0.5\ v_0$

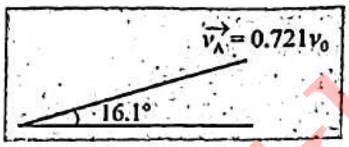
$\therefore$ Velocity of 'B' after collision $v = 0.693v_0$ (horizontally)

and Velocity of 'A' after collision $v_A = \sqrt{v_n^2 + v_t^2} = 0.721v_0$

$$\alpha = \tan^{-1}\left(\frac{v_n}{v_t}\right) = 46.1^\circ$$



Hence, v_A makes an angle of (46.1° – 30°) or 16.1° with horizontal as shown



13. Find the mass of the rocket as a function of time, if it moves with a constant acceleration a, in absence of external forces. The gas escapes with a constant velocity u relative to the rocket and its initial mass was m_0.

Sol. Using, $\vec{F}_{net} = \vec{V}_{rel}\left(\frac{-dm}{dt}\right)$

or $F_{net} = -u\frac{dm}{dt}$ ... (*i*)

and $F_{net} = ma$ ... (*ii*)

Solving equation (*i*) and (*ii*)

$$ma = -u\frac{dm}{dt}$$

$$\int_{m_0}^{m}\frac{dm}{m} = \int_0^t \frac{-adt}{u} = \frac{-at}{u}$$

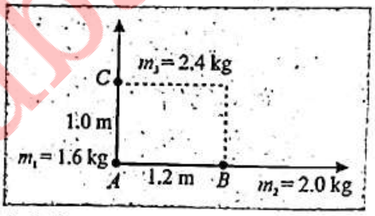
$$\Rightarrow \{\ln(m)\}_{m_0}^{m} = -\frac{a}{u}(t)$$

or $\ln\left(\frac{m}{m_0}\right) = \frac{at}{u}$ or $\frac{m}{m_0} = e^{-at/u} \Rightarrow m = m_0e^{-at/u}$

School Level Problems

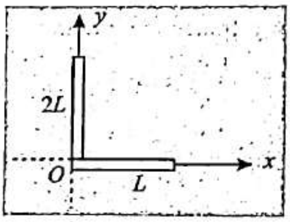
SINGLE CORRECT TYPE QUESTIONS

1. Three-point masses m_1, m_2 and m_3 are placed at the corners of a thin massless rectangular sheet (1.2 m × 1.0 m) as shown. Centre of mass will be located at the point



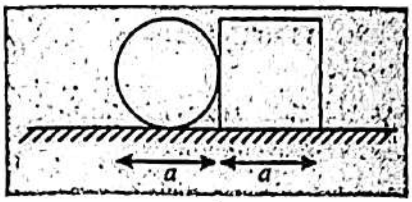
(*a*) (0.8, 0.6)m (*b*) (0.6, 0.8)m
(*c*) (0.4, 0.4)m (*d*) (0.5, 0.6)m

2. Figure shows a composite system of two uniform rods of lengths as indicated. Then the coordinates of the centre of mass of the system of rods are



(*a*) $\left(\frac{L}{2}, \frac{2L}{3}\right)$ (*b*) $\left(\frac{L}{4}, \frac{2L}{3}\right)$ (*c*) $\left(\frac{L}{6}, \frac{2L}{3}\right)$ (*d*) $\left(\frac{L}{6}, \frac{L}{3}\right)$

3. A circular plate of diameter 'a' is kept in contact with a square plate of side a as shown. The density of the material and the thickness are same everywhere. The centre of mass of composite system will be



(*a*) Inside the circular plate (*b*) Inside the square plate
(*c*) At the point of contact (*d*) Outside the system

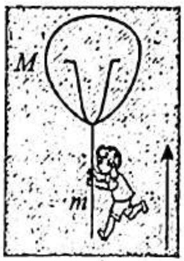
4. A man of mass m starts moving on a plank of mass M with constant velocity v with respect to plank. If the plank lies on a smooth horizontal surface, then velocity of plank with respect to ground is

(*a*) $\frac{Mv}{m+M}$ (*b*) $\frac{mv}{M}$
(*c*) $\frac{Mv}{m}$ (*d*) $\frac{mv}{m+M}$

5. A shell following a parabolic path explodes somewhere in its flight. The centre of mass of fragments will move in

(*a*) Vertical direction (*b*) Any direction
(*c*) Horizontal direction (*d*) Same parabolic path

6. A man of mass m is suspended in air by holding the rope of a balloon of mass M. As the man climbs up the rope, the balloon

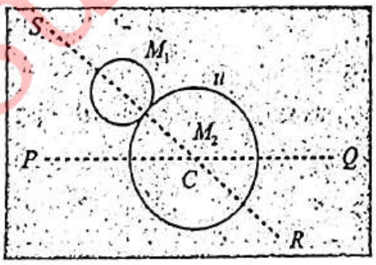


(*a*) Moves upward (*b*) Moves downward
(*c*) Remains stationary (*d*) Cannot say

7. A ball of mass m is thrown upward and another ball of same mass is thrown downward so as to move freely under gravity. The acceleration of centre of mass is

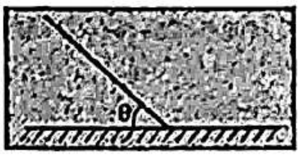
(*a*) g (*b*) $\frac{g}{2}$ (*c*) $2g$ (*d*) Zero

8. An object of mass M_1 moving horizontally with speed u collides elastically with another object of mass M_2 at rest. Select correct statement.



(*a*) The momentum of system is conserved only in direction PQ
(*b*) Momentum of M_1 is conserved in direction perpendicular to SR
(*c*) Momentum of M_2 will change in direction normal to CR
(*d*) All of these

9. A ball of mass m moving with speed u collides with a smooth horizontal surface at angle θ with it as shown in figure. The magnitude of impulse imparted to surface by ball is [Coefficient of restitution of collision is e]



(*a*) $mu(1+\theta)\cos\theta$ (*b*) $mu(1-\theta)\sin\theta$
(*c*) $mu(1-e)\cos\theta$ (*d*) $mu(1+e)\sin\theta$

10. Two particles A and B initially at rest move towards each other under a mutual force of attraction. At the instant when velocity of A is v and that of B is $2v$, the velocity of centre of mass of the system

(*a*) v (*b*) $2v$
(*c*) $3v$ (*d*) Zero

Directions: These questions consist of two statements each, printed as Assertion and Reason. While answering these questions, you are required to choose any one of the following four responses.

(*a*) Both Assertion and Reason are True and the Reason is a correct explanation of the Assertion.
(*b*) Both Assertion and Reason are True but Reason is not a correct explanation of the Assertion.
(*c*) Assertion is True but the Reason is False.
(*d*) Assertion and Reason are False

11. **Assertion:** In inelastic collision, a part of kinetic energy converts into heat energy, sound energy and light energy etc.

Reason: The force of interaction in an inelastic collision is non-conservative in nature.

12. **Assertion:** Centre of mass of a system may or may not lie inside the system.

Reason: The position of centre of mass depends on distribution of mass within the system.

VERY SHORT ANSWER TYPE QUESTIONS

13. Where does the centre of mass of a system having two particles lie if one particle is heavier than the other?

14. Where does the centre of mass of a uniform circular ring lie?

15. Where does the centre of mass of a uniform triangular lamina lie?

16. Two bodies of masses 3 kg and 6 kg are moving with velocities 2 m/s and 5 m/s towards each other. Calculate the velocity of their centre of mass.

17. Differentiate between the centre of mass and centre of gravity of a rigid body.

18. Explain why a bomb at rest when explodes, its centre of mass remains at rest.

19. An object of mass m_1 moving with speed v collides with another object of mass m_2 at rest and stick to it. Find the impulse imparted to the second object.

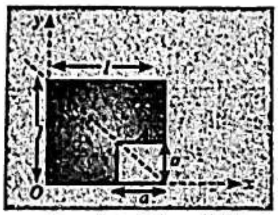
SHORT ANSWER TYPE QUESTIONS

20. $(n-1)$ equal point masses each of mass m are placed at the vertices of a regular n-polygon. The vacant vertex has a position vector $\vec{a}$ with respect to centre of polygon. Find position vector of centre of mass.

21. Obtain an expression for the position vector of the centre of mass of a two-particle system in two dimensions.

LONG ANSWER TYPE QUESTIONS

22. The figure shows a square metal plate of side l from which a square plate of side a has been cut as shown in the figure. Find the ratio so that the centre of mass of the remaining -shaped plate coincides with the point .



23. Discuss the motion of centre of mass and law of conservation of linear momentum with the help of examples.

CASE STUDY BASED QUESTIONS

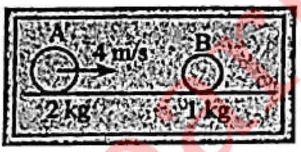
24. While dealing with collisions between particles, you must have deal from an Inertial reference frame. Often, we choose that frame to be fixed in the laboratory in which the collision is observed. So, it is called the Laboratory reference frame or Lab frame. From Lab frame we define an elastic collision as a collision in which KE before and after collision is conserved and a perfectly inelastic collision as a collision in which after collision the two colliding bodies have same velocity vector along the line of action of impulse during collision.

If we discuss the head-on collision between two particles from center of mass reference frame, then the velocity of center of mass (CM) will be taken to be zero in any type of collision i.e, velocity of CM before and after collision 'will both be zero. 'Since before, collision velocity of CM was zero (as our frame is) fixed to CM) and no external impulse acts, it will remain zero forever.

(i) From CM frame, after a head-on elastic collision:
- (a) velocity of particles change in direction but not in magnitude
- (b) velocity of particles gets interchanged
- (c) velocity remains unchanged
- (d) momentum of each particle remains conserved

(ii) From CM frame, after a perfectly inelastic head-on collision
- (a) velocity of particles change in direction but not in magnitude
- (b) velocity of particles gets interchanged
- (c) velocity remains unchanged
- (d) momentum of each particle becomes zero

(iii) Two particles of mass 2 kg and 1 kg as shown in the figure, make a perfectly inelastic collision. Then if we are dealing with center of mass reference frame, the velocity of B before collision is:



- (a) zero
- (b) 8 m/s leftwards
- (c) 4 m/s rightwards
- (d) $\frac{8}{3}$ m/s leftwards

(iv) After collision, the velocity of B in CM reference frame will be:
- (a) zero
- (b) 4 m/s leftwards
- (c) $\frac{8}{3}$ m/s rightwards
- (d) $\frac{8}{3}$ m/s leftwards

(v) What is a perfectly inelastic collision?
- (a) A collision in which kinetic energy before and after the collision is conserved
- (b) A collision in which the two colliding bodies have the same velocity vector along the line of action of impulse during collision
- (c) A collision in which kinetic energy before and after the collision is not conserved
- (d) A collision in which momentum is not conserved

25. One particle of mass 1 kg is moving along positive x-axis with velocity 3 m/s. Another particle of mass 2 kg is moving along y-axis with 6 m/s. At time $t = 0$, 1 kg mass is at (3 m, 0) and 2 kg at (0, 9m) $x-y$ plane is the horizontal plane. (Surface is smooth)

(i) What is the approximate total initial kinetic energy of the system at $t = 0$?
- (a) 40.5 J
- (b) 20.5 J
- (c) 36.5 J
- (d) 45.5 J

(ii) The centre of mass of the two particles is moving in a straight line:
- (a) $y = x + 2$
- (b) $y = 4x + 2$
- (c) $y = 2x - 4$
- (d) $y = 2x + 4$

Prarambh (Topicwise)

CALCULATION OF COM

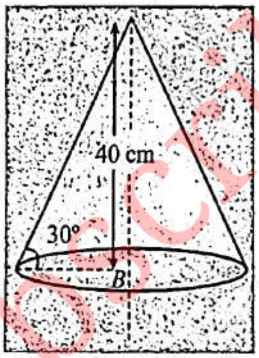
1. The centre of mass of a body:
   (a) Lies always at the geometrical centre
   (b) Lies always inside the body
   (c) Lies always outside the body
   (d) Can lie inside or outside the body

2. A body has its centre of mass at the origin. The x-coordinates of the particles
   (a) May be all positive
   (b) May be all negative
   (c) Must be all non-negative
   (d) May be positive for some particles and negative in other particles

3. All the particles of a body are situated at a distance R from the origin. The distance of the centre of mass of the body from the origin is
   (a) $= R$ (b) $\leq R$ (c) $> R$ (d) $\geq R$

4. Where will be the centre of mass on combining two masses m and $M (M > m)$:
   (a) near m
   (b) near M
   (c) Outside the line joining m and M
   (d) None of these

5. A uniform solid cone of height 40 cm is shown in figure. The distance of centre of mass of the cone from point B (centre of the base) is:



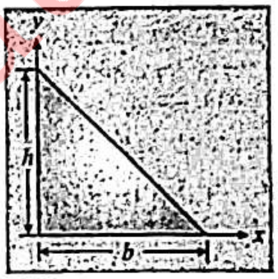
   (a) 20 cm (b) 10/3 cm (c) 20/3 cm (d) 10 cm

6. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å ($1 \text{ Å} = 10^{-10}$ m). The approximate location of the centre of mass of the molecule (distance from hydrogen atom) assuming the chlorine atom to be about 35.5 times massive as hydrogen is
   (a) 1Å (b) 2.5 Å (c) 1.24 Å (d) 1.5 Å

7. Centre of mass is a point
   (a) Which is geometric centre of a body
   (b) From which distance of particles are same
   (c) Where the whole mass of the body is supposed to be concentrated
   (d) Which is the origin of reference frame

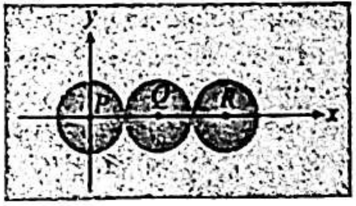
8. Choose the correct statement about the centre of mass (COM) of a system of two particles
   (a) The COM lies on the line joining the two particles midway between them
   (b) The COM lies on the line joining them at a point whose distance from each particle is inversely proportional to the mass of that particle
   (c) The COM lies on the line joining them at a point whose distance from each particle is proportional to the square of the mass of that particle
   (d) The COM is on the line joining them at a point whose distance from each particle is proportional to the mass of that particle

9. The centre of mass of triangle shown in figure has coordinates



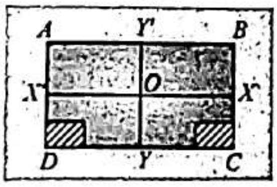
   (a) $x = \frac{h}{2}, y = \frac{b}{2}$ (b) $x = \frac{b}{2}, y = \frac{h}{2}$
   (c) $x = \frac{b}{3}, y = \frac{h}{3}$ (d) $x = \frac{h}{3}, y = \frac{b}{3}$

10. Three identical spheres, each of mass 1 kg are kept as shown in figure, touching each other, with their centres on a straight line. If their centres are marked P, Q, R respectively, the distance of centre of mass of the system from P (origin) is



   (a) $\frac{\overline{PQ}+\overline{PR}+\overline{QR}}{3}$ (b) $\frac{\overline{PQ}+\overline{PR}}{3}$
   (c) $\frac{\overline{PQ}+\overline{QR}}{3}$ (d) $\frac{\overline{PR}+\overline{QR}}{3}$

11. A uniform square plate $ABCD$ has a mass of 10 kg. If two point masses of 3 kg each are placed at the corners C and D as shown in the adjoining figure, then the centre of mass shifts to a point which lies on:



(a) OC (b) OD
(c) OY (d) OX

DISPLACEMENT, VELOCITY, ACCELERATION OF COM

12. Two particles whose masses are 10 kg and 30 kg and their position vectors are $\hat{i}+\hat{j}+\hat{k}$ and $-\hat{i}-\hat{j}-\hat{k}$ respectively would have the centre of mass at:

(a) $-\frac{(\hat{i}+\hat{j}+\hat{k})}{2}$ (b) $\frac{(\hat{i}+\hat{j}+\hat{k})}{2}$
(c) $-\frac{(\hat{i}+\hat{j}+\hat{k})}{4}$ (d) $\frac{(\hat{i}+\hat{j}+\hat{k})}{4}$

13. A bomb travelling in a parabolic path under the effect of gravity, explodes in mid air. The centre of mass of fragments will:

(a) Move vertically upwards and then downwards
(b) Move vertically downwards
(c) Move in irregular path
(d) Move in the parabolic path which the unexploded bomb would have travelled.

14. Two balls are thrown in air. The acceleration of the centre of mass of the two balls while in air (neglect air resistance)

(a) Depends on the direction of the motion of the balls
(b) Depends on the masses of the two balls
(c) Depends on the speeds of the two balls
(d) Is equal to g

15. The motion of the centre of mass of a system of two particles is unaffected by their internal forces:

(a) Irrespective of the actual directions of the internal forces
(b) Only if they are long the line joining the particles
(c) Only if they are at right angles to the line joining the particles
(d) Only if they are obliquely inclined to the line joining the particles.

16. Two objects of masses 200 gm and 500 gm possess velocities $10\hat{i}$ m/s and $(3\hat{i}+5\hat{j})$ m/s respectively. The velocity of their centre of mass in m/s is:

(a) $5\hat{i}-25\hat{j}$ (b) $\frac{5}{7}\hat{i}-25\hat{j}$
(c) $5\hat{i}+\frac{25}{7}\hat{j}$ (d) $25\hat{i}-\frac{5}{7}\hat{j}$

17. 2 bodies of different masses of 2kg and 4kg are moving with velocities 20m/s and 10m/s towards each other due to mutual gravitational attraction. What is the velocity of their centre of mass?

(a) 5 m/s (b) 6 m/s (c) 8 m/s (d) zero

18. Two bodies of masses 2 kg and 4 kg are moving with velocities 2 m/s and 10m/s respectively along same direction. Then the velocity of their centre of mass will be

(a) 8.1 m/s (b) 7.3 m/s
(c) 6.4m/s (d) 5.3 m/s

19. Two balls A and B of masses 100 gm and 250 gm respectively are connected by a stretched spring of negligible mass and placed on a smooth table. When the balls are released simultaneously, the initial acceleration of B is 10 cm/sec^2 westward. What is the magnitude and direction of initial acceleration of the ball A?

(a) 25 cm/sec^2 Eastward
(b) 25 cm/sec^2 Northward
(c) 25 cm/sec^2 Westward
(d) 25 cm/sec^2 Southward

LAW OF CONSERVATION OF MOMENTUM

20. Two bodies of masses m_1 and m_2 have equal kinetic energies. If p_1 and p_2 are their respective momentum, then ratio $p_1:p_2$ is equal to

(a) $m_1:m_2$ (b) $m_2:m_1$
(c) $\sqrt{m_1}:\sqrt{m_2}$ (d) $m_1^2:m_2^2$

21. A bullet of mass m is being fired from a stationary gun of mass M. If the velocity of the bullet is v, the velocity of the gun is:

(a) $\frac{Mv}{m+M}$ (b) $\frac{-mv}{M}$
(c) $\frac{(M+m)v}{M}$ (d) $\frac{M+m}{Mv}$

22. A bomb at rest has mass 60 kg. It explodes and a fragment of 40 kg has kinetic energy 96 joule. Then kinetic energy of other fragment is:

(a) 180 J (b) 190 J
(c) 182 J (d) 192 J

IMPULSE

23. A force of 50 dynes is acted on a body of mass 5 gm which is at rest for an interval of 3 sec, then impulse is:

(a) 0.16×10^{-3} N-s (b) 0.98×10^{-3} N-s
(c) 1.5×10^{-3} N-s (d) 2.5×10^{-3} N-s

24. The area of F-t curve is A, where 'F' is the force on one mass due to the other. If one of the colliding bodies of mass M is at rest initially, its speed just after the collision is:

(a) A/M (b) M/A
(c) AM (d) $\sqrt{\frac{2A}{M}}$

25. If two balls, each of mass 0.06 kg. moving in opposite directions with speed of 4 m/s, collide and rebound with the same speed, then the impulse imparted to each ball due to other (in kg-m/s) is:

(*a*) 0.48 (*b*) 0.53

(*c*) 0.81 (*d*) 0.92

COLLISIONS (OBLIQUE AND HEAD ON)

26. A block moving in air explodes in two parts then just after explosion

(*a*) The total momentum must be conserved

(*b*) The total kinetic energy of two parts must be same as that of block before explosion.

(*c*) The total momentum must change

(*d*) The total kinetic energy must not be increased

27. In head on elastic collision of two bodies of equal masses, it is not possible that

(*a*) The velocities are interchanged

(*b*) The speeds are interchanged

(*c*) The momenta are interchanged

(*d*) The faster body speeds up and the slower body slows down

28. Two identical blocks A and B, each of mass 'm' resting on smooth floor are connected by a light spring of natural length L and spring constant K, with the spring at its natural length. A third identical block 'C' (mass m) moving with a speed v along the line joining A and B collides with A. The maximum compression in the spring is (consider all collisions are elastic)

(*a*) $v\sqrt{\frac{m}{2k}}$ (*b*) $m\sqrt{\frac{v}{2k}}$

(*c*) $\sqrt{\frac{mv}{k}}$ (*d*) $\frac{mv}{2k}$

29. A ball of mass 3 kg collides with a wall with velocity 10 m/sec at an angle of 30° with the wall and after collision reflects at the same angle with the same speed. The change in momentum of ball in MKS unit is-

(*a*) 20 (*b*) 30

(*c*) 15 (*d*) 45

30. A ball of mass 'm', moving with uniform speed, collides elastically with another stationary ball. The incident ball will lose maximum kinetic energy when the mass of the stationary ball is

(*a*) m (*b*) $2m$

(*c*) $4m$ (*d*) infinity

31. The coefficient of restitution e for a perfectly elastic collision is

(*a*) 1 (*b*) 0 (*c*) ∞ (*d*) –1

32. A lead ball strikes a wall and falls down, a tennis ball having the same mass and velocity strikes the wall and bounces back. Check the correct statement

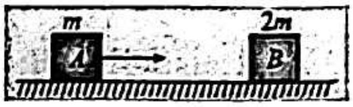
(*a*) The momentum of the lead ball is greater than that of the tennis ball

(*b*) The lead ball suffers a greater change in momentum compared with the tennis ball

(*c*) The tennis ball suffers a greater change in momentum as compared with the lead ball

(*d*) Both suffer an equal change in momentum

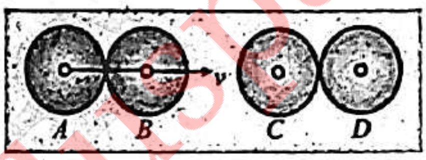
33. In the figure shown the block A collides head on with another block B at rest. Mass of B is twice the mass of A. The block A stops after collision. The co-efficient of restitution is:



(*a*) 0.5 (*b*) 1

(*c*) 0.25 (*d*) it is not possible

34. Two identical smooth spheres A and B are moving with same velocity and collide with similar spheres C and D. then after collision (Consider one dimensional collision and all collision are elastic)



(*a*) D will move with greater speed

(*b*) C and D will move with same velocity v

(*c*) C will stop and D will move with velocity v

(*d*) All spheres A, B, C & D will move with velocity $v/2$

35. A body of mass m having an initial velocity v, makes head on elastic collision with a stationary body of mass M. After the collision, the body of mass m comes to rest and only the body having mass M moves. This will happen only when

(*a*) $m >> M$ (*b*) $m << M$

(*c*) $m = M$ (*d*) $m = \frac{1}{2}M$

36. Two equal masses moving along the same straight line with velocities + 3m/s and – 5m/s respectively collide elastically. Their velocities after the collision will be respectively

(*a*) + 4 m/s for both (*b*) – 3 m/s and + 5 m/s

(*c*) – 4 m/s and + 4 m/s (*d*) – 5 m/s and + 3 m/s

37. A body falls on a surface of coefficient of restitution 0.6 from a height of 1 m. Then the body rebounds to a height of

(*a*) 0.6 m (*b*) 0.4 m

(*c*) 1 m (*d*) 0.36 m

38. A ball of mass m falls vertically to the ground from a height h_1 and rebound to a height h_2. The change in momentum of the ball on striking the ground is

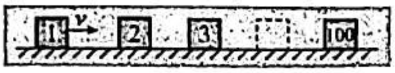
(*a*) $mg(h_1 - h_2)$ (*b*) $m\left(\sqrt{2gh_1} + \sqrt{2gh_2}\right)$

(*c*) $m\sqrt{2g(h_1 + h_2)}$ (*d*) $m\sqrt{2g}(h_1 + h_2)$

39. Two spheres approaching each other collides elastically. Before collision the speed of A is 5 m/s and that of B is 10 m/s. Their masses are 1 kg and 0.5 kg. After collision velocities of A and B are respectively- (Take initial velocity of A as positive)

(a) 5 m/s, 10 m/s (b) 10m/s, –5 m/s
(c) –10 m/s, –5 m/s (d) –5 m/s, 10 m/s

40. There are hundred identical sliders equally spaced on a frictionless track as shown in the figure. Initially all the sliders are at rest. Slider 1 is pushed with velocity v towards slider 2. In a collision the sliders stick together. The final velocity of the set of hundred stucked sliders will be:

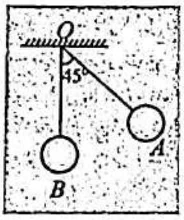


(a) $\frac{v}{99}$ (b) $\frac{v}{100}$
(c) zero (d) v

41. A space craft of mass M is moving with velocity V and suddenly explodes into two pieces. A part of it (of mass m) comes to rest, then the speed of other part will be

(a) $\frac{MV}{M-m}$ (b) $\frac{MV}{M+m}$
(c) $\frac{mV}{M-m}$ (d) $\frac{(M+m)V}{m}$

42. The bob A of a simple pendulum is released when the string makes an angle of 45° with the vertical. It hits another bob B of the same material and same mass kept at rest on the table. If the collision is elastic



(a) Both A and B rise to the same height
(b) Both A and B come to rest at B
(c) Both A and B move with the same velocity of A
(d) A comes to rest and B moves with the velocity of A

43. A completely inelastic collision is one in which the two colliding particles

(a) Are separated after collision
(b) Remain together after collision
(c) Split into small fragments flying in all directions
(d) None of the above

44. A body of mass 2kg is moving with velocity 10 m/s towards east. Another body of same mass and same velocity moving towards north collides with former and coalesces and moves towards north-east. Its velocity is

(a) 10 m/s (b) 5 m/s
(c) 2.5 m/s (d) $5\sqrt{2}$ m/s

VARIABLE MASS SYSTEMS

45. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 m/s². The initial thrust of the blast is-

(a) 14.0×10^5 N (b) 1.76×10^5 N
(c) 3.5×10^5 N (d) 7.0×10^5 N

46. Fuel is consumed at the rate of 100 kg/sec in a rocket. The exhaust gases are ejected at a speed of 4.5×10^4 m/s. What is the thrust experienced by the rocket?

(a) 3×10^6 N (b) 4.5×10^6 N
(c) 6×10^6 N (d) 9×10^6 N

47. A 6000 kg rocket is set for vertical firing. The exhaust speed is 1000 m/sec. How much gas must be ejected each second to supply the thrust needed to give the rocket an initial upward acceleration of 20 m/sec²? (Consider $g = 9.8$ m sec⁻² acceleration due to gravity)

(a) 92.4 kg/sec (b) 178.8 kg/sec
(c) 143.2 kg/sec (d) 47.2 kg/sec

Prabal (JEE Main Level)

1. A thin uniform wire is bent to form the two equal sides AB and AC of triangle ABC, where $AB = AC = 5$ cm. The third side BC, of length 6cm, is made from uniform wire of twice the density of the first. The distance of centre of mass from A is:

(a) $\frac{34}{11}$ cm (b) $\frac{11}{34}$ cm
(c) $\frac{34}{9}$ cm (d) $\frac{11}{45}$ cm

2. A body of mass 1 kg moving in the x-direction, suddenly explodes into two fragments of mass 1/8 kg and 7/8 kg. An instant later, the smaller fragment is 0.14 m above the x-axis. The position of the heavier fragment is:

(a) $\frac{1}{50}$ m above x-axis (b) $\frac{1}{50}$ m below x-axis
(c) $\frac{7}{50}$ m below x-axis (d) $\frac{7}{50}$ m above x-axis

3. A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless surface. The man walks to other end of the plank. If the mass of the plank is $\frac{M}{3}$, then the distance that the man moves relative to ground is:

(a) $\frac{3L}{4}$ (b) $\frac{L}{4}$ (c) $\frac{4L}{5}$ (d) $\frac{L}{3}$

4. A particle of mass $3m$ is projected from the ground at some angle with horizontal. The horizontal range is R. At the highest point of its path it breaks into two pieces m and $2m$. The smaller mass comes to rest and larger mass finally falls at a distance x from the point of projection where x is equal to

(a) $\frac{3R}{4}$ (b) $\frac{3R}{2}$ (c) $\frac{5R}{4}$ (d) $3R$

5. Two particles of mass 1 kg and 0.5 kg are moving in the same direction with speed of 2m/s and 6m/s respectively on a smooth horizontal surface. The speed of centre of mass of the system is:

(a) $\frac{10}{3}$ m/s (b) $\frac{10}{7}$ m/s (c) $\frac{11}{2}$ m/s (d) $\frac{12}{3}$ m/s

6. Two particles having mass ratio $n : 1$ are interconnected by a light inextensible string that passes over a smooth pulley. If the system is released, then the acceleration of the centre of mass of the system is:

(a) $(n-1)^2 g$ (b) $\left(\frac{n+1}{n-1}\right)^2 g$

(c) $\left(\frac{n-1}{n+1}\right)^2 g$ (d) $\left(\frac{n+1}{n-1}\right) g$

7. Internal forces in a system can change

(a) Linear momentum only

(b) Kinetic energy only

(c) Both kinetic energy and linear momentum

(d) Neither the linear momentum nor the kinetic energy of the system.

8. A man of mass 'm' climbs on a rope of length L suspended below a balloon of mass M. The balloon is stationary with respect to ground. If the man begins to climb up the rope at a speed v_{rel} (relative to rope). In what direction and with what speed (relative to ground) will the balloon move?

(a) Downwards, $\frac{mv_{rel}}{m+M}$ (b) Upwards, $\frac{Mv_{rel}}{m+M}$

(c) Downwards, $\frac{mv_{rel}}{M}$ (d) Downwards, $\frac{(M+m)v_{rel}}{M}$

9. There are some passengers inside a stationary railway compartment. The track is frictionless. The centre of mass of the compartment itself (without the passengers) is C_1, while the centre of mass of the compartment plus passengers system is C_2. If the passengers move about inside the compartment along the track,

(a) Both C_1 and C_2 will move with respect to the ground

(b) Neither C_1 nor C_2 will over with respect to the ground

(c) C_1 will move but C_2 will be stationary with respect to the ground

(d) C_2 will move but C_1 will be stationary with respect to the ground

10. A shell is fired from a canon with a velocity V at an angle θ with the horizontal direction. At the highest point in its path, it explodes into two pieces of equal masses. One of the pieces come to rest. The speed of the other piece immediately after the explosion is

(a) $3V\cos\theta$ (b) $2V\cos\theta$

(c) $\frac{3}{2}V\cos\theta$ (d) $V\cos\theta$

11. A small sphere is moving at a constant speed in a vertical circle. Below is a list of quantities that could be used to describe some aspect of the motion of the sphere

I. Kinetic energy

II. Gravitational potential energy

III. Momentum

Which of these quantities will change as this sphere moves around the circle?

(a) I and II only

(b) I and III only

(c) III only

(d) II and III only

12. A nucleus of mass number A originally at rest emits α-particle with speed v. The recoil speed of daughter nucleus is:

(a) $\frac{4v}{A-4}$ (b) $\frac{4v}{A+4}$

(c) $\frac{v}{A-4}$ (d) $\frac{v}{A+4}$

13. A super-ball is to bounce elastically back and forth between two rigid walls at a distance d from each other. Neglecting gravity and assuming the velocity of super-ball to be v_0 horizontally, the average force being exerted by the super-ball on each wall is:

(a) $\frac{1}{2}\frac{mv_0^2}{d}$ (b) $\frac{mv_0^2}{d}$

(c) $\frac{2mv_0^2}{d}$ (d) $\frac{4mv_0^2}{d}$

14. A force exerts an impulse I on a particle changing its speed from u to $2u$. The applied force and the initial velocity are oppositely directed along the same line. The work done by the force is

(a) $\frac{3}{2}Iu$ (b) $\frac{1}{2}Iu$

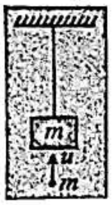
(c) Iu (d) $2Iu$

15. A particle of mass $4m$ which is at rest explodes into three fragments. Two of the fragments each of mass m are found to move with a speed 'v' each in mutually perpendicular directions. The minimum energy released in the process of explosion is:

(a) $(2/3)\,mv^2$ (b) $(3/2)\,mv^2$

(c) $(4/3)\,mv^2$ (d) $(3/4)\,mv^2$

16. A bullet of mass m moving vertically upwards instantaneously with a velocity 'u' hits the hanging block of mass 'm' and gets embedded in it. As shown in the figure the height through which block rises after the collision (assume sufficient space above block) is



(*a*) $u^2/2g$ (*b*) u^2/g (*c*) $u^2/8g$ (*d*) $u^2/4g$

17. In an inelastic collision-

(*a*) Momentum is conserved but kinetic energy is not

(*b*) Momentum is not conserved but kinetic energy is conserved

(*c*) Neither momentum nor kinetic energy is conserved

(*d*) Both the momentum and kinetic energy are conserved

18. Two perfectly elastic balls of same mass m are moving with velocities u_1 and u_2. They collide elastically n times. The kinetic energy of the system finally is:

(*a*) $\frac{1}{2}\frac{m}{n}u_1^2$ (*b*) $\frac{1}{2}\frac{m}{n}(u_1^2+u_2^2)$

(*c*) $\frac{1}{2}m(u_1^2+u_2^2)$ (*d*) $\frac{1}{2}mn(u_1^2+u_2^2)$

19. A ball hits the floor and rebounds after an inelastic collision. In this case-

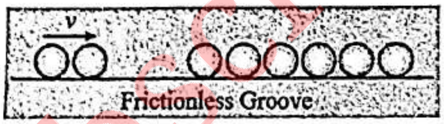
(*a*) The momentum of the ball just after the collision is the same as that just before the collision

(*b*) The mechanical energy of the ball remains the same in the collision

(*c*) The total momentum of the ball and the earth is conserved.

(*d*) The total mechanical energy of the ball and the earth is conserved

20. Six steel ball of identical size are lined up along a straight frictionless groove. Two similar balls moving with a speed v along the groove collide with this row on the extreme left hand, then (all collision are elastic).



(*a*) All the balls will start moving to the right with speed 1/8 each

(*b*) All the six balls initially at rest will move on with speed $v/6$ each and two identical balls will come to rest

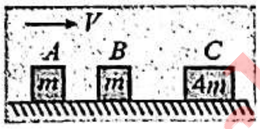
(*c*) Two balls from the extreme right end will move on with speed v each and the remaining balls will remain at rest

(*d*) One ball from the right end will move on with speed $2v$, the remaining balls will be at rest.

21. A sphere of mass m moving with a constant velocity hits another stationary sphere of the same mass. If e is the coefficient of restitution, then ratio of speed of the first sphere to the speed of the second sphere after collision will be:

(*a*) $\left(\frac{1-e}{1+e}\right)$ (*b*) $\left(\frac{1+e}{1-e}\right)$ (*c*) $\left(\frac{e+1}{e-1}\right)$ (*d*) $\left(\frac{e-1}{e+1}\right)$

22. Three blocks are initially placed as shown in the figure. Block A has mass m and initial velocity V to the right. Block B with mass m and block C with mass $4m$ are both initially at rest. Neglect friction. All collisions are elastic. The final velocity of block A is



(*a*) $0.60V$ to the left (*b*) $1.4V$ to the left

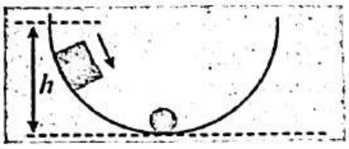
(*c*) V to the left (*d*) $0.4V$ to the left

23. Two billiard balls undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed v towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of $v/3$ in the same direction. What type of collision has occured?

(*a*) Inelastic (*b*) Elastic

(*c*) Completely inelastic (*d*) None of these

24. A block of mass m starts from rest and slides down a frictionless semi-circular track from a height h as shown. When it reaches the lowest point of the track, it collides with a stationary piece of putty also having mass m. If the block and the putty stick together and continue to slide, the maximum height that the block-putty system could reach is



(*a*) $h/4$ (*b*) $h/2$

(*c*) h (*d*) Independent of h

25. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two identical particles, after collision, is

(*a*) $\sqrt{2}v_0$ (*b*) $\frac{v_0}{2}$ (*c*) $\frac{v_0}{\sqrt{2}}$ (*d*) $\frac{v_0}{4}$

26. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly:

(*a*) 4.70×10^3 N/m^2 (*b*) 2.35×10^2 N/m^2

(*c*) 4.70×10^2 N/m^2 (*d*) 2.35×10^3 N/m^2

27. A particle of mass m is moving in a straight line with momentum p. Starting at time $t = 0$, a force $F = kt$ acts in the same direction on the moving particle during time interval T so that its momentum changes from p to $3p$. Here k is a constant. The value of T is:

(*a*) $2\sqrt{\frac{k}{p}}$ (*b*) $2\sqrt{\frac{p}{k}}$

(*c*) $\sqrt{\frac{2k}{p}}$ (*d*) $\sqrt{\frac{2p}{k}}$

28. A particle of mass 'm' is moving with speed '$2v$' and collides with a mass '$2m$' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the original direction. The speed of each of the moving particles will be:

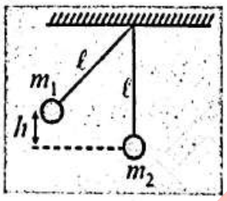
(*a*) $v/\left(2\sqrt{2}\right)$ (*b*) $2\sqrt{2}v$

(*c*) $\sqrt{2}v$ (*d*) $v/\sqrt{2}$

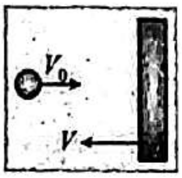
INTEGER TYPE QUESTIONS

29. A bomb initially at rest explodes by itself into three equal mass fragments. The velocities of two fragments are $\left(3\hat{i}+2\hat{j}\right)$ m/s and $\left(-\hat{i}-4\hat{j}\right)$ m/s. The speed of the third fragment is $x\sqrt{2}$ m/s, find the value of x

30. In the arrangement shown, the pendulum on the left is pulled aside. It is then released and allowed to collide with other pendulum which is at rest. A perfectly inelastic collision occurs and the system rises to a height $h/4$. The ratio of the masses (m_1/m_2) of the pendulum is:



31. A particle of mass m moves with velocity $V_0 = 20$ m/sec towards a large wall that is moving with velocity $V = 5$ m/sec. towards a particle as shown. If the particle collides with the wall elastically, the speed (in ms^{-1}) of the particle just after the collision is:



32. An object of mass 5 kg and speed 10 ms^{-1} explodes into two pieces of equal mass. One piece comes to rest. The kinetic energy (in joule) added to the system during the explosion is-

33. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically. If $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m?

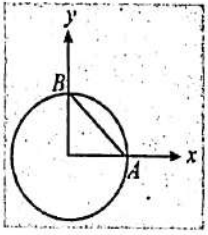
34. A piece of wood of mass 0.03 kg is dropped from the top of 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height (in m) to which the combined system reaches above the top of the building before falling below is ($g = 10$ ms^{-2})

35. A wedge of mass $M = 4m$ lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by: $\frac{2v^2}{Ng}$ Find the value of N.

Parikshit (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. An object comprises of a uniform ring of radius R and its uniform chord AB (not necessarily made of the same material) as shown. Which of the following can not be the centre of mass of the object?



(*a*) $(R/3, R/3)$ (*b*) $(R/3, R/2)$

(*c*) $(R/4, R/4)$ (*d*) $(R/\sqrt{2}, R/\sqrt{2})$

2. If the net external forces acting on a system is zero, then the centre of mass

(*a*) Must not move (*b*) Must not accelerate

(*c*) May move (*d*) May accelerate

3. An external force $\overline{F}\,(\overline{F} \neq 0)$ acts on a system of particles. The velocity and the acceleration of the centre of mass are found to be v_{cm} and a_{cm}, then it is possible that

(*a*) $v_{cm} = 0, a_{cm} = 0$ (*b*) $v_{cm} = 0, a_{cm} \neq 0$

(*c*) $v_{cm} \neq 0, a_{cm} = 0$ (*d*) $v_{cm} \neq 0, a_{cm} \neq 0$

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4. Two blocks A and B each of mass 'm' are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal plane. Block C also of mass m moves on the floor with a speed 'v' along the line joining A and B and collides elastically with A. Which of the following is/are correct?

(*a*) KE of the AB system at maximum compression of the spring is zero

(*b*) The KE of AB system at maximum compression is $(1/4)\, mv^2$

(*c*) The maximum compression of spring is $v\sqrt{m/k}$

(*d*) The maximum compression of spring is $v\sqrt{m/2k}$

5. In an elastic collision, in absence of external force, which of the following is/are correct?

(*a*) The linear momentum is conserved

(*b*) The potential energy is conserved in collision

(*c*) The final kinetic energy is less than the initial kinetic energy

(*d*) The final kinetic energy is equal to the initial kinetic energy

6. A block moving in air explodes in two parts, then, just after explosion (neglect change in momentum due to gravity)

(*a*) The total momentum of two parts must be same to the momentum of the block before explosion.

(*b*) That total kinetic energy of two parts must be same as that of block before explosion.

(*c*) The total momentum must change.

(*d*) The total kinetic energy must increase.

7. A set of n-identical cubical blocks lie at rest parallel to each other along a line on a smooth horizontal surface. The separation between the near surfaces of any two adjacent blocks is L. The block at one end is given a speed V towards the next one at time $t = 0$. All collisions are completely inelastic, then

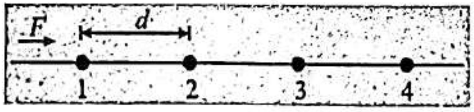
(*a*) The last block starts moving at $t = n(n-1)\dfrac{L}{2V}$

(*b*) The last block starts moving at $t = (n-1)\dfrac{L}{V}$

(*c*) The centre of mass of the system will have a final speed v/n

(*d*) The centre of mass of the system will have a final speed v

8. The figure shows a string of equally spaced beads of mass m, separated by distance d. The beads are free to slide without friction on a thin wire. A constant force F acts on the first bead initially at rest till it makes collision with the second bead. The second bead then collides with the third and so on. Suppose all collisions are elastic, then:



(*a*) Speed of the first bead immediately before and immediately after its collision with the second bead is $\sqrt{\dfrac{2Fd}{m}}$ and zero respectively.

(*b*) Speed of the first bead immediately before and immediately after its collision with the second bead is $\sqrt{\dfrac{2Fd}{m}}$ and $\dfrac{1}{2}\sqrt{\dfrac{2Fd}{m}}$ respectively.

(*c*) Speed of the second bead immediately after its collision with third bead is zero.

(*d*) The average speed of the first bead is $\dfrac{1}{2}\sqrt{\dfrac{2Fd}{m}}$

9. A shell explodes in a region of negligible gravitational field, giving out n fragments of equal mass m. Then its total (Assume situation just before and just after collision)

(*a*) Kinetic energy is smaller than that before the explosion

(*b*) Kinetic energy is greater than that before the explosion

(*c*) Momentum and kinetic energy depend on n

(*d*) Momentum is equal to that before the explosion.

10. Two identical balls are interconnected with a massless and inextensible thread. The system is in gravity free space with the thread just taut. Each ball is imparted a velocity v, one towards the other ball and the other perpendicular to the first, at $t = 0$. Then,

(*a*) The thread will become taut at $t = (L / v)$.

(*b*) The thread will become taut at some time $t < (L / v)$.

(*c*) The thread will always remain taut for $t > (L/v)$

(*d*) The kinetic energy of the system will always remain mv^2.

11. A particle moving with kinetic energy = 3 joule makes an elastic head on collision with a stationary particle which has twice its mass during the impact. Then,

(*a*) The minimum kinetic energy of the system is 1 joule

(*b*) The maximum elastic potential energy of the system is 2 joule.

(*c*) Momentum and total kinetic energy of the system are conserved at every instant.

(*d*) The ratio of kinetic energy to potential energy of the system first decreases and then increases.

12. In an inelastic collision,

(*a*) The velocity of both the particles may be same after collision.

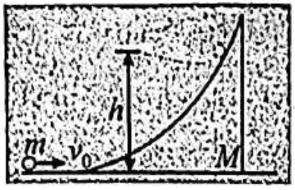
(*b*) Kinetic energy is not conserved

(*c*) Linear momentum of the system is conserved.

(*d*) Velocity of separation will be less than velocity of approach.

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 13 to 19): A particle of mass m moving horizontal with v_0 strikes a smooth wedge of mass M, as shown in figure. After collision, the ball starts moving up the inclined face of the wedge and rises to a height h.



13. The final velocity of the wedge v_2 is

(*a*) $\dfrac{mv_0}{M}$ (*b*) $\dfrac{mv_0}{M+m}$

(*c*) v_0 (*d*) insufficient data

14. When the particle has risen to a height h on the wedge, then choose the correct alternative(s)

(*a*) The particle is stationary with respect to ground

(*b*) Both are stationary with respect to the centre of mass

(*c*) The kinetic energy of the centre of mass remains constant

(*d*) The kinetic energy with respect to centre of mass is converted into potential energy

15. The maximum height h attained by the particle is

(*a*) $\left(\dfrac{m}{m+M}\right)\dfrac{v_0^2}{2g}$ (*b*) $\left(\dfrac{m}{M}\right)\dfrac{v_0^2}{2g}$

(*c*) $\left(\dfrac{M}{m+M}\right)\dfrac{v_0^2}{2g}$ (*d*) None of these

16. Identify the correct statement(*s*) related to the situation when the particle starts moving downward.

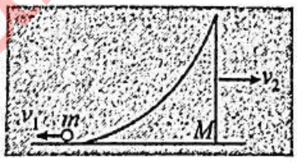
(*a*) The centre of mass of the system remains stationary

(*b*) Both the particle and the wedge remain stationary with respect to centre of mass

(*c*) When the particle reaches the horizontal surface its velocity relative to the wedge is v_0

(*d*) None of these

17. Suppose the particle when reaches the horizontal surfaces, its velocity with respect to ground is v_1 and that of wedge is v_2. Choose the correct statement (s)



(*a*) $mv_1 = Mv_2$ (*b*) $Mv_2 - mv_1 = mv_0$

(*c*) $v_1 + v_2 = v_0$ (*d*) $v_1 + v_2 < v_0$

18. From above Q. 17 choose the correct statement(s) related to particle m.

(*a*) Its kinetic energy is $K_f = \left(\dfrac{mM}{m+M}\right)gh$

(*b*) $v_1 = v_0\left(\dfrac{M-m}{M+m}\right)$

(*c*) The ratio of its final kinetic energy to its initial kinetic energy is $\dfrac{K_f}{K_i} = \left(\dfrac{M}{m+M}\right)^2$.

(*d*) It moves opposite to its initial direction of motion

19. Choose the correct statement related to the wedge M.

(*a*) Its kinetic energy is $K_f = \left(\dfrac{4m^2}{m+M}\right)gh$

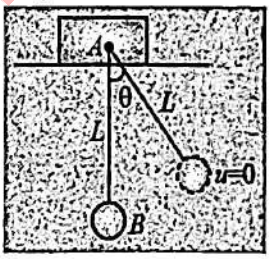
(*b*) $v_2 = \left(\dfrac{2m}{m+M}\right)v_0$

(*c*) Its gain in kinetic energy is

$$\Delta K = \left(\frac{4mM}{(m+M)^2}\right)\left(\frac{1}{2}mv_0^2\right)$$

(*d*) Its final velocity is more that the velocity of centre of mass

Comprehension (Q. 20 to 23): A small ball B of mass m is suspended with light inelastic string of length L from a block A of same mass m which can move on smooth horizontal surface as shown in the figure. The ball is displaced by angle θ from equilibrium position & then released.



20. The displacement of block when ball reaches the equilibrium position is

(*a*) $\dfrac{L\sin\theta}{2}$ (*b*) $L\sin\theta$

(*c*) L (*d*) None of these

21. Tension in string when it is vertical, is

(*a*) mg (*b*) $mg(2-\cos\theta)$

(*c*) $mg(3-2\cos\theta)$ (*d*) $mg(5-4\cos\theta)$

22. Maximum velocity of block during subsequent motion of the system after release of ball is

(*a*) $[gl(l-\cos\theta)]^{1/2}$ (*b*) $[2gl(l-\cos\theta)]^{1/2}$

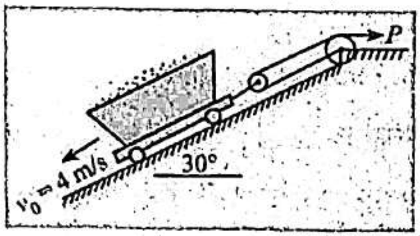
(*c*) $[gl\cos\theta]^{1/2}$ (*d*) $[gl(l-\sin\theta)]^{1/2}$

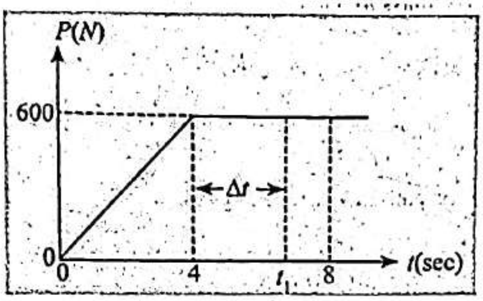
23. The displacement of centre of mass of $A+B$ system till the string becomes vertical is

(*a*) zero (*b*) $\dfrac{L}{2}(1-\cos\theta)$

(*c*) $\dfrac{L}{2}(1-\sin\theta)$ (*d*) None of these

Comprehension (Q. 24 to 26): The loaded 150 kg skip is rolling down the incline at 4 m/s when a force P is applied to the cable as shown at time $t = 0$. The force P is increased uniformly with the time until it reaches 600 N at $t = 4$s, after which time it remains constant at this value.





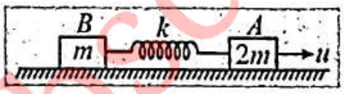
24. The time t_1 at which the skip reverses its direction is nearly.
(*a*) 10.87s (*b*) 8.87s (*c*) 6.67s (*d*) 4.67s

25. The velocity v of the skip at $t = 8$ s. (treating the skip as a particle) is
(*a*) 2 m/s (*b*) 4.0 m/s (*c*) 6.0 m/s (*d*) 8.0 m/s

26. Total impulse provided by force P till the time the skip reverses its direction is
(*a*) 1400 N-s (*b*) 2800 N-s
(*c*) 5600 N-s (*d*) 2400 N-s

MATCH THE COLUMN TYPE QUESTIONS

27. Two blocks A and B of mass m and $2m$ respectively are connected by a massless spring of spring constant k. This system lies over a smooth horizontal surface. At $t = 0$ the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant. In each situation of column-I, certain statements are given and corresponding results are given in column-II. Match the statements in column-I corresponding results in column-II.

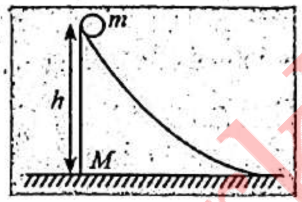


	Column-I		Column-II
A.	The velocity of block A	p.	can never be zero
B.	The velocity of block B	q.	may be zero at certain instants of time
C.	The kinetic energy of system of two blocks	r.	is minimum at maximum compression of spring
D.	The potential energy of spring	s.	is maximum at maximum extension of spring

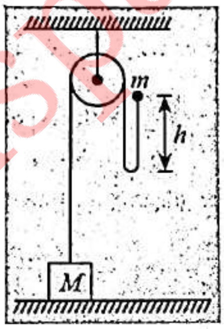
(*a*) A-(p); B-(q); C-(p, r); D-(q, s)
(*b*) A-(q); B-(p); C-(p, r); D-(q, s)
(*c*) A-(p); B-(q); C-(p, s); D-(q, r)
(*d*) A-(p, q); B-(q); C-(q, r); D-(q, s)

INTEGER TYPE QUESTIONS

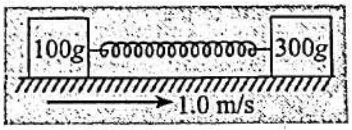
28. System is released from rest. If all surfaces in contact are smooth, the speed (in m/s) acquired by M when small ball m just touches the floor is (take: $m = 1$ kg, $M = 2$ kg, $h = 30$ cm) [Take $g = 10$ m/s^2]



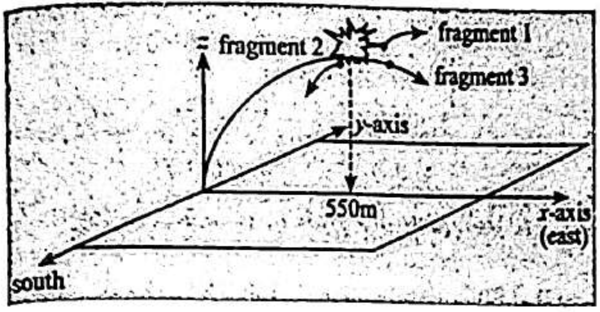
29. Two masses M and m are tied with a string and arranged as shown. What is the velocity (in m/s) of block M when it loses contact with the ground? [$m = 1$ kg and $M = 2$kg. $h = 90$ cm.]



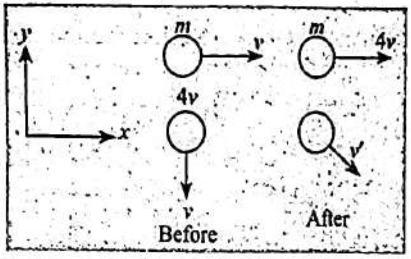
30. The carts in figure are sliding to the right at 1.0 m/s on a smooth level ground. The spring between them has a spring constant of 120 N/m and is compressed 40 cm. The carts slide past a flame that burns through the string holding them together. The carts are not attached to the spring. Afterward, what is the speed (in m/s) of 300g cart after it loses contact with the spring.



31. A Diwali rocket explodes at the topmost point of its trajectory, at horizontal distance 550 m from the point of projection see figure. When the fragments strike the ground one of the fragments is found at a location 550 m east and 120m north of the launch point. Second fragment is found at a location 550m east and 60 m south of the launch point. First two fragments are of equal mass m and third fragment has mass $2m$. If all the three fragments struck the ground simultaneously, what is the location of the third fragment with respect to origin when it strikes ground? Taking x-axis as east and y-axis as north, and point of projection as origin express your answer as $x\hat{i} + y\hat{j}$ (in m). Find $|x| + |y|$.

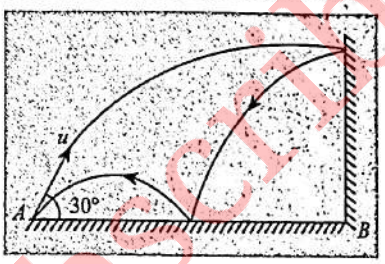


32. Two particles of masses m and $4m$, moving in vacuum at right angles to each other experience same force $\bar{F}$ for time T simultaneously. Consequently the particle m moves with velocity $4v$ in its original direction. Find the new magnitude of the velocity v' (in m/s) of the particle of mass $4m$. Given $v = 100$ m/s.



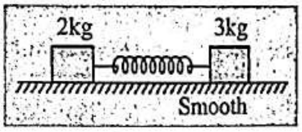
33. A big Diwali rocket is projected vertically upward so as to attain a maximum height of 160m. The rocket explodes just as it reaches the top of its trajectory sending out luminous particles in all possible directions all with same speed v. The display, consisting of the luminous particles, spreads out as an expanding, brilliant sphere. The bottom of this sphere just touches the ground when its radius is 80m. With what speed (in m/s) are the luminous particles ejected by the explosion?

34. A particle is projected with speed u making an angle 30° with the horizontal from a point A on the ground. The vertical wall is smooth. The particle hits the wall at highest point of its trajectory, after which it rebounds from the smooth ground and reaches point A as shown in the diagram. If coefficient of restitution e is same for all the collision then find the value of $10e$.

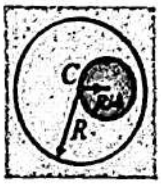


NUMERICAL TYPE QUESTIONS

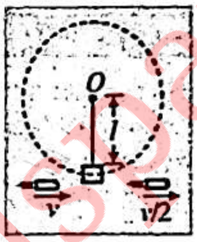
35. A 2kg mass and 3 kg mass are used to compress opposite ends of a spring ($k = 750$ N/m) by a distance of 40 cm from natural length and then is released from rest. If the speeds of the two masses as they leave the spring are v_2 and v_3, find $v_2 + v_3$ (in m/s)



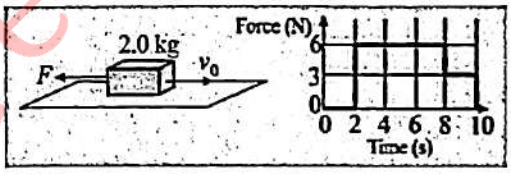
36. For years together, people thought that hell is located inside earth. Assume that hell is a spherical bubble created inside earth. It is completely evacuated to make sinful people feel suffocated. It has a radius $R/4$ and is located as shown. (C is earth's centre). What is the distance of centre of mass of the resulting body from C (in km). Assume that earth's radius is 6300 km.



37. A bullet of mass m and velocity v passes through a pendulum bob of mass M and emerges with velocity $v/2$ (figure). The pendulum bob is at the end of a string of length l. What is the minimum value of v (in m/s) such that the pendulum bob will swing through a complete circle? (Take: $l = 2$ m, $M = 1$kg, $m = 10$ gm.)

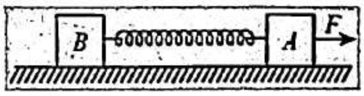


38. A block of mass 2kg is sliding on a smooth surface. At $t = 0$, its speed is $v_0 = 2$ m/s. At $t = 0$, a time-varying force starts acting on the block in the direction opposite to v_0. Find the speed of object (in m/s) at $t = 10$



39. A 80 kg man is riding on a 40 kg cart travelling at a speed of 2.5m/s on a frictionless surface. He jumps off the cart such that his velocity just after jump is zero with respect to ground. What is the work done by the man (in J) on the system during the jump?

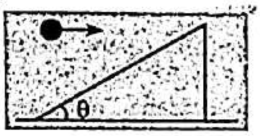
40. Two blocks of mass 2kg each are connected by a spring of constant 1000 N/m. Initially, the spring is at the natural length. A force of 10 N is applied to the 1st block. At 0.1 sec, the extension in the spring is seen to be 1 cm. What is the displacement of the block A (in cm) till that time as seen from ground.



41. An object of mass m is projected vertically upwards with a speed of v_0. At the same moment another object of mass M, which is initially above the projected one, is dropped from a height of h. The two point like objects collide completely inelastically, and they stick to each other. Find kinetic energy (in J) of combined mass just before it hits the ground. (Given: $m = 1$ kg, $v_0 = 20$ m/s, $M = 3$ kg, $h = 20$ m, $g = 10$ m/s^2)

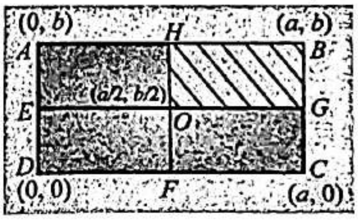
42. A bomb moving in certain direction suddenly explodes into three identical pieces which fly off with the same speed. Two of the three pieces go in the same direction while the third one goes perpendicular to the other two. If K_f and K_i are final and initial kinetic energies of the system, find the ratio of K_f to K_i.

43. An inclined plane is placed on a horizontal smooth surface. The plane is struck by an elastic ball whose velocity is horizontal just before the impact. The ball bounces off the inclined plane and then lands on it again at the point of first impact. Find the ratio of the masses of the ball and the inclined plane. (Angle $\theta = 30°$)



SINGLE CORRECT TYPE QUESTIONS

44. A uniform rectangular thin sheet $ABCD$ of mass M has length a and breadth b, as shown in the figure. If the shaded portion $HBGO$ is cut-off, the coordinates of the centre of mass of the remaining portion will be:



(*a*) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (*b*) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

(*c*) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (*d*) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

45. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$ undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively. If $m_2 = 0.5\ m_1$ and $v_3 = 0.5\ v_1$, then v_1 is:

(*a*) $v_4 - \frac{v_2}{4}$ (*b*) $v_4 - \frac{v_2}{2}$

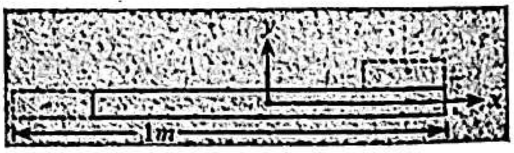
(*c*) $v_4 - v_2$ (*d*) $v_4 + v_2$

46. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively:

(*a*) (0.28, 0.89) (*b*) (0, 0)

(*c*) (0, 1) (*d*) (0.89, 0.28)

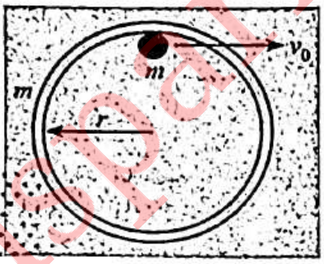
47. First 10 cm piece of a uniform meter stick is cut and kept on the other end as shown in figure. Find the shift in the x-coordinate of the centre of mass.



(*a*) 5 cm (*b*) 10 cm

(*c*) 9 cm (*d*) 45 cm

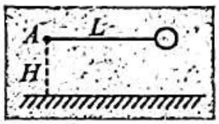
48. A small body of mass m is at rest inside a thin, narrow tyre of mass m and radius r which is lying on a horizontal table with its plane horizontal. The small body is given a tangential initial velocity v_0 at $t = 0$, as shown in figure. If after time T the velocity of tyre becomes zero, then find magnitude of velocity of ball at $t = T/6$. (Neglect friction everywhere)



(*a*) $\frac{v_0}{\sqrt{2}}$ (*b*) $\frac{v_0}{\sqrt{3}}$

(*c*) $\frac{\sqrt{3}}{2}v_0$ (*d*) $\frac{v_0}{2}$

49. A is a fixed point at a height H above a perfectly inelastic smooth horizontal plane. A light inextensible string of length L (> H) has one end attached to A and other is to a heavy particle. The particle is held at the level of A with string just taut and released from rest. Find the height of the particle above the plane when it is next instantaneously at rest.



(*a*) $\frac{H^5}{L^4}$ (*b*) $\frac{H^4}{L^3}$

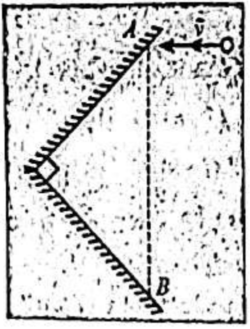
(*c*) $\frac{H^3}{L^2}$ (*d*) $\frac{H^2}{L}$

50. A cart loaded with sand having total mass $m_0 = 1800$ kg moves on a straight horizontal road starting from rest under the action of a force of 120 N. The sand spills through a small hole in the bottom at a rate of 0.5 kg/sec. What will be the velocity of cart after 20 min?

(*a*) 220 m/sec (*b*) 240 $\ell n\left(\frac{3}{2}\right)$

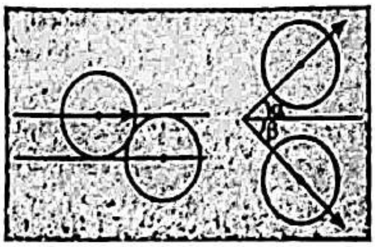
(*c*) 120 m/sec (*d*) None of these

51. AB is an L shaped obstacle fixed on a horizontal smooth table. A ball strikes it at A, gets deflected and restrikes it at B. If the velocity vector before collision is $\vec{v}$ and coefficient of restitution of each collision is 'e', then the velocity of ball after its second impact is



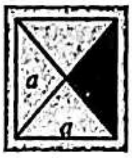
(a) $e^2\vec{v}$
(b) $-e^2\vec{v}$
(c) $-e\vec{v}$
(d) Data insufficient

52. A moving 2 kg ball with a speed of 2 m s^{-1} hits obliquely a stationary identical ball inelastically as shown. If the first ball move away with angle $\alpha = 30°$ to the original path, find the total KE lost after the collision.



(a) 4/3 J
(b) 5/4 J
(c) 6/5 J
(d) 7/6 J

53. From a square sheet of uniform density, a portion is removed shown as shaded region in the figure. Find distance of the centre of mass of the remaining portion from centre of the square if the side of the square is a.



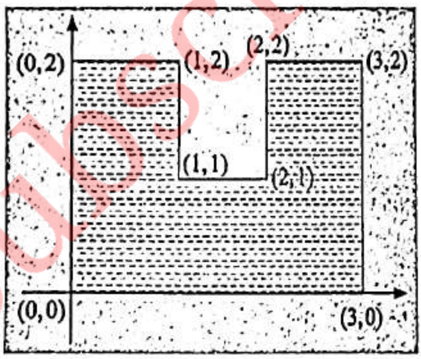
(a) $\frac{a}{3}$
(b) $\frac{a}{6}$
(c) $\frac{a}{9}$
(d) $\frac{a}{8}$

PYQ's (Past Year Questions)

CALCULATION OF COM

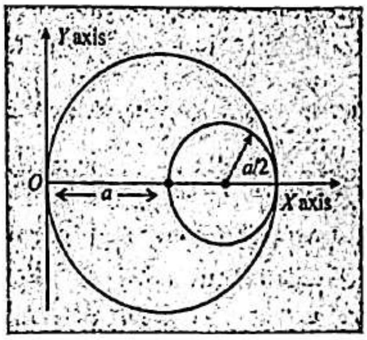
1. The identical spheres each of mass $2M$ are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 4 m each. Taking point of intersection of these two sides as origin, the magnitude of position vector of the centre of mass of the system is $\frac{4\sqrt{2}}{x}$, where the value of x is ____________ **[01 Feb, 2024 (Shift-I)]**

2. A uniform thin metal plate of mass 10 kg with dimensions as shown. The ratio of x and y coordinates of center of mass of plate is $n/9$. The value of n is______. **[08 April, 2024 (Shift-I)]**



3. The distance of centre of mass from end A of a one dimensional rod (AB) having mass density $\varrho = \varrho_0\left(1-\frac{x^2}{L^2}\right)$ kg/m and length L (in meter) is $\frac{3L}{\alpha}m$. The value of α is ______. (where x is the distance from end A) **[28 July, 2022 (Shift-II)]**

4. A circular hole of radius $\left(\frac{a}{2}\right)$ is cut of a circular disc of radius 'a' as shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be: **[24 Feb, 2021 (Shift-II)]**



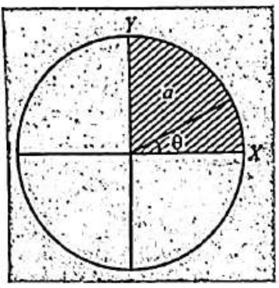
(a) $\frac{5}{6}a$
(b) $\frac{1}{6}a$
(c) $\frac{10}{11}a$
(d) $\frac{2}{3}a$

5. The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$ where x is __

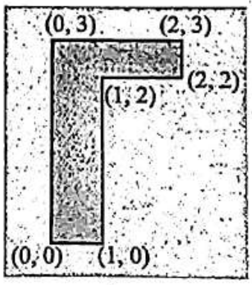
(Round off to the Nearest Integer)

[17 March, 2021 (Shift-II)]

[a is an area as shown in the figure]

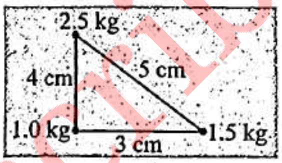


6. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4kg. (The coordinates of the same are shown in figure) are: **[08 Jan, 2020 (Shift-I)]**



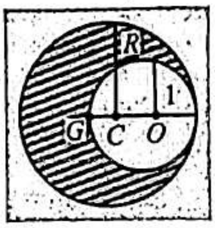
(a) 1.25 m, 1.50 m)
(b) (0.75 m, 0.75 m)
(c) (0.75 m, 1.75 m)
(d) (1 m, 1.75 m)

7. Three point particles of masses 10 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point: **[07 Jan, 2020 (Shift-I)]**



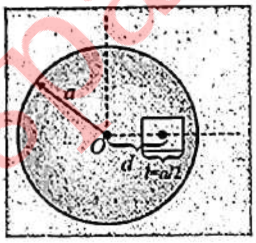
(a) 0.6 cm right and 2.0 cm above 1kg mass
(b) 2.0 cm right and 0.9 cm above 1 kg mass
(c) 0.9 cm right and 2.0 cm above 1kg mass
(d) 1.5 cm right and 1.2 cm above 1kg mass

8. As shown in figure. When a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e. on the surface of the cavity. R can be determined by the equation: **[08 Jan, 2020 (Shift-II)]**

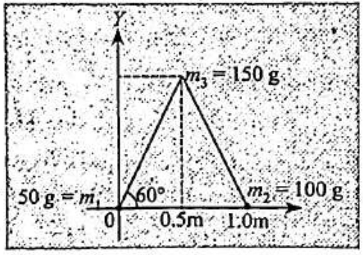


(a) $(R^2+R+1)(2-R)=1$ (b) $(R^2+R-1)(2-R)=1$
(c) $(R^2-ssR+1)(2-R)=1$ (d) $(R^2-R-1)(2-R)=1$

9. A square shaped hole of side $l=\frac{a}{2}$ is carved out at a distance $d=\frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{x}$, value of x (to the nearest integer) is ______. **[02 Sep, 2020 (Shift-II)]**

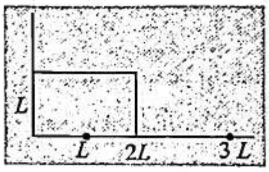


10. Three particles of masses 50 g, 100g and 150g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be: **[12 April, 2019 (Shift-II)]**



(a) $\left(\frac{\sqrt{3}}{7}m, \frac{7}{12}m\right)$
(b) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{8}m\right)$
(c) $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$
(d) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$

11. The position vector of the centre of mass $\vec{r}$ cm of an asymmetric uniform bar of negligible area of cross-section as shown in figure is: **[12 Jan, 2019 (Shift-I)]**



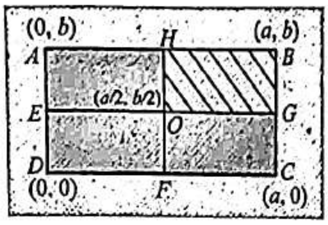
(a) $\vec{r}\text{ (cm)} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$

(b) $\vec{r}\text{ (cm)} = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$

(c) $\vec{r}\text{ (cm)} = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$

(d) $\vec{r}\text{ (cm)} = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$

12. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut off, the coordinates of the centre of mass of the remaining portion will be: **[08 April, 2019 (Shift-II)]**



(a) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (b) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

(c) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (d) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

DISPLACEMENT, VELOCITY AND ACCELERATION OF COM

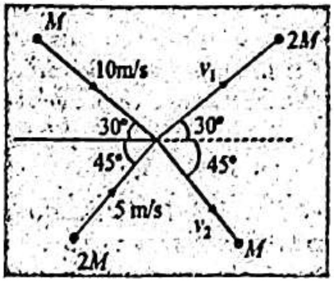
13. In a system two particles of masses $m_1 = 3\text{kg}$ and $m_2 = 2\text{kg}$ are placed at certain distance from each other. The particle of mass m_1 is moved towards the center of mass of the system through a distance 2 cm. In order to keep the center of mass of the system at the original position, the particle of mass m_2 should move towards the center of mass by the distance ____ cm. **[04 April, 2024]**

14. Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-3\hat{i} - 2\hat{j} + \hat{k}$ respectively. The magnitude of position vector of centre of mass of this system will be similar to the magnitude of vector: **[29 July, 2022 (Shift-I)]**

(a) $\hat{i} - 2\hat{j} + \hat{k}$ (b) $-3\hat{i} - 2\hat{j} + \hat{k}$

(c) $-2\hat{i} + 2\hat{k}$ (d) $-2\hat{i} - \hat{j} + \hat{k}$

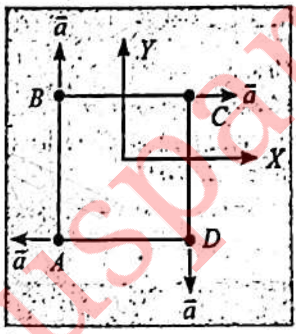
15. Two particles, of masses M and $2M$, moving as shown, with speeds of 10 m/s and 5 m/s collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2 respectively. The value of v_1 and v_2 are nearly: **[10 April, 2019 (Shift-I)]**



(a) 3.2 m/s and 6.3 m/s (b) 3.2 m/s and 12.6 m/s

(c) 6.5 m/s and 6.3 m/s (d) 6.5 m/s and 3.2 m/s

16. Four particles A, B, C and D with masses mA = m, mB = 2m, mC = 3m and mD = 4m are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is: **[08 April, 2019 (Shift-I)]**



(a) $\frac{a}{5}(\hat{i} - \hat{j})$ (b) Zero

(c) $\frac{a}{5}(\hat{i} \times \hat{j})$ (d) $a(\hat{i} + \hat{j})$

17. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is: **[12 April, 2019 (Shift-I)]**

(a) 0.20 ms^{-1} (b) 0.14 ms^{-1}

(c) 0.47 ms^{-1} (d) 0.28 ms^{-1}

IMPULSE, CONSERVATION OF MOMENTUM

18. A body of mass 1000 kg is moving horizontally with a velocity 6 m/s. If 200 kg extra mass is added, the final velocity (in m/s) is: **[27 Jan, 2024 (Shift-I)]**

(a) 6 (b) 2 (c) 3 (d) 5

19. A stationary particle breaks into two parts of masses m_A and m_B which move with velocities v_A and v_B respectively. The ratio of their kinetic energies $(K_B : K_A)$ is: **[08 April, 2024 (Shift-I)]**

(a) $v_B : v_A$ (b) $m_B : m_A$

(c) $m_B v_B : m_A v_A$ (d) 1 : 1

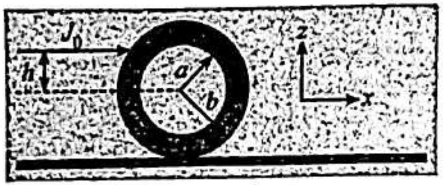
20. A block moving horizontally on a smooth surface with a speed of 40 m/s splits into two parts with masses in the ratio of 1:2. If the smaller part moves at 60 m/s in the same direction, then the fractional change in kinetic energy is:

[31 Aug, 2021 (Shift-II)]

(*a*) 2/3 (*b*) 1/3 (*c*) 1/4 (*d*) 1/8

21. An annular disk of mass M, inner radius a and outer radius b is placed on a horizontal surface with coefficient of friction μ, as shown in the figure. At some time, an impulse $J_0\hat{x}$ is applied at a height h above the center of the disk. If $h = h_m$ then the disk rolls without slipping along the x-axis. Which of the following statement(*s*) is(are) correct?

[JEE Adv, 2023]



(*a*) For $\mu \neq 0$ and $a \to 0$, $h_m = b/2$

(*b*) For $\mu \neq 0$ and $a \to b$, $h_m = b$

(*c*) For $h = h_m$, the initial angular velocity does **not** depend on the inner radius a.

(*d*) For $\mu = 0$ and $h = 0$, the wheel always slides without rolling.

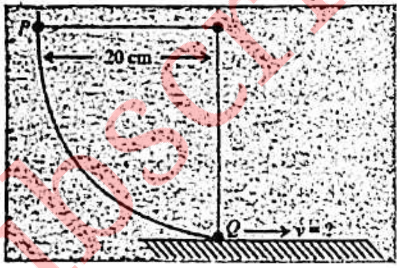
COLLISIONS (OBLIQUE AND HEAD ON)

22. A body starts falling freely from height H hits an inclined plane in its path at height h. As a result of this perfectly elastic impact, the direction of the velocity of the body becomes horizontal. The value of $\frac{H}{h}$ for which the body will take the maximum time to reach the ground is ____.

[31 Jan, 2024 (Shift-I)]

23. As per the given figure, a small ball P slides down the quadrant of a circle and hits the other ball Q of equal mass which is initially at rest. Neglecting the effect of friction and assume the collision to be elastic, the velocity of ball Q after collision will be: ($g = 10\ m/s^2$)

[30 Jan, 2023 (Shift-I)]

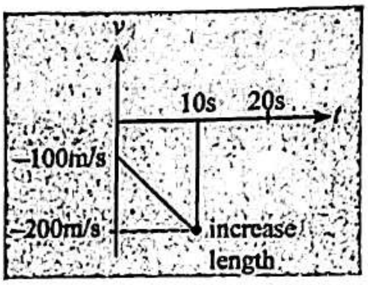


(*a*) 0 (*b*) 0.25 m/s (*c*) 2 m/s (*d*) 4 m/s

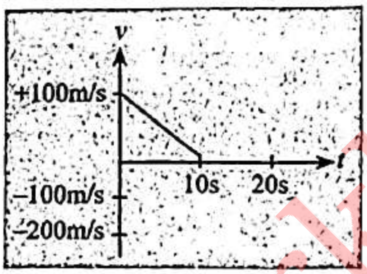
24. A bullet is shot vertically downwards with an initial velocity of 100 m/s from a certain height. Within 10s, the bullet reaches the ground and instantaneously comes to rest due to the perfectly inelastic collision. The velocity -time curve for total time $t = 20$ s will be:
(Take $g = 10 m/s^2$).

[27 July, 2022 (Shift-I)]

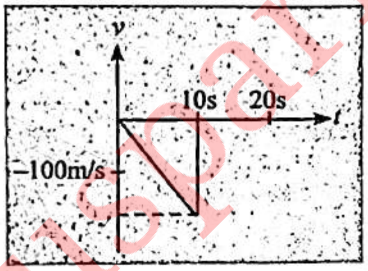
(*a*)



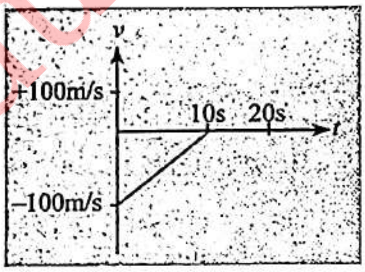
(*b*)



(*c*)



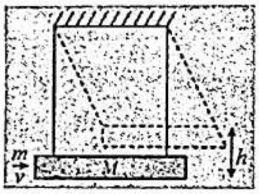
(*d*)



25. What percentage of kinetic energy of a moving particle is transferred to a stationary particle when it strikes the stationary particle of 5 times its mass? (Assume the collision to be head-on elastic collision) **[27 June, 2022 (Shift-I)]**

(*a*) 50.0 % (*b*) 66.6 % (*c*) 55.5 % (*d*) 33.3 %

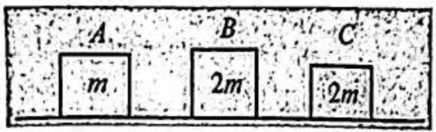
26. A large block of wood of mass $M = 5.99$ kg is hanging from two long massless cords. A bullet of mass $m = 10$ g is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance $h = 9.8$ cm before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is:
(take $g = 9.8\ ms^{-2}$) **[16 March, 2021 (Shift-II)]**



(*a*) 831.5 m/s (*b*) 811.5 m/s
(*c*) 841.5 m/s (*d*) 821.5 m/s

27. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. The masses of A, B and C are m, 2 m and 2 m respectively. A moves towards B with a speed of 9 m/s and makes an elastic collision with it. Thereafter B makes a completely inelastic collision with C. All motions occur along same straight line. The final speed of C is:

[27 July, 2021 (Shift-I)]

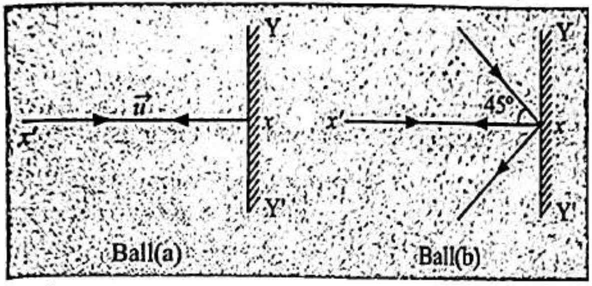


(*a*) 4 m/s (*b*) 6 m/s

(*c*) 9 m/s (*d*) 3 m/s

28. Two billiard balls of equal mass 30g strike a rigid wall with same speed of 108 kmph (as shown) but at different angles. If the balls get reflected with the same speed then the ratio of the magnitude of impulses imparted to ball '*a*' and ball '*b*' by the wall along '*X*' direction is:

[25 July, 2021 (Shift-I)]



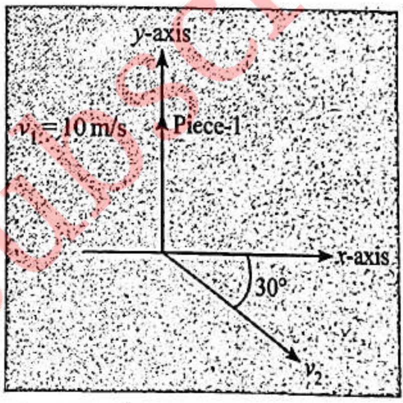
(*a*) $\sqrt{2}:1$ (*b*) $1:\sqrt{2}$

(*c*) 1 : 1 (*d*) 2 : 1

29. A ball of mass 10kg, moving with a velocity $10\sqrt{3}$m/s along the x-axis, hits another ball of mass 20kg which is at rest. After the collision, first ball comes to rest while second ball disintegrates into two equal pieces. One piece starts moving along y–axis with a speed of 10 m/s. The second piece starts moving at an angle of 30° with respect to the x-axis.

The velocity of the ball moving at 30° with x-axis is x m/s.

The configuration of pieces after collision is shown in the figure below.



The value of x to the nearest integer is _______.

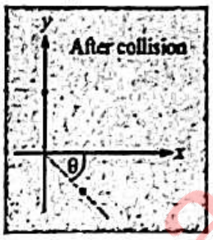
[18 March, 2021 (Shift-I)]

30. A ball of mass 10 kg moving with a velocity $10\sqrt{3}\text{ms}^{-1}$ along X–axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y–axis at a speed of 10m/s. The second piece starts moving at a speed of 20m/s at an angle θ (degree) with respect to the X–axis.

The configuration of pieces after collision is shown in the figure.

The value of θ to the nearest integer is _______.

[16 March, 2021 (Shift-I)]



31. Three particles P, Q and R are moving along the vectors $\vec{A}=\hat{i}+\hat{j}, \vec{B}=\hat{j}+\hat{k}$ and $\vec{C}=-\hat{i}+\hat{j}$ respectively. They strike on point and start to move in different directions. Now particle P is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of x is ________.

[22 July, 2021 (Shift-II)]

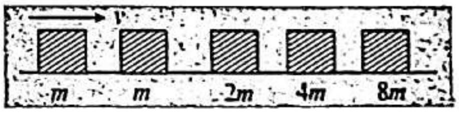
32. A block of mass 1.9 kg is at rest at the edge of a table, of height 1m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take $g = 10$ m/s^2. Assume there is no rotational motion and loss of energy after the collision is negligible.]

[03 Sep, 2020 (Shift-II)]

(a) 19 J (*b*) 23 J (c) 20 J (*d*) 21 J

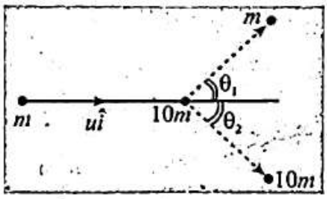
33. Blocks of masses m, $2m$, $4m$ and $8m$ are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass $8m$ starts moving, the total energy loss is p% of the original energy. Value of 'p' is close to

[04 Sep, 2020 (Shift-I)]

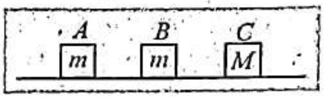


(*a*) 37 (*b*) 77 (c) 87 (*d*) 94

34. A particle of mass m is moving along the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass $10m$ at rest and then moves with half its initial kinetic energy(see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$, then value of n is ________. **[02 Sep, 2020 (Shift-II)]**

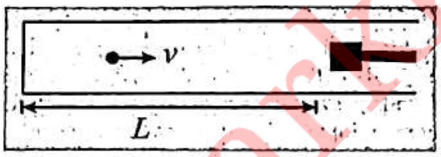


35. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically if $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m? **[09 Jan, 2019 (Shift-I)]**



(*a*) 5
(*b*) 2
(*c*) 4
(*d*) 3

36. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$, the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L}v_0$ where dL is the infinitesimal displacement of the piston. Which of the following statement (s) is/are correct? **[JEE Adv, 2019]**



(*a*) The rate at which the particle strikes the piston is v/L

(*b*) After each collision with the piston, the particle speed increases by $2V$

(*c*) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

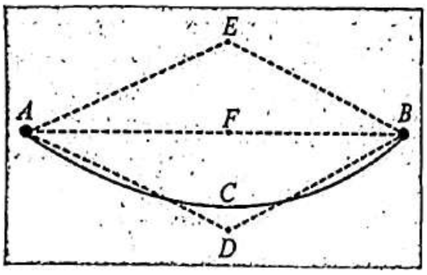
(*d*) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

THE 'DISTRACTION JOURNAL'

- During study sessions, write down every distraction or thought that pulls you away. Review it later to find patterns.
- This activity will help you identify and minimize distractions.

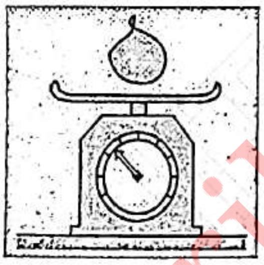
PW Challengers

1. A uniform rope suspended between two level nails A and B assume a curved shape ABC as shown in the figure. The midpoint C of the rope is pulled vertically down to a position D to make a V shape and released. When the rope again acquires the curved shape ACB, the midpoint is pulled vertically up to a position E to make an inverted V shape. If in these two pulling processes, total work done by the pulling agency is W, how much work will be done by the pulling agency in lifting vertically the midpoint from equilibrium to a position F in level with the nails?



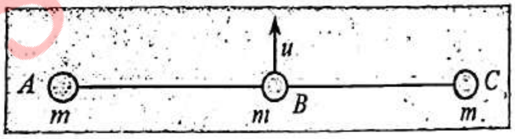
(*a*) $\frac{W}{2}$ (*b*) $\frac{2W}{5}$ (*c*) $\frac{W}{3}$ (*d*) $\frac{W}{4}$

2. When a flexible bag filled with sand is dropped on the pan of a spring balance from a height of $h = 4$ cm above the pan, pointer of the balance suddenly shoots to a reading $m_1 = 6.0$ kg and after several oscillations finally settles to a reading $m_2 = 2.0$ kg. If force constant of the spring is $k = 1.5$ kN/m and dry friction is absent, find mass of the pan. Assume acceleration of free fall $g = 10$ m/s^2.



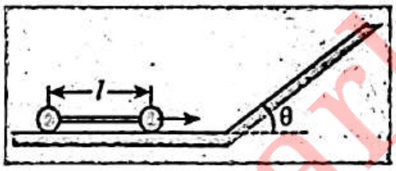
(*a*) 1 kg (*b*) 1.5 kg (*c*) 2 kg (*d*) 2.5 kg

3. A disc of mass m is connected with two identical discs each of mass m with the help of two light inextensible threads each of lengths l. Initially the system rests on a frictionless horizontal floor with centres of the discs in a straight line and the threads straight. Now the middle disc is projected along the floor with a velocity u perpendicular to the threads. Find tensile forces in the threads when the end discs are about to collide.



(*a*) $\frac{mu^2}{3l}$ (*b*) $\frac{mu^2}{6l}$ (*c*) $\frac{mu^2}{9l}$ (*d*) $\frac{2mu^2}{3l}$

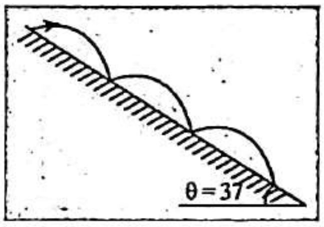
4. A toy similar to letter H in shape is made by fixing two identical cylinders at the ends of a light rod of length l. The toy is placed on a frictionless horizontal floor, which terminates into a uniform frictionless slope of inclination θ. Cylinders are parallel to the line joining the floor and the slope. The velocity given to the toy towards the slope so that it can rise completely on the slope. Collisions of the cylinders with the slope are perfectly inelastic. The minimum velocity that should be given to the toy towards the slope so that it can rise completely on the slope is



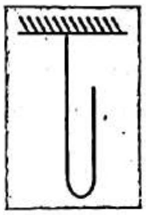
(*a*) $\sqrt{\frac{gl\sin\theta}{1+\cos^2\theta}}$ (*b*) $\sqrt{\frac{2gl\sin\theta}{1+\cos^2\theta}}$

(*c*) $\sqrt{\frac{2gl\sin\theta}{\cos^2\theta}}$ (*d*) $\sqrt{\frac{gl\sin\theta(1+\cos^2\theta)}{2\cos^2\theta}}$

5. A ball is projected horizontally from a large inclined plane which is inclined at 37° with the horizontal. The ball follows identical trajectories between every two successive bounces. If e is the coefficient of restitution for collisions and μ the coefficient of friction between the plane and the ball then $\frac{\mu}{e}$ is $\frac{n}{100}$. Find n.

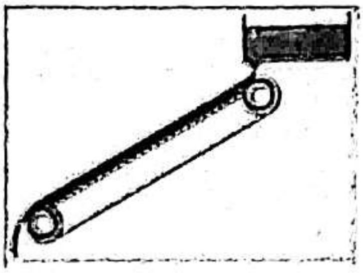


6. Ends of a thin uniform inextensible rope of mass m and length l are fastened to the ceiling close to each other. If one of the ends is released and allowed to fall freely as shown in the figure, find the largest force that the fastening at the other end must be capable of bearing so that rope remains fastened there.



(*a*) mg (*b*) $\frac{3mg}{2}$ (*c*) $2mg$ (*d*) $3mg$

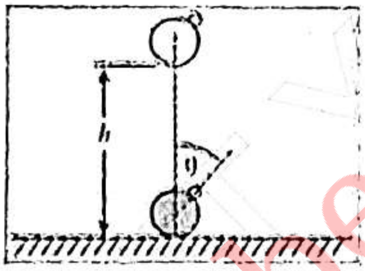
7. A conveyor belt is inclined at an angle θ with the horizontal. Length of a straight portion of the belt is l, resistive forces in the belt mechanism are negligible and radii of the end cylinders are negligible as compared to the length l. Sand falls on the belt with a negligible speed at a constant rate at the top and leaves at the bottom. Friction between the belt and the sand is high enough to stop sliding of sand particles almost in no time. Inertia of the belt mechanism is negligible as compared to that of the total sand on it. Find the steady speed acquired by the belt. Acceleration due to gravity is g.



(a) $\sqrt{\frac{g}{2}l\sin\theta}$ (b) $\sqrt{gl\sin\theta}$

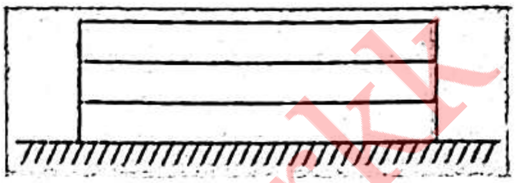
(c) $\sqrt{2gl\sin\theta}$ (d) $\sqrt{2gl\tan\theta}$

8. A heavy steel ball held in contact with an extremely light steel ball at a height $h = 5\sqrt{3}$ m above a hard horizontal floor. The line joining the centres of the balls makes angle $\theta = 30^\circ$ with the vertical. If all the collisions are elastic and sizes of the balls are negligible as compared to the height h, find the horizontal range of the lighter ball.



(a) 60m (b) 90m (c) 120m (d) 150m

9. Three identical long metal bars each of mass m are placed in a stack on a horizontal floor as shown in the figure. Surfaces of the plates as well as of the floor are lubricated with oil. Force $\vec{F}$ of viscous drag from lubricating oil on a surface is given by equation $\vec{F} = -k\vec{v}_{rel}$, where k is a positive constant and $\vec{v}_{rel}$ is velocity of the surface relative to the other. If the upper bar is given a velocity v towards the right by a sharp hit, find the displacement of upper bar when all of them stop.



(a) $\frac{mv}{k}$ (b) $\frac{2mv}{k}$

(c) $\frac{3mu}{k}$ (d) $\frac{4mu}{k}$

10. In rocket propulsion, kinetic energy imparted to the burnt fuel as ejected gases is wasted as far as propelling is concerned. If at an instant v is the speed of the rocket and u is the speed of the ejected gases relative to the rocket, develop suitable expression for efficiency of propulsion.

(a) $\frac{v^2}{2(x^2+v^2)}$

(b) $\frac{uv}{2(u^2+v^2)}$

(c) $\frac{2uv}{u^2+v^2}$

(d) $\frac{uv}{u^2+v^2}$

> "Concentrate all your thoughts upon the work at hand. The sun's rays do not burn until brought to a focus."
>
> **– Alexander Graham Bell**

ANSWER KEY

CONCEPT APPLICATION

1. (d) 2. (b) 3. (c) 4. (d) 5. (d) 6. (b) 7. (a) 8. (a) 9. (d) 10. (b)
11. (c) 12. (b) 13. (b) 14. (c) 15. (d) 16. (a) 17. (a, d) 18. (a) 19. (d) 20. (c)
21. [illegible] 22. (b) 23. (c) 24. (b) 25. (c) 26. (d) 27. [illegible] 28. [illegible] 29. [illegible]
30. (d) 31. [3]

SCHOOL LEVEL PROBLEMS

1. (c) 2. (c) 3. (b) 4. (d) 5. (d) 6. (b) 7. (a) 8. (b) 9. (d) 10. (d)
11. (d) 12. (c) 24. [illegible] 25. [illegible]

PRARAMBH (TOPICWISE)

1. (d) 2. (d) 3. (b) 4. (b) 5. (d) 6. (c) 7. (c) 8. (b) 9. (c) 10. (b)
11. (c) 12. (c) 13. (d) 14. (d) 15. (c) 16. (c) 17. (d) 18. (b) 19. (a) 20. (c)
21. (b) 22. (d) 23. (c) 24. (d) 25. (c) 26. (a) 27. (d) 28. (a) 29. (b) 30. (a)
31. (c) 32. (c) 33. (c) 34. (b) 35. (c) 36. (d) 37. (d) 38. (b) 39. (d) 40. (b)
41. (c) 42. (d) 43. (b) 44. (d) 45. (d) 46. (b) 47. (b)

PRABAL (JEE MAIN LEVEL)

1. (c) 2. (b) 3. (b) 4. (c) 5. (a) 6. (c) 7. (b) 8. (a) 9. (c) 10. (b)
11. (d) 12. (a) 13. (b) 14. (b) 15. (b) 16. (c) 17. (a) 18. (c) 19. (c) 20. (c)
21. (c) 22. (a) 23. (b) 24. (a) 25. (a) 26. (d) 27. (b) 28. (b) 29. [2] 30. [1]
31. [illegible] 32. [250] 33. [4] 34. [illegible] 35. [5]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (b, d) 2. (b, c) 3. (b, d) 4. (b, d) 5. (a, d) 6. (a, d) 7. (a, c) 8. (a, c) 9. (b, d) 10. (a, d)
11. (a, b, d) 12. (a, b, c, d) 13. (b) 14. (b, d) 15. (c) 16. (c) 17. (b, c) 18. (b, d) 19. (a, b, c, d) 20. (a)
21. (d) 22. (a) 23. (b) 24. (c) 25. (b) 26. (b) 27. (a) 28. [1] 29. [2] 30. [5]
31. [illegible] 32. [125] 33. [0.020] 34. [5] 35. [10.00] 36. [25.00] 37. [2000.00] 38. [19.00]
39. [illegible] 40. [1.75] 41. [650.00] 42. [1.80] 43. [2.00] 44. (d) 45. (c) 46. (d) 47. (c)
48. (c) 49. (a) 50. (b) 51. (c) 52. (a) 53. (c)

PYQ's (PAST YEAR QUESTIONS)

1. [5] 2. [15] 3. [4] 4. (a) 5. [4] 6. (c) 7. (c) 8. (a) 9. [22] 10. (d)
11. (c) 12. (d) 13. [3] 14. (d) 15. (c) 16. (a) 17. (a) 18. (d) 19. (a) 20. (d)
21. (a, b, c, d) 22. [2] 23. (c) 24. (a) 25. (c) 26. (a) 27. (d) 28. (a) 29. [20] 30. [30]
31. [5] 32. (d) 33. (d) 34. [10] 35. (c) 36. (b, c)

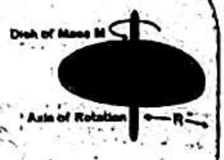
PW CHALLENGERS

1. (d) 2. (c) 3. (c) 4. (d) 5. [75] 6. (c) 7. (b) 8. (c) 9. (c) 10. (c)

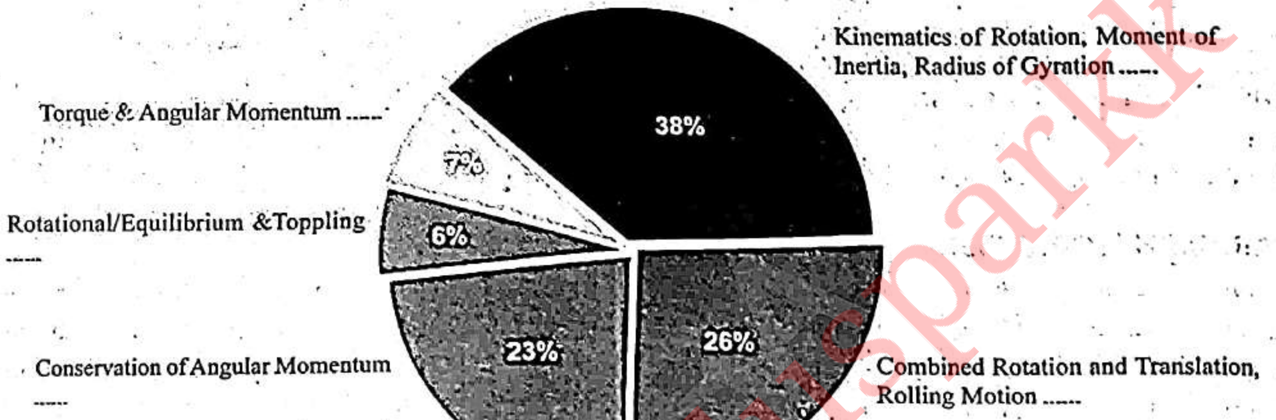
CHAPTER

8

Rotational Motion



Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

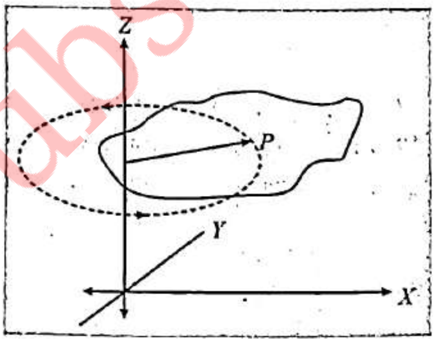
RIGID BODY

A system of particles is said to be a rigid body if distance between any two particles of the body remains constant no matter what force is applied on it. Remember, rigid body is an idealized concept. No body is perfectly rigid but many body come close to it. An iron rod, steel ball, stone etc, are almost rigid bodies.

Up until now, we dealt only with translatory motion of rigid bodies. A rigid body, however, can also rotate.

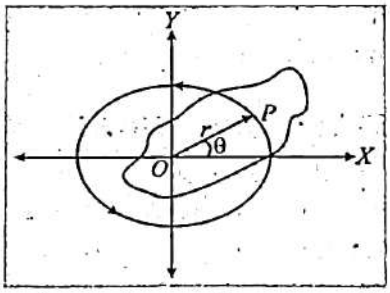
Kinematics of rotation about a fixed axis

We shall now consider rotational motion of a rigid body about an axis fixed in an inertial frame. (see Fig.)



Let P represent an arbitrary particle in the rigid body described by the position vector $\vec{r}$. A rigid body moves in pure rotation, if every particle of the body moves in a circle, the centers of which are on a straight line called the axis of rotation (the z-axis in this figure). If we draw a perpendicular from any point in the body to the axis, each such line will sweep through the same angle in any given time interval as another such line. Therefore, we can describe the pure rotation of rigid body by considering the motion of any one of the particles.

Let us pass a plane through P at right angles to the axis of rotation. The circle in which particle P moves lies on the X-Y plane (see Fig.)



From the discussion in the preceding paragraph it is clear that location of entire body is known if we know the location of single particle such as P. Thus for kinematical purposes we need to consider the circular motion of a single particle.

The angular displacement point of particle is θ and the angular velocity ω is given by $\omega = \frac{d\theta}{dt}$.

If the angular velocity is not constant then the particle will have angular acceleration denoted by α and is equal to $\alpha = \frac{d\omega}{dt}$.

The unit of angular velocity ω and angular acceleration α are rad/sec and rad/sec^2 respectively.

The rotation of a rigid body about a fixed axis has a direct correspondence to the translation motion of a particle along a fixed direction. The kinematics variable in the translation case are x, v and a while in case of rotation they are θ, ω and α. The correspondence is

$x \to \theta$

$v \to \omega$

$a \to \alpha$

If the rigid body has constant angular acceleration, we can derive the following formulae in direct analogy to translation motion.

$$\omega_f = \omega_i + \alpha t \qquad \text{... (i)}$$

$$\theta_f - \theta_i = \omega_i t + (1/2)\alpha t^2 \qquad \text{... (ii)}$$

$$\omega_f^2 - \omega_i^2 = 2\alpha\left(\theta_f - \theta_i\right) \qquad \text{... (iii)}$$

where the subscript i & f correspond to initial and final values.

The linear velocity of the particle P is ωr and its tangential and centripetal acceleration are αr and $\omega^2 r$ respectively.

The angular velocity and angular acceleration are actually vector quantities. We are able to treat them as scalar quantities because we consider rotation about a fixed axis of rotation. But if the axis of rotation is changing direction then we will have to take their vector nature into consideration.

The magnitude of angular velocity $\vec{\omega}$ is $d\theta/dt$ but its direction is along the axis of rotation. By convention, if the fingers of right hand curl around the axis of rotation in the direction of rotation of the body, the extended thumb points out in the direction of $\vec{\omega}$. In figure shown above the direction of $\vec{\omega}$ is in the +ve Z-direction. If the body was rotating in the clockwise direction then its direction will be along the –ve Z axis.

Notes: Nothing moves in the direction of $\vec{\omega}$ as opposed to the linear velocity $\vec{v}$ in translatory motion which points in the direction of motion.

The direction of angular acceleration $\vec{\alpha}$ can now be determined from $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$. In vector form, the following relations hold good for the motion of a particle in the body.

$$\vec{v} = \vec{\omega} \times \vec{r}$$

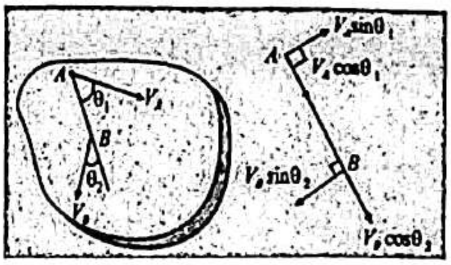
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$

$$= \vec{a}_t + \vec{a}_r$$

in which $\vec{a}_t = \vec{\alpha} \times \vec{r}$ and $\vec{a}_r = \vec{\omega} \times \vec{v}$ are tangential and radial accelerations of the particle

For every pair of particles in a rigid body, there is no velocity of separation or approach between the particles i.e. with respect to any particle A of a rigid body, the motion of any other particle B of that rigid body is in circular motion.

Let velocities of points A and B with respect to ground be $\vec{V}_A$ and $\vec{V}_B$ respectively in the figure below.



If the body shown in figure is rigid, $V_A \cos\theta_1 = V_B \cos\theta_2$

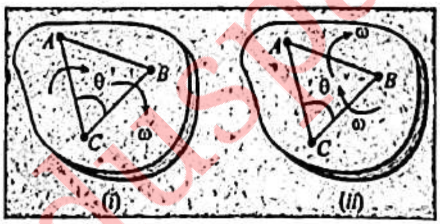
The relative velocity of B with respect to A is

$V_{BA} = V_A \sin\theta_1 + V_B \sin\theta_2$

B will appear to move in a circle to an observer fixed at A.

With respect to any point of the rigid body the angular velocity of all other points of that rigid body is same.

Suppose A, B, C is a rigid system hence during any motion sides AB, BC and CA must rotate through the same angle. Hence all the sides rotate by the same rate.

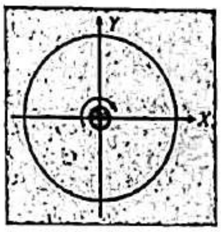


From figure (*i*) angular velocity of A and B w.r.t. C is ω

From figure (*ii*) angular velocity of A and C w.r.t. B is ω

Train Your Brain

Example 1: A wheel rotates at a rate of 100 rev/min in the clock wise direction (see Fig.). What is the magnitude and direction of the constant angular acceleration so that it stops in 10 seconds? How many revolution will it cover before stopping?



Sol. Let z-axis point out of the plane of the paper.

$$\omega_i = \frac{2\pi \times 100}{60} = \frac{10\pi}{3} \text{ rad / sec}$$

Curving our right hand fingers in the direction of motion of the body we find that thumb points in the –ve z-direction. Thus

$\vec{\omega}_i = -\frac{10\pi}{3}\hat{k}$ where $\hat{k}$ is the unit vector in the z-direction.

Here, $\vec{\omega}_f = 0, \Rightarrow \vec{\alpha} = \frac{\vec{\omega}_f - \vec{\omega}_i}{10} = \frac{0 - \left(-\frac{10\pi}{3}\right)\hat{k}}{10}$

$= \frac{\pi}{3}(\text{rad / s}^2)\hat{k}$

So, α points out of the plane of the paper and its magnitude is $\frac{\pi}{3}$ rad / s^2

Angular displacement θ is given by the eqn.

$$\omega^2_f - \omega^2_i = 2\alpha\theta \Rightarrow \theta = \frac{0-\left(\frac{10\pi}{3}\right)^2}{2\left(\frac{\pi}{3}\right)} = \frac{50\pi}{3}\text{ rad}$$

Number of revolutions $= \frac{50\pi}{3\times 2\pi} = \frac{50}{6}$, clock wise.

Example 2: A turn table of radius 50 cm starts from rest and accelerates with constant angular acceleration of 1.0 rad/s^2. Compute the tangential, radial and resultant acceleration of a point on the rim after $t = 2$ sec.

Sol. We have $\alpha = 1.0$ rad/s^2, $\omega_i = 0$, $r = 0.5$ m

After $t = 2$ sec, $\omega = \omega_i + 2\alpha = 2$ rad/s

Tangential acc $= a_t = \alpha r = (1.0)(0.5) = 0.5$ m/s^2

Radial acc $a_r = \omega^2 r = (4.0)(0.5) = 2.0$ m/s^2

Since, a_t and a_r are perpendicular to each other, the resultant acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{2^2 + 0.5^2} = \frac{\sqrt{17}}{2}\text{ m/s}^2$$

Concept Application

1. A disc of radius 10 cm is rotating about its axis at an angular speed of 20 rad/s. Find the linear speed (m/s) of
   (*a*) a point on the rim
   (*b*) the middle point of radius
2. A disc rotates about its axis with a constant angular acceleration of 4 rad/s^2. Find the radial (cm/s^2) and tangential accelerations (cm/s^2) of a particle at a distance of 1 cm from the axis at the end of the first second after the disc starts rotating.
3. A wheel starting from rest is uniformly accelerated at 4 rad/s^2 for 10 seconds. It is allowed to rotate uniformly for the next 10 seconds and is finally brought to rest in the next 10 seconds. Find the total angle (rad) rotated by the wheel.

MOMENT OF INERTIA

Like the centre of mass, the moment of inertia is a property of an object that is related to its mass distribution. The moment of inertia (denoted by I) is an important quantity in the study of system of particles that are rotating. The role of the moment of inertia in the study of rotational motion is analogous to that of mass in the study of linear motion. Moment of inertia gives a measurement of the resistance of a body to a change in its rotational motion.

If a body is at rest, the larger the moment of inertia of a body the more difficult it is to put that body into rotational motion. Similarly, the larger the moment of inertia of a body, the more difficult to stop its rotational motion. The moment of inertia is calculated about some axis (usually the rotational axis).

Moment of Inertia Depends on:

(*i*) density of the material of body

(*ii*) shape & size of body

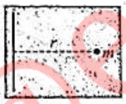
(*iii*) axis of rotation

In totality we can say that it depends upon distribution of mass relative to axis of rotation.

Note: Moment of inertia does not change if the mass is shifted such that its distance from axis of rotation does not change.

MOMENT OF INERTIA OF A SINGLE PARTICLE

For a very simple case the moment of inertia of a single particle about an axis is given by,



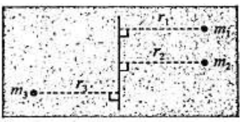
$$I = mr^2 \qquad \text{... }(i)$$

Here, m is the mass of the particle and r its distance from the axis under consideration.

MOMENT OF INERTIA OF A SYSTEM OF PARTICLES

The moment of inertia of a system of particles about an axis is given by,

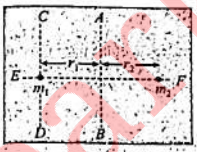
$$I = \sum_i m_i r_i^2 \qquad \text{... }(ii)$$



where r_i is the perpendicular distance of the i^{th} particle from the axis, which has a mass m_i.

Train Your Brain

Example 3: Two heavy particles having masses m_1 & m_2 are situated in a plane perpendicular to line AB at a distance of r_1 and r_2 respectively.



(*i*) What is the moment of inertia of the system about axis AB?

(*ii*) What is the moment of inertia of the system about an axis passing through m_1 and perpendicular to the line joining m_1 and m_2?

(*iii*) What is the moment of inertia of the system about an axis passing through m_1 and m_2?

Sol. (*i*) Moment of inertia of particle on left is $I_1 = m_1 r_1^2$.

Moment of Inertia of particle on right is $I_2 = m_2 r_2^2$.

Moment of Inertia of the system about AB is

$I = I_1 + I_2 = m_1 r_1^2 + m_2 r_2^2$

(*ii*) Moment of inertia of particle on left is $I_1 = 0$

Moment of Inertia of the system about CD is

$I = I_1 + I_2 = 0 + m_2(r_1 + r_2)^2$

(*iii*) Moment of inertia of particle on left is $I_1 = 0$

Moment of inertia of particle on right is $I_2 = 0$

Moment of Inertia of the system about EF is

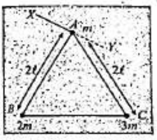
$I = I_1 + I_2 = 0 + 0$

Example 4: Three light rods, each of length 2ℓ, are joined together to form a triangle. Three particles A, B, C of masses m, $2m$, $3m$ are fixed to the vertices of the triangle. Find the moment of inertia of the resulting body about

(*a*) an axis through A and perpendicular to the plane ABC,

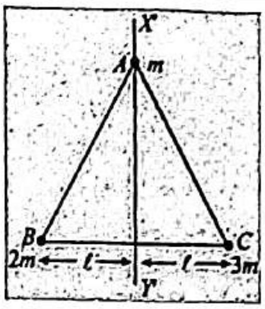
(*b*) an axis passing through A and the midpoint of BC.

Sol.

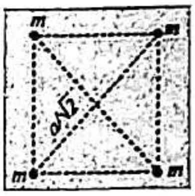


(*a*) B and C are at a distant 2ℓ from the axis XY so the moment of inertia about XY is $2m\,(2\ell)^2 + 3m\,(2\ell)^2 = 20\,m\ell^2$

(*b*) The moment of inertia of the body about $X'Y'$ is $m(0)^2 + 2m(\ell)^2 + 3m(\ell)^2 = 5\,m\ell^2$



Example 5: Four particles each of mass m are kept at the four corners of a square of edge length a. Find the moment of inertia of the system about a line perpendicular to the plane of the square and passing through the centre of the square.

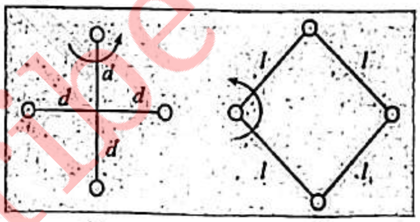


Sol. The perpendicular distance of every particle from the given line is $a/\sqrt{2}$. The moment of inertia of one particle is, therefore, $m(a/\sqrt{2})^2 = \frac{1}{2}ma^2$.

The moment of inertia of the system is, therefore, $4 \times \frac{1}{2}ma^2 = 2\,ma^2$.

Concept Application

4. Four particles with equal masses m are connected by rods of negligible mass. The moment of inertia, about the indicated axis, "(perpendicular to the plane)" of
   (*a*) the cross in figure (*a*) is xmd^2 then the value of x is
   (*b*) the square in figure (*b*) is xml^2 then the value of x is

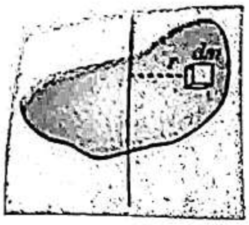


(*a*) (*b*)

5. Three particles of masses 1gm, 2gm & 3gm are at 1cm, 2cm, & 3cm from the axis of rotation respectively, then the moment of inertia of the system is _____ gm cm^2.
   (*a*) 63 (*b*) 60 (*c*) 39 (*d*) 36

MOMENT OF INERTIA OF RIGID BODIES

For a continuous mass distribution such as found in a rigid body, we replace the summation of $I = \sum_i m_i r_i^2$ by an integral. If the system is divided into infinitesimal element of mass dm and r is the perpendicular distance of a mass element from the axis of rotation, then the moment of inertia is,

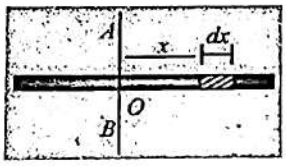


(A) Uniform rod about a perpendicular bisector

Consider a uniform rod of mass M and length l figure and suppose the moment of inertia is to be calculated about the bisector AB. Take the origin at the middle point O of the rod. Consider an element of the rod between a distance x and $x + dx$ from the origin. As the rod is uniform,

mass per unit length of the rod = M/l.

So that the mass of the element = $(M/l)dx$.



The perpendicular distance of the element from the line AB is x. The moment of inertia of this element about AB is

$$dI = \frac{M}{l}dx\, x^2.$$

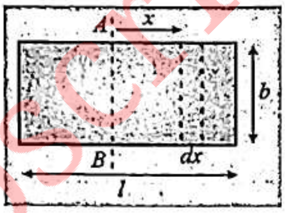
When $x = -l/2$, the element is at the left end of the rod. As x is changed from $-l/2$ to $l/2$, the elements cover the whole rod.

Thus, the moment of inertia of the entire rod about AB is

$$I = \int_{-l/2}^{l/2} \frac{M}{l}x^2 dx = \left[\frac{M}{l}\frac{x^3}{3}\right]_{-l/2}^{l/2} = \frac{Ml^2}{12}$$

(B) Moment of inertia of a rectangular plate about a line parallel to an edge and passing through the centre

The situation is shown in figure. Draw a line parallel to AB at a distance x from it and another at a distance $x + dx$. We can take the strip enclosed between the two lines as the small element.



It is "small" because the perpendiculars from different points of the strip to AB differ by not more than dx. As the plate is uniform,

its mass per unit area = $\frac{M}{bl}$

Mass of the strip = $\frac{M}{bl}b\,dx = \frac{M}{l}dx$.

The perpendicular distance of the strip from $AB = x$.

The moment of inertia of the strip about

$AB = dI = \frac{M}{l}dx\,x^2$. The moment of inertia of the given plate is, therefore,

$$I = \int_{-l/2}^{l/2} \frac{M}{l}x^2 dx = \frac{Ml^2}{12}$$

The moment of inertia of the plate about the line parallel to the other edge and passing through the centre may be obtained from the above formula by replacing l by b and thus,

$$I = \frac{Mb^2}{12}.$$

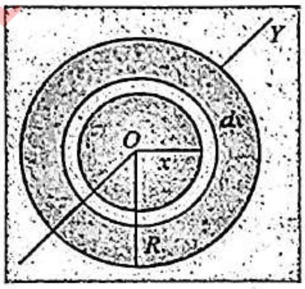
(C) Moment of inertia of a circular ring about its axis (the line perpendicular to the plane of the ring through its centre)

Suppose the radius of the ring is R and its mass is M. As all the elements of the ring are at the same perpendicular distance R from the axis, the moment of inertia of the ring is

$$I = \int r^2\, dm = \int R^2\, dm = R^2 \int dm = MR^2.$$

(D) Moment of inertia of a uniform circular plate about its axis

Let the mass of the plate be M and its radius R. The centre is at O and the axis OY is perpendicular to the plane of the plate.



Draw two concentric circles of radii x and $x + dx$, both centred at O and consider the area of the plate in between the two circles.

This part of the plate may be considered as a circular ring of radius x. As the periphery of the ring is $2\pi x$ and its width is dx, the area of this elementary ring is $2\pi x dx$. The area of the plate is πR^2. As the plate is uniform,

Its mass per unit area = $\frac{M}{\pi R^2}$

Mass of the ring $= \frac{M}{\pi R^2} 2\pi x\, dx = \frac{2Mx\,dx}{R^2}$

Using the result obtained above for a circular ring, the moment of inertia of the elementary ring about OY is

$$dI = \left[\frac{2Mx\,dx}{R^2}\right]x^2.$$

The moment of inertia of the plate about OY is

$$I = \int_0^R \frac{2M}{R^2}x^3\, dx = \frac{MR^2}{2}.$$

(E) Moment of inertia of a hollow cylinder about its axis

Suppose the radius of the cylinder is R and its mass is M. As every element of this cylinder is at the same perpendicular distance R from the axis, the moment of inertia of the hollow cylinder about its axis is

$$I = \int r^2\,dm = R^2 \int dm = MR^2$$

(F) Moment of inertia of a uniform hollow sphere about a diameter

Let M and R be the mass and the radius of the sphere, O its centre and OX the given axis (figure). The mass is uniformly spread over the surface of the sphere and the inside is hollow.

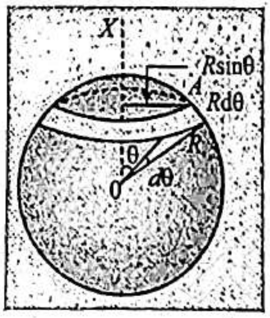
Let us consider a radius OA of the sphere at an angle θ with the axis OX and rotate this radius about OX. The point A traces a circle on the sphere. Now change θ to $\theta + d\theta$ and get another circle of somewhat larger radius on the sphere. The part of the sphere between these two circles, shown in the figure, forms a ring of radius $R\sin\theta$. The width of this ring is $Rd\theta$ and its periphery is $2\pi R\sin\theta$.

Hence, the area of the ring $= (2\pi R\sin\theta)\,(Rd\theta)$.

Mass per unit area of the sphere $= \dfrac{M}{4\pi R^2}$.

The mass of the ring

$$= \frac{M}{4\pi R^2}(2\pi R\sin\theta)(Rd\theta) = \frac{M}{2}\sin\theta\,d\theta.$$



The moment of inertia of this elemental ring about OX is

$$dI = \left(\frac{M}{2}\sin\theta\,d\theta\right)(R\sin\theta)^2 = \frac{M}{2}R^2\sin^3\theta\,d\theta$$

As θ increases from 0 to π, the elemental rings cover the whole spherical surface. The moment of inertia of the hollow sphere is, therefore,

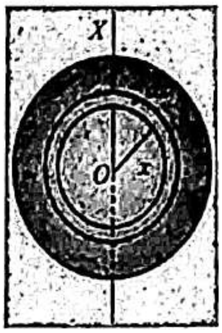
$$I = \int_0^{\pi} \frac{M}{2}R^2\sin^3\theta\,d\theta = \frac{MR^2}{2}\left[\int_0^{\pi}(1-\cos^2\theta)\sin\theta\,d\theta\right]$$

$$= \frac{MR^2}{2}\left[\int_{\theta=0}^{\pi} -(1-\cos^2\theta)\,d(\cos\theta)\right]$$

$$= \frac{-MR^2}{2}\left[\cos\theta - \frac{\cos^3\theta}{3}\right]_0^{\pi} = \frac{2}{3}MR^2$$

(G) Moment of inertia of a uniform solid sphere about a diameter

Let M and R be the mass and radius of the given solid sphere. Let O be centre and OX the given axis. Draw two spheres of radii x and $x + dx$ concentric with the given solid sphere. The thin spherical shell trapped between these spheres may be treated as a hollow sphere of radius x.



The mass per unit volume of the solid sphere

$$= \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3}$$

The thin hollow sphere considered above has a surface area $4\pi x^2$ and thickness dx. Its volume is $4\pi x^2\,dx$ and hence its mass is

$$= \left(\frac{3M}{4\pi R^3}\right)(4\pi x^2\,dx) = \frac{3M}{R^3}x^2\,dx$$

Its moment of inertia about the diameter OX is, therefore,

$$dI = \frac{2}{3}\left[\frac{3M}{R^3}x^2\,dx\right]x^2 = \frac{2M}{R^3}x^4\,dx$$

At $x = 0$, the shell is formed at the centre of the solid sphere. As x increases from 0 to R, the shells cover the whole solid sphere.

The moment of inertia of the solid sphere about OX is, therefore,

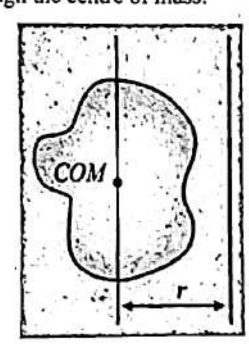
$$I = \int_0^R \frac{2M}{R^3}x^4\,dx = \frac{2}{5}MR^2.$$

THEOREMS OF MOMENT OF INERTIA

There are two important theorems on moment of inertia, which, in some cases enable the moment of inertia of a body to be determined about an axis, if its moment of inertia about some other axis is known. Let us now discuss both of them.

Theorem of Parallel axes

A very useful theorem, called the parallel axes theorem relates the moment of inertia of a rigid body about two parallel axes, one of which passes through the centre of mass.



Two such axes are shown in figure for a body of mass M. If r is the distance between the axes, I_{COM} and I are the respective moments of inertia about them, these moments are related by,

$$I = I_{COM} + Mr^2$$

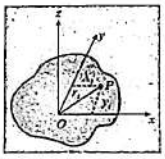
- Theorem of parallel axis is applicable for any type of rigid body whether it is a two dimensional or three dimensional

Theorem of Perpendicular Axis

The theorem states that the moment of inertia of a plane lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about two axes perpendicular to each other, in its own plane and intersecting each other, at the point where the perpendicular axis passes through it.

Let x and y axes be chosen in the plane of the body and z-axis perpendicular, to this plane, three axes being mutually perpendicular, then the theorem states that.

$$I_z = I_x + I_y$$



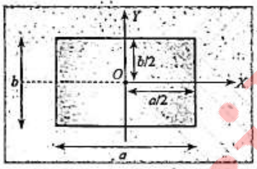
Important Point in Perpendicular Axis Theorem

(*i*) This theorem is applicable only for the plane bodies (two dimensional).

(*ii*) In theorem of perpendicular axes, all the three axes (x, y and z) intersect each other and this point may be any point on the plane of the body (it may even lie outside the body).

(*iii*) Intersection point may or may not be the centre of mass of the body.

Moment of inertia of a uniform rectangular plate about an axis passing through its center of mass and perpendicular to it. Consider a rectangular plate of length a and width b lying in the X-Y plane. We wish to find moment of inertia about z-axis.



$$I_X = \frac{Mb^2}{12}, I_Y = \frac{Ma^2}{12}$$

Applying perpendicular axis theorem, we get

$$I_z = I_X + I_Y$$

$$= \frac{M}{12}(a^2 + b^2)$$

RADIUS OF GYRATION

The radius of gyration is the perpendicular distance from axis of rotation of a body to a point where total mass of the body is supposed to be concentrated.

If I is the moment of inertia and K is radius of gyration, then

$$I = MK^2$$

I = Moment of Inertia of a body

M = Mass of a body

K = Radius of gyration

$$K = \sqrt{\frac{I}{M}}$$

Length K is the geometrical property of the body and axis of rotation.

S.I. Unit of K is metre.

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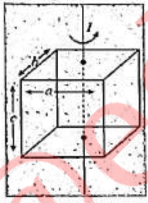
Example 6: Find the moment of Inertia of a cuboid along the axis as shown in the figure.

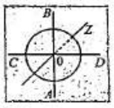


Sol. Moment of inertia will not change if we shift mass parallel to axis of rotation.

If we compress the cuboid parallel to the axis, it becomes a rectangular plate. Hence

$$I = \frac{M(a^2+b^2)}{12}$$

Example 7: Find the moment of inertia of uniform ring of mass M and radius R about a diameter.

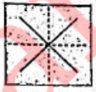


Sol. Let AB and CD be two mutually perpendicular diameters of the ring. Take them as X and Y-axis and the line perpendicular to the plane of the ring through the centre as the Z-axis. The moment of inertia of the ring about the Z-axis is $I = MR^2$. As the ring is uniform, $M.I.$ about all its diameters will be same and so $I_x = I_y$. From perpendicular axes theorem,

$$I_z = I_x + I_y. \text{ Hence } I_x = \frac{I_z}{2} = \frac{MR^2}{2}$$

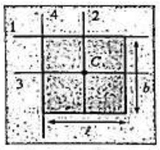
Similarly, the moment of inertia of a uniform disc about its diameter is $MR^2/4$.

Example 8: Two uniform identical rods each of mass M and length ℓ are joined perpendicularly to form a cross as shown in figure. Find the moment of inertia of the cross about a bisector as shown dotted in the figure.



Sol. Consider the line perpendicular to the plane of the figure through the centre of the cross. The moment of inertia of each rod about this line is $\frac{M\ell^2}{12}$ and hence the moment of inertia of the cross is $\frac{M\ell^2}{6}$. The moment of inertia of the cross about the two bisector are equal by symmetry and according to the theorem of perpendicular axes, the moment of inertia of the cross about the bisector will be $\frac{M\ell^2}{12}$.

Example 9: In the figure shown find moment of inertia of a plate having mass M, length ℓ and width b about axis 1, 2, 3 and 4. Assume that C is centre and mass is uniformly distributed



Sol. Moment of inertia of the plate about axis 1 (by taking rods perpendicular to axis 1)

$$I_1 = \frac{mb^2}{12} + m\left(\frac{b}{2}\right)^2 = \frac{mb^2}{3}$$

Moment of inertia of the plate about axis 2 (by taking rods perpendicular to axis 2)

$$I_2 = M\ell^2/12$$

Moment of inertia of the plate about axis 3 (by taking rods perpendicular to axis 3)

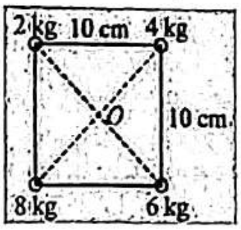
$$I_3 = \frac{Mb^2}{12}$$

Moment of inertia of the plate about axis 4 (by taking rods perpendicular to axis 4)

$$I_4 = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{3}$$

Concept Application

6. Four point masses 2 kg, 4 kg, 6 kg and 8 kg are placed at four corners of a square of side 10 cm. The radius of gyration of system about an axis passing through centre O and perpendicular to square plane is



(*a*) 10 cm　　(*b*) $10\sqrt{2}$ cm

(*c*) $\frac{10}{\sqrt{2}}$ cm　　(*d*) $20\sqrt{2}$ cm

7. Moment of inertia of a thin rod of mass M and length l about an axis passing through centre and inclined at an angle θ with the axis.

(*a*) $\frac{Ml^2}{12}\sin^2\theta$　　(*b*) $\frac{Ml^2}{3}\sin^2\theta$

(*c*) $\frac{Ml^2}{12}\cos^2\theta$　　(*d*) $\frac{Ml^2}{3}\cos^2\theta$

8. I_1, I_2 are moments of inertia of two solid spheres of same mass about axes passing through their centres If first is made of wood and the second is made of steel, then

(*a*) $I_2 = I_2$　　(*b*) $I_1 < I_2$

(*c*) $I_1 > I_2$　　(*d*) $I_1 \leq I_2$

9. The radius of gyration of a rotating uniform metallic disc is independent of the following physical quantity.

(*a*) Position of axis of rotation

(*b*) Mass of disc

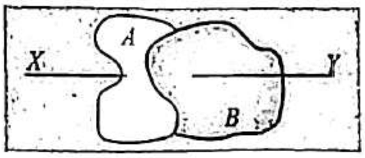
(*c*) Radius of disc

(*d*) Temperature of disc

MOMENT OF INERTIA OF COMPOUND BODIES

Consider two bodies A and B, rigidly joined together. The moment of inertia of the compound body, about an axis XY, is required. If I_A is the moment of inertia of body A about XY. I_B is the moment of inertia of body B about XY. Then, moment of Inertia of compound body about an axis XY is $I = I_A + I_B$

Extending this argument to cover any number of bodies rigidly joined together, we see that the moment of inertia of the compound body, about a specified axis, is the sum of the moments of inertia of the separate parts of the body about the same axis.



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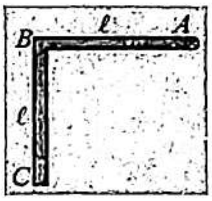


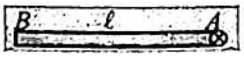
Example 10: Two rods each having length ℓ and mass m joined together at point B as shown in figure. Then find out moment of inertia about an axis passing through A and perpendicular to the plane of page as shown in figure.

Sol. We find the resultant moment of inertia I by dividing in two parts such as

I = M.I of rod AB about A + M.I of rod BC about A

$$I = I_1 + I_2 \quad \ldots (i)$$

first calculate I_1:



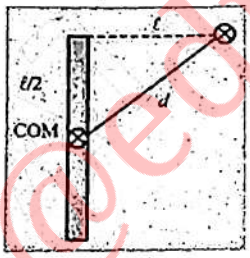
$$I_1 = \frac{m\ell^2}{3} \quad \ldots (ii)$$

Calculation of I_2:

Use parallel axis theorem

$$I_2 = I_{CM} + md^2$$

$$\frac{m\ell^2}{12} + m\left(\frac{\ell^2}{4} + \ell^2\right)$$



$$= \frac{m\ell^2}{12} + \frac{5\ell^2}{4}m \quad \ldots (iii)$$

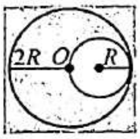
Put value from eq. (*ii*) & (*iii*) into (*i*), we get

$$\Rightarrow I = \frac{m\ell^2}{3} + \frac{m\ell^2}{12} + \frac{5\ell^2 m}{4}$$

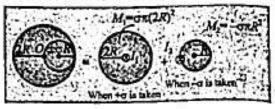
$$I = \frac{m\ell^2}{12}(4+1+15)$$

$$\Rightarrow I = \frac{5m\ell^2}{3}$$

Example 11: A uniform disc having radius $2R$ and mass density σ as shown in figure. If a small disc of radius R is cut from the disc as shown. Then find out the moment of inertia of remaining disc around the axis that passes through O and is perpendicular to the plane of the page.



Sol. We assume that in remaining part a disc of radius R and mass density $\pm\sigma$ is placed. Then



Total Moment of Inertia $I=I_1+I_2$

$$I_1=\frac{M_1(2R)^2}{2}$$

$$I_1=\frac{\sigma\pi 4R^2.4R^2}{2}=8\,\pi\,\sigma\,R^4$$

To calculate I_2 we use parallel axis theorem.

$$I_2=I_{CM}+M_2R^2$$

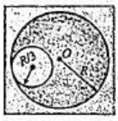
$$I_2=\frac{M_2R^2}{2}+M_2R^2$$

$$I_2=\frac{3}{2}M_2R^2=\frac{3}{2}(-\sigma\pi R^2)R^2$$

Now $I=I_1+I_2$

$$I=8\pi\sigma R^4-\frac{3}{2}\sigma\pi R^4\,;\ I=\frac{13}{2}\sigma\pi R^4$$

Example 12: A uniform disc of radius R has a round disc of radius $R/3$ cut as shown in Fig. The mass of the remaining (shaded) portion of the disc equals M. Find the moment of inertia of such a disc relative to the axis passing through geometrical centre of original disc and perpendicular to the plane of the disc.



Sol. Let the mass per unit area of the material of disc be σ. Now the empty space can be considered as having density $-\sigma$ and σ.

Now $I_0=I_\sigma+I_{-\sigma}$

$I_\sigma=(\sigma\pi R^2)R^2/2=M.I$ of σ about O

$$I_{-\sigma}=\frac{-\sigma\pi(R/3)^2(R/3)^2}{2}+[-\sigma\pi(R/3)^2](2R/3)^2$$

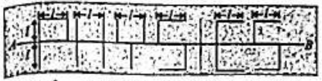
$=M.I$ of $-\sigma$ about O

$$\therefore\ I_0=\frac{4}{9}\sigma\pi R^4$$

where $\sigma=\dfrac{M}{\pi R^2-\pi\left(\dfrac{R^2}{9}\right)}$; $\sigma=\dfrac{9M}{8\pi R^2}$

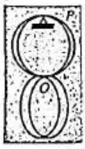
Concept Application

10. Find out the moment of inertia of the following structure (written as Physics) about axis AB made of thin uniform rods of mass per unit length λ.



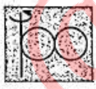
(a) $5\lambda l^3$ (b) $13\lambda l^3$ (c) $10\lambda l^3$ (d) $12\lambda l^3$

11. Two identical rings each of mass m with their planes mutually perpendicular, radius R are welded at their point of contact O. If the system is free to rotate about an axis passing through the point P perpendicular to the plane of the paper, then the moment of inertia of the system about this axis is equal to



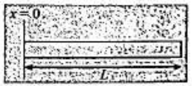
(a) $6.5\,mR^2$ (b) $12\,mR^2$
(c) $6\,mR^2$ (d) $11.5\,mR^2$

12. Two identical hollow spheres of mass M and radius R are joined together, and the combination is rotated about an axis tangential to one sphere and perpendicular to the line connecting them. The rotational inertia of the combination is



(a) $10\,MR^2$ (b) $\frac{4}{3}MR^2$
(c) $\frac{32}{3}MR^2$ (d) $\frac{34}{3}MR^2$

13. Mass M is distributed over the rod of length L. If linear mass density (λ) of the rod is linearly increasing with length as $\lambda=Kx$, where 'x' is measured from one end as shown in figure and K is constant, then $M.I$ of the rod about an axis passing through the end and perpendicular to rod where, linear mass density is zero.



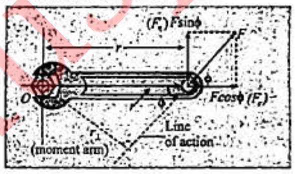
(a) $\frac{ML^2}{3}$ (b) $\frac{ML^2}{12}$ (c) $\frac{2}{3}ML^2$ (d) $\frac{ML^2}{2}$

MOMENT OF FORCE OR TORQUE

Rotational Analogue of Force

Let us take the example of opening or closing of a door. A door is a rigid body which can rotate about a fixed vertical axis passing through the hinges. What makes the door rotate? It is clear that unless a force is applied the door does not rotate. But any force does not do the job. A force applied to the hinge line cannot produce any rotation at all, whereas a force of given magnitude applied at right angles to the door at its outer edge is most effective in producing rotation. It is not the force alone, but how and where the force is applied is important in rotational motion. The quantitative measure of the tendency of a force to cause or change the rotational motion of a body is called torque. Consider an example to understand this.

In the figure below, the wrench is trying to open the nut. Now the ability of wrench to open the nut will depend not only on the applied force, but the distance at which force is applied. This gives birth to a new physical quantity called torque.



If only radial force F_r were present, the nut could not be turned. Thus the force causing the rotation is tangential force F_T only. The magnitude of the torque about an axis due to a force is given by

τ = (Force causing the rotation) × (distance of point of application of force from the axis)

$=(F\sin\phi)\,r$

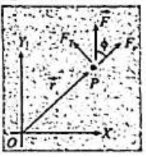
we may also write $\tau=F(r\sin\phi)=F(r_\perp)$

$\therefore\ \tau=r\,F\sin\phi=(r\sin\phi)F$

$=rF_T=r_\perp F$

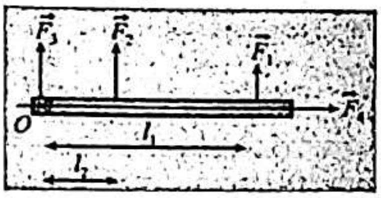
where $r_\perp$ is known as moment arm (lever arm)

Thus, if a force acts on a single particle at a point P whose position with respect to the origin O is given by the position vector $\vec{r}$, (see figure), the moment of force (torque) acting on the particle with respect to the origin O is defined as the vector product;

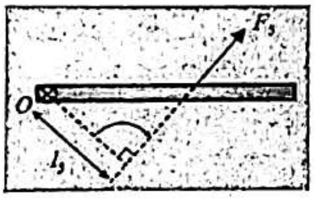


$\vec{\tau} = \vec{r} \times \vec{F}$

Direction of torque is found by sliding the force vector at the axis of rotation and using right hand thumb rule.



Torque about O due to F_1, F_2, F_3, F_4 and F_5 and their moment arm or force arm are shown in the figure.



(Torque) $\tau_1 = F_1 l_1$

$\tau_2 = F_2 l_2$

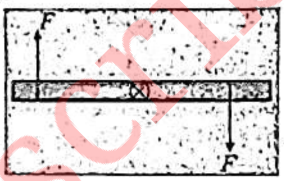
$\tau_3 = 0$

$\tau_4 = 0$

$\tau_5 = F_5 l_5$

Important Notes

- The SI unit of torque is newton-metre. This is also the unit of work and energy. Usually, the unit of work or energy is written as "joule". But since torque is a different physical quantity it is not expressed in Joule.
- Torque is always defined with reference to a given point (or given line). On changing the reference point (or line) the torque may change.
- Torque is a vector quantity, whose direction is perpendicular to the plane of force and position vector and its direction is given by right hand screw rule.
- There can be non-zero torque on body even when net force is zero (couple).



- **Force couple:** A pair of forces each of same magnitude and in opposite direction is called a force couple. Torque due to a couple about any point is same and its magnitude = Magnitude of one force × distance between their lines of action

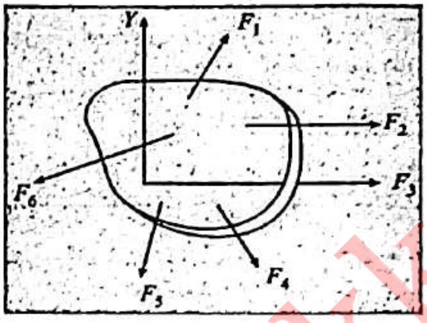
Rotational Equilibrium of a Rigid Body

A system is in mechanical equilibrium if it is in translational as well as rotational equilibrium. For this:

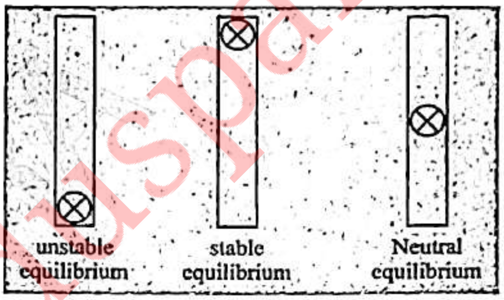
$\vec{F}_{net} = 0$, $\vec{\tau}_{net} = 0$ (about every point)

It can be proved that

If $\vec{F}_{net} = 0$ then $\vec{\tau}_{net}$ is same about every point



Hence necessary and sufficient condition for equilibrium is $\vec{F}_{net} = 0, \vec{\tau}_{net} = 0$ about any one point, which we can choose as per our convenience. ($\vec{\tau}_{net}$ will automatically be zero about every point)



The equilibrium of a body is called stable if the body tries to regain its equilibrium position after being slightly displaced and released. It is called unstable if it gets further displaced after being slightly displaced and released. If it can stay in equilibrium even after being slightly displaced and released, it is said to be in neutral equilibrium.

Train Your Brain

Example 13: Given that, $\vec{r} = 2\hat{i} + 3\hat{j}$ m and $\vec{F} = 2\hat{i} + 6\hat{k}$ N. The magnitude of torque will be-

(a) $\sqrt{405}$ N.m (b) $\sqrt{410}$ N.m

(c) $\sqrt{504}$ N.m (d) $\sqrt{510}$ N.m

Sol. (c) We know that, $\vec{\tau} = \vec{r} \times \vec{F}$

$$\Rightarrow \vec{\tau} = (2\hat{i} + 3\hat{j}) \times (2\hat{i} + 6\hat{k}) \text{ N.m}$$

$$= 12(-\hat{j}) + 6(-\hat{k}) + 18\hat{i} = -12\hat{j} - 6\hat{k} + 18\hat{i}$$

[Note : $\hat{i} \times \hat{i} = 0, \hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{i} = -\hat{k}$ etc]

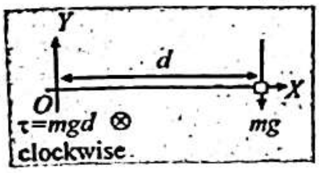
$$\text{Now, } |\vec{\tau}| = \sqrt{(-12)^2 + (-6)^2 + (18)^2}$$

$$= \sqrt{144 + 36 + 324} = \sqrt{504} \text{ N.m}$$

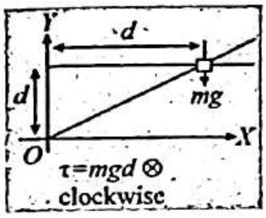
Example 14: A particle is falling freely along line $x = d$. Find torque on this particle due to gravity, about origin when it

(*i*) Crosses X axis (*ii*) is at $y = d$

Sol. (*i*)

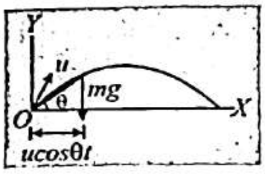


(*ii*)



Example 15: Find torque due to gravity at any time t about point of projection O, if a body is projected with velocity u at an angle θ with the horizontal.

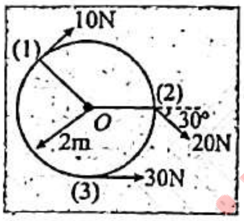
Sol.



At every instant force mg is in downward direction and perpendicular distance will be displacement along x-axis i.e. $x = (u\cos\theta)t$

$\tau_O = mg\,u\cos\theta\,t$, it is increasing with time.

Example 16: Find net torque about axis of rotation passing through O.



Sol.

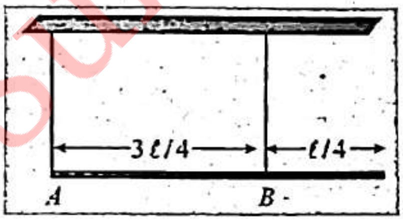
$\tau_1 = 10 \times 2 \quad \otimes \quad = 20\,\otimes$

$\tau_2 = 20 \times 2\sin 30° \quad \otimes \quad = 20\,\otimes$

$\tau_3 = 30 \times 2 \quad \odot \quad = 60\,\odot$

$\Sigma\vec{\tau} = 20\,\odot$

Example 17: A uniform rod of length ℓ, mass m is hung from two strings of equal length from a ceiling as shown in figure. Determine the tensions in the strings ?



Sol. $T_A + T_B = mg$ ...(*i*)

Torque about point A is zero

So, $T_B \times \frac{3\ell}{4} = mg\frac{\ell}{2}$ ...(*ii*)

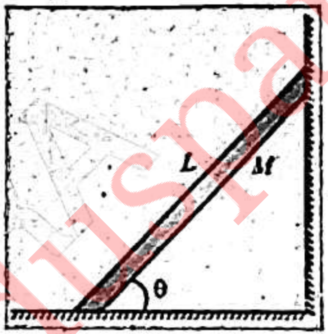
From eq. (*i*) & (*ii*),

$T_A = mg/3, T_B = 2mg/3.$

Example 18: A uniform rod of length L and mass m is placed on a rough horizontal surface and rests against a smooth wall as shown in the figure.

(*a*) Find the frictional force acting on it.

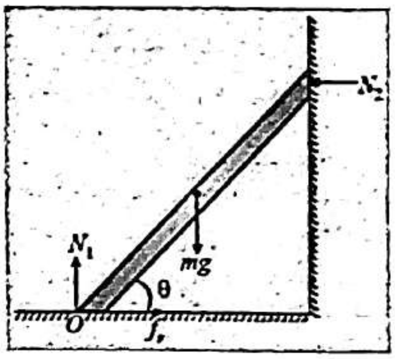
(*b*) If coefficient of static friction is $\mu_s = \frac{\sqrt{3}}{2}$, then find the maximum angle 'θ' that the rod can make with the horizontal.



Sol. Balancing force in horizontal and vertical directions

$N_1 - mg = 0$

$f_s - N_2 = 0$



F.B.D of the rod

Balancing torque about O,

$mgL/2\cos\theta = N_2 L\sin\theta$

$\Rightarrow N_2 = \frac{mg}{2}\cot\theta$ or $f_s = \frac{mg}{2}\cot\theta$

$\frac{mg}{2}\cot\theta \le \mu_s N_1 = \mu_s mg$

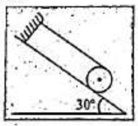
$\cot\theta \le 2\mu_s$ or $\cot\theta \le \sqrt{3}$

$\Rightarrow \theta_{max} = 30°$

Concept Application

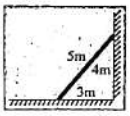
14. The beam and pans of a balance have negligible mass. An object weighs W_1, when placed in one pan and W_2 when placed in the other pan. The weight W of the object is:
(a) $\sqrt{W_1W_2}$ (b) $\sqrt{(W_1^2+W_2^2)/2}$
(c) $\sqrt{W_1^2+W_2^2+W_1W_2}$ (d) $\left(W_1^{-1}+W_2^{-1}\right)^{-1}/2$

15. A thin hoop of weight 500 N and radius 1 m rests on a rough inclined plane as shown in the figure. The minimum coefficient of friction needed for this configuration is:



(a) $\frac{1}{3\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{1}{2}$ (d) $\frac{1}{2\sqrt{3}}$

16. A uniform ladder of length 5 m is placed against the wall as shown in the figure. If coefficient of friction μ is the same for both the wall and the floor then minimum value of μ for it not to slip is



(a) $\mu = 1/2$ (b) $\mu = 1/4$
(c) $\mu = 1/3$ (d) $\mu = 1/5$

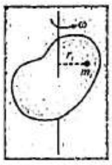
17. In case of torque of a couple, if the axis is changed by displacing it parallel to itself, then torque will
(a) Increase (b) Decrease
(c) Remain constant (d) None of these

KINETIC ENERGY OF A RIGID BODY ROTATING ABOUT A FIXED AXIS

Suppose a rigid body is rotating about a fixed axis with angular speed ω.

Then, kinetic energy of the rigid body will be:

$$K = \sum_i \frac{1}{2}m_iv_i^2 = \sum_i \frac{1}{2}m_i(\omega r_i)^2$$



$$= \frac{1}{2}\omega^2\sum_i m_ir_i^2 = \frac{1}{2}I\omega^2 \text{ (as } \sum_i m_ir_i^2 = I\text{)}$$

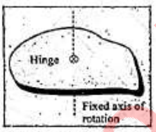
Thus, KE = $\frac{1}{2}I\omega^2$

Sometimes it is called the rotational kinetic energy.

DYNAMICS OF ROTATION ABOUT A FIXED AXIS

If I_{Hinge} = moment of inertia about the axis of rotation (since this axis passes through the hinge, hence the name I_{Hinge}).

$\vec{\tau}_{\text{ext}}$ = resultant external torque acting on the body about axis of rotation, and



α = angular acceleration of the body. Then

$$(\vec{\tau}_{\text{ext}})_{\text{Hinge}} = I_{\text{Hinge}}\,\vec{\alpha}$$

The momentum of the body is given by $\vec{P} = M\,\vec{v}_{CM}$; and acceleration of the center of mass is governed by $\vec{F}_{\text{external}} = M\,\vec{a}_{CM}$

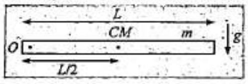
Since the center of mass will move in a circular path, it is convenient and very useful to resolve net external force along radial (centripetal) and tangential directions.

$$F_C = ma_C = m\frac{v^2}{r_{CM}} = m\omega^2 r_{CM}$$

$$F_t = ma_t = m\alpha r_{CM}$$

Train Your Brain

Example 19: A uniform rod of mass m and length L is hinged at point O is free to rotate about the horizontal axis in the vertical plane. If it is released from rest in the position shown, find its angular velocity ω and velocity of its center of mass when it has turned by 90°? Neglect friction due to hinge.



Sol. Loss in potential energy $= mg\,\Delta h_{cm} = mg\frac{L}{2}$

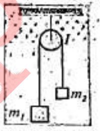
Gain in kinetic energy $= \frac{1}{2}I_o\omega^2 = \frac{1}{2}\frac{mL^2}{3}\omega^2$

Applying energy conservation, we get

$$\frac{1}{2}\frac{mL^2}{3}\omega^2 = mg\frac{L}{2} \Rightarrow \omega = \sqrt{\frac{3g}{L}}$$

$$V_{cm} = \frac{\omega L}{2} = \frac{\sqrt{3gL}}{2}$$

Example 20: The pulley shown in figure has a moment of inertia I about its axis and its radius is r. Calculate the magnitude of the acceleration of the two blocks. Assume that the string is light and does not slip on the pulley.



Sol. Suppose the tension in the left string is T_1 and that in the right string in T_2. Suppose the block of mass m_1 goes down with an acceleration a and the other block moves up with the same acceleration a. This is also the tangential acceleration of the rim of the wheel as the string does not slip over the rim. The angular acceleration of the wheel is, therefore, $\alpha = a/r$. The equations of motion for the mass m_1, the mass m_2 and the pulley are as follows:

$m_1g - T_1 = m_1a$ ... (i)
$T_2 - m_2g = m_2a$ ... (ii)
$T_1r - T_2r = I\alpha = Ia/r$ ... (iii)

Putting T_1 and T_2 from (i) and (ii) into (iii),

$$[m_1(g-a) - m_2(g+a)]\,r = I\frac{a}{r}$$

which gives $a = \frac{(m_1-m_2)gr^2}{I+(m_1+m_2)r^2}$.

Alternative Method: (Using energy conservation)

Let the velocity of m_1 and m_2 be v when they have undergone displacement S. Loss in potential energy = $m_1gs - m_2gs$. Gain in kinetic energy $= \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2$

Applying energy conservation :

$$\Rightarrow \frac{1}{2}\left(m_1+m_2+\frac{I}{r^2}\right)v^2 = (m_1-m_2)gs$$

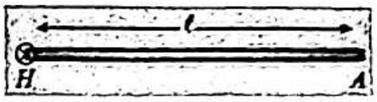
$$\Rightarrow v^2 = 2\frac{(m_1-m_2)gr^2}{((m_1+m_2)r^2+I)}s \quad \text{... (i)}$$

But $v^2 = 2\,as$ ... (ii)

Comparing (*i*) and (*ii*) we get

$$a = \frac{(m_1 - m_2)gr^2}{I + (m_1 + m_2)r^2}$$

Example 21: A uniform rod of mass M and length ℓ can rotate in vertical plane about a smooth horizontal axis hinged at point H.



(*i*) Find angular acceleration α of the rod just after it is released from initial horizontal position from rest?

(*ii*) Calculate the acceleration (tangential and radial) of point A at this moment.

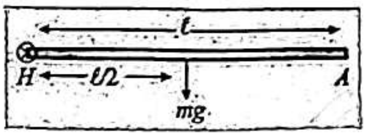
(*iii*) Calculate net hinge force acting at this moment.

(*iv*) Find α and ω when rod becomes vertical.

(*v*) Find hinge force when rod become vertical.

Sol. (*i*) $\tau_H = I_H \alpha$

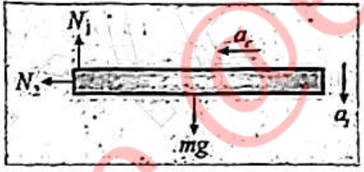
$$mg.\frac{\ell}{2} = \frac{m\ell^2}{3}\alpha \Rightarrow \alpha = \frac{3g}{2\ell}$$



(*ii*) $a_{tA} = \alpha\ell = \frac{3g}{2\ell}.\ell = \frac{3g}{2}$

$a_{CA} = \omega^2 r = 0.\ell = 0$ $\quad$ ($\because$ $\omega = 0$ just after release)

(*iii*) Suppose hinge exerts normal reaction in component form as shown



In vertical direction

$$F_{ext} = ma_{CM} \Rightarrow mg - N_1 = m.\frac{3g}{4}$$

(we get the value of a_{CM} from previous example)

$$\Rightarrow N_1 = \frac{mg}{4}$$

In horizontal direction

$$F_{ext} = ma_{CM} \Rightarrow N_2 = 0$$

($\therefore$ a_{CM} in horizontal $= 0$ as $\omega = 0$ just after release).

(*iv*) Torque $= 0$ when rod becomes vertical.

so $\alpha = 0$

using energy conservation

$$\frac{mg\ell}{2} = \frac{1}{2}I\omega^2 \left(I = \frac{m\ell^2}{3}\right)$$

$$\omega = \sqrt{\frac{3g}{\ell}}$$

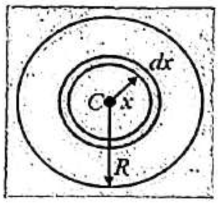
(v) When rod becomes vertical

$$\alpha = 0,\ \omega = \sqrt{\frac{3g}{\ell}}$$

$$F_H - mg = \frac{m\omega^2\ell}{2}\ ;\ F_H = \frac{5mg}{2}$$

Example 22: A uniform disc of radius R and mass M is spinned to an angular speed ω_0 in its own plane about its centre and then placed on a rough horizontal surface such that plane of the disc is parallel to the horizontal plane. If co-efficient of friction between the disc and the surface is μ then how long will it take for the disc to come to stop.

Sol. Consider a differential circular strip of the disc of radius x and thickness dx Mass of this strip is $dm = 2\rho\pi x dx$.



where $\rho = \frac{M}{\pi R^2}$. Frictional force on this strip is along the tangent and is equal to $dF = \mu\rho 2\pi x dx g$

Torque on the strip due to frictional force is equal to $d\tau = \mu\rho g 2\pi x^2 dx$, disc is supposed to be the combination of number of such strips hence torque on the disc is given by

$$\tau = \int d\tau = \mu\rho g 2\pi \int_0^R x^2 dx = \mu\rho g 2\pi \frac{R^3}{3}$$

$$\Rightarrow \tau = \mu Mg(2/3)R \Rightarrow \alpha = \frac{2\mu MgR}{3\left(\frac{MR^2}{2}\right)} = \frac{4}{3}\frac{\mu g}{R}$$

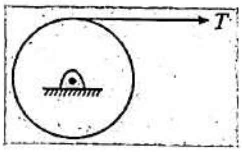
The α is opposite to the ω

$$\because\ \omega(t) = \omega_0 + \alpha t$$

$$\Rightarrow 0 = \omega_0 - \frac{4\mu g}{3R}t \Rightarrow t = \frac{3\omega_0 R}{4\mu g}.$$

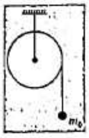
Concept Application

18. A wheel 4 m in diameter rotates about a fixed frictionless horizontal axis, about which its moment of inertia is 10 kg m^2. A constant tension of 40 N is maintained on a rope wrapped around the rim of the wheel. If the wheel starts from rest at $t = 0$ s, find the length of rope unwounded till $t = 3$s.



(a) 36.0 m (b) 72.0 m (c) 18.0 m (d) 720 m

19. A uniform solid cylinder of mass m can rotate freely about its fixed axis, which is kept horizontal. A particle of mass m_o hangs from the end of a light string which is tightly wound round the cylinder. When system is allowed to move, acceleration with which the particle descends

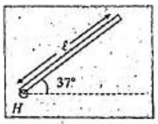


(a) $\frac{mg}{m+2m_o}$ (b) $\frac{2m_o g}{m+2m_o}$

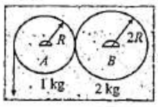
(c) $\frac{m_o g}{m+m_o}$ (d) $\frac{2mg}{m+2m_o}$

20. A uniform rod of mass m and length ℓ can rotate in vertical plane about a smooth horizontal axis hinged at point H. The angular acceleration α is $\frac{xg}{5\ell}$, of the rod just after it is released from initial position making an angle of 37° with horizontal from rest then the value of x is.

The force exerted by the hinge is $\frac{\sqrt{y}mg}{5}$, just after the rod is released from rest then the value of y is.

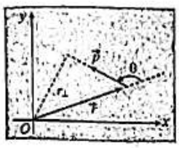


21. Two discs A and B touch each other as in figure. A rope lightly wound on A is pulled down at 2 m/s². Find the friction force (N) between A and B if slipping is absent.



ANGULAR MOMENTUM AND ITS CONSERVATION

Consider a particle that has linear momentum $\vec{p}$ and is located at position $\vec{r}$ relative to origin O, as shown in the figure. Its angular momentum about the origin is defined as



$\vec{L}=\vec{r}\times\vec{p}$

The magnitude of momentum is given by

$L=rp\sin\theta$

The direction of angular momentum is given by the right hand rule.

The SI unit of angular momentum is kg m²/s.

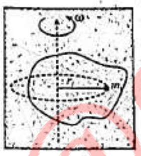
Note: Angular momentum is defined always with respect to a point

The total angular momentum $\vec{L}$ of a system of particles relative to a given origin is the sum of the angular momentum of the particles.

$$\vec{L}=\sum\vec{r}_i\times\vec{p}_i$$

Angular momentum of a body rotating about a fixed axis: Let the rigid body rotate about z-axis as shown in the figure.

The angular momentum of the i^{th} particle about axis of rotation is $L_{iz}=r_ip_i$



$L_z=\Sigma r_ip_i$

Also $p_i=m_iv_i=m_ir_i\omega$

or $L_z=\Sigma m_ir_i^2\,\omega$ or $L_z=I\omega$

where $I=\Sigma m_ir_i^2$

If the mass of the body is distributed symmetrically about axis of rotation then angular momentum and angular velocity point in the same direction and $\vec{L}=I\vec{\omega}$

RELATION BETWEEN ANGULAR MOMENTUM AND TORQUE FOR A SINGLE PARTICLE

As force changes the linear momentum of a particle, torque changes the angular momentum of a particle. The rate of change of angular momentum with time is given by

$$\frac{d\vec{L}}{dt}=\frac{d}{dt}(\vec{r}\times m\vec{v}) \text{ or } \frac{d\vec{L}}{dt}=\frac{d\vec{r}}{dt}\times m\vec{v}+\vec{r}\times m\frac{d\vec{v}}{dt}$$

or $\frac{d\vec{L}}{dt}=\vec{v}\times m\vec{v}+\vec{r}\times\vec{F}$

or $\frac{d\vec{L}}{dt}=\vec{r}\times\vec{F}=\vec{\tau}$ because $\vec{v}\times m\vec{v}$ is zero

Thus, $\vec{\tau}=\frac{d\vec{L}}{dt}$

The above equation is the rotational analog of equation $\vec{F}=\frac{d\vec{P}}{dt}$

Relationship between angular momentum and torque for a system of particles:

The total angular momentum of the system is $\vec{L}=\Sigma\vec{L}_i$

Differentiating with respect to time, we get

$$\frac{d\vec{L}}{dt}=\sum\frac{d\vec{L}_i}{dt}=\sum\vec{\tau}_i$$

where $\vec{\tau}_i$ is the torque acting on the i^{th} particle:

$\vec{\tau}_i=\vec{r}_i\times\vec{F}_i$

Now, $\vec{F}_i=\vec{F}_{i,ext}+\vec{F}_{i,int}$

where $\vec{F}_{i,ext}$ and $\vec{F}_{i,int}$ are vector sums of forces acting on the the i^{th} particle due to external forces and by other particles of the system respectively.

Therefore $\vec{\tau}=\Sigma\vec{r}_i\times\vec{F}_{i,ext}+\Sigma\vec{r}_i\times\vec{F}_{i,int}=\vec{\tau}_{i,ext}+\vec{\tau}_{i,int}$

If we assume forces acting between any two particles of the system are equal and opposite and also act along line joining the particles (stronger version of Newton's third law), then torque due to each pair will cancel out and net torque $\vec{\tau}_{int}$ due to internal forces will be zero.

Thus, we have $\frac{d\vec{L}}{dt}=\Sigma\vec{\tau}_{ext}$

Note that it holds for any system of particles whether it is a rigid body or any system of particles which may have all kinds of internal motion.

Conservation of Angular momentum: If the net external torque on a system is zero, the total angular momentum is constant in magnitude and direction.

That is, if $\vec{\tau}_{ext}=\vec{0}$ then $\frac{d\vec{L}}{dt}=\vec{0}$

$\vec{L}$ = constant

For a rigid body rotating about a fixed axis,

$L_f=L_i$

or $I_f\omega_f=I_i\omega_i$

ANGULAR IMPULSE

In complete analogy with the linear momentum, angular impulse is defined as

$\vec{J}=\int\vec{\tau}_{ext}dt$

Using Newton's second law for rotation motion,

$\vec{\tau}_{ext}=\frac{d\vec{L}}{dt}$

$\therefore \vec{J} = \Delta\vec{L} = \vec{L}_f - \vec{L}_i$

The net angular impulse acting on a rigid body is equal to the change in angular momentum of the body. This is called the impulse – momentum theorem for rotational dynamics.

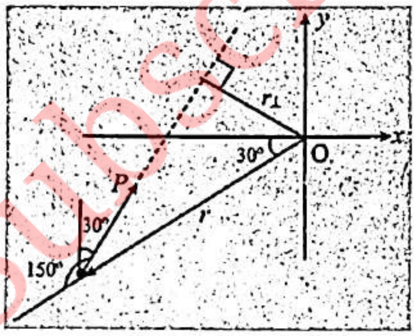
Finally, we present an analogy between rotational dynamics and linear dynamics.

	Quantity	Linear	Rotational
1.	Inertia	m	$\sum m_i r_i^2$ or $\int r^2 dm$
2.	Newton's Second Law	$\vec{F}_{ext} = m\vec{a}$ $\vec{F}_{ext} = \frac{d\vec{p}}{dt}$	$\vec{\tau}_{ext} = I\vec{\alpha}$ $\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$
3.	Work	$W_{lin} = \int \vec{F}.d\vec{s}$	$W_{rot} = \int \vec{\tau}.d\vec{\theta}$
4.	Kinetic Energy	$K_{lin} = \frac{1}{2}mv^2$	$K_{rot} = \frac{1}{2}I\omega^2$
5.	Work Energy Theorem	$W_{lin} = \Delta K_{lin}$	$W_{rot} = \Delta K_{rot}$
6.	Impulse	$\vec{I} = \int \vec{F}_{ext} dt$	$\vec{J} = \int \vec{\tau}_{ext} dt$
7.	Momentum	$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
8.	Impulse Momentum Theorem	$\vec{I} = \Delta\vec{p}$	$\vec{J} = \Delta\vec{L}$
9.	Power	$P = \vec{F}.\vec{v}$	$P = \vec{\tau}.\vec{\omega}$

Train Your Brain

Example 23: What is the angular momentum of a particle of mass $m = 2$ kg that is located 15 m from the origin in the direction 30° south of west and has a velocity $v = 10$ m/s in the direction 30° east of north?

Sol. In the figure (a), the x – axis points east. We know $r = 15$ m; $p = mv = 20$ kg m/s. The angle between r and p is $(180° - 30°) = 150°$



Thus,

$L = rp \sin\theta = (15)(20) \sin 150° = 150$ kg m²/s

In unit vector notation,

$$\vec{r} = -15 \cos 30° \,\hat{i} - 15 \sin 30° \,\hat{j} \text{ m}$$

$$\vec{p} = 20 \sin 30° \,\hat{i} + 20 \cos 30° \,\hat{j} \text{ kg m/s}$$

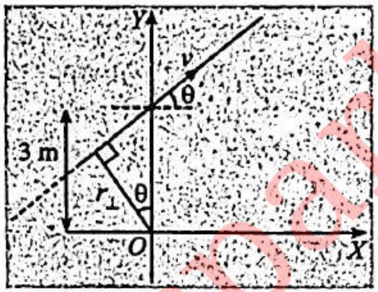
Therefore,

$$\vec{L} = \left(-\frac{15\sqrt{3}}{2}\hat{i} - \frac{15}{2}\hat{j}\right) \times (10\hat{i} + 10\sqrt{3}\hat{j})$$

$$= -150 \,\hat{k} \text{ kg m}^2/\text{s}$$

Example 24: A particle of mass m moves along the line $y = 2x + 3$ with uniform velocity v. Find it's angular momentum about the origin when it crosses y-axis.

Sol.



$$L_o = mvr_\perp$$

$$r_\perp = 3 \cos\theta$$

$$\text{Slope} = \tan\theta = 2$$

$$\cos\theta = \frac{1}{\sqrt{5}}$$

$$L_o = \frac{3mv}{\sqrt{5}}, \otimes$$

The angular momentum will remain constant as $r_\perp = 3 \cos\theta$ = constant.

Example 25: A disc of moment of inertia 4 kg m² is spinning freely at 3 rad/s. A second disc of moment of inertia 2 kg m², slides down the spindle and they rotate together.

(a) What is the angular velocity of the combination?

(b) What is the change in kinetic energy of the system?

Sol. (a) Since there are no external torques acting, we may apply the conservation of angular momentum.

$$I_f\omega_f = I_i\omega_i$$

$$(6 \text{ kg m}^2)\,\omega_f = (4 \text{ kg m}^2)(3 \text{ rad/s})$$

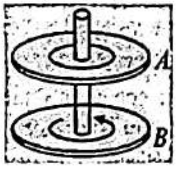
$$\text{Thus}, \omega_f = 2 \text{ rad/s}$$

(b) The kinetic energies before and after the collision are

$$K_i = \frac{1}{2}I_i\omega_i^2 = 18\text{ J};\; K_f = \frac{1}{2}I_f\omega_f^2 = 12\text{ J}$$

The change is $\Delta K = K_f - K_i = -6$ J.

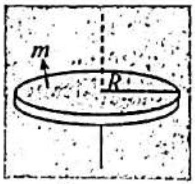
In order for the two discs to spin together at the same rate, there had to be friction between them. The lost kinetic energy is converted to thermal energy.



Example 26: A person of mass m stands at the edge of a circular platform of radius R and moment of inertia I. A platform is at rest initially. But the platform rotate when the person jumps off from the platform tangentially with velocity u with respect to platform. Determine the angular velocity of the platform.

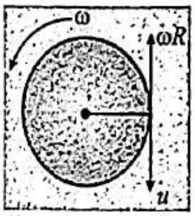
Sol. Let the angular velocity of platform is ω,

the velocity of person with respect to ground is v_m.



$v_{mD} = v_{mG} - V_{DG}$; $u = v_m + \omega R$; $v_m = u - \omega R$

Now from angular momentum conservation



$L_i = L_f$

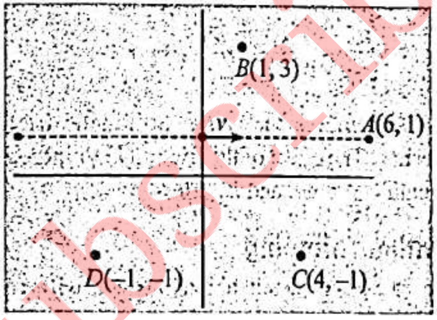
$0 = mv_m R - I\omega$

$\Rightarrow \quad I\omega = m(u - \omega R) \cdot R$

$\Rightarrow \quad \omega = \dfrac{muR}{I + mR^2}$

Concept Application

22. A particle of mass m is moving with a constant velocity v as shown in the figure. Magnitude of its angular momentum is minimum about point



(*a*) A (*b*) B (*c*) C (*d*) D

23. A particle is projected at time $t = 0$ from a point P with a speed v_o at an angle of 45° to the horizontal. Direction and the magnitude of the angular momentum of the particle about the point P at time $t = v_o/g$ is $\dfrac{mv_0^3}{xg}$ then the value of x is

24. If the radius of earth shrinks to ηR ($\eta < 1$), where R is the radius of earth, then the time period of rotation of earth about its axis will become

(*a*) $24\,\eta^2$ hour (*b*) $\dfrac{24}{\eta}$ hour

(*c*) $24\,\eta$ hour (*d*) $\dfrac{24}{\eta^2}$ hour

25. A playground merry-go-round is at rest, pivoted about a frictionless axis. A child of mass m runs along a path tangential to the rim with speed v and jumps on to the merry-go-round. If R is the radius of the merry-go-round and I is its moment of inertia, then the angular velocity of the merry-go-round and the child is –

(*a*) $\dfrac{mvR}{mR^2 + I}$ (*b*) $\dfrac{mvR}{I}$

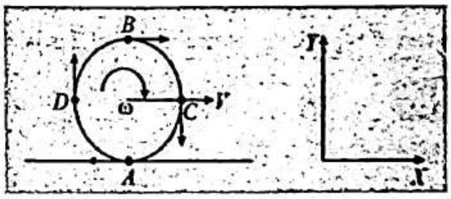
(*c*) $\dfrac{mR^2 + I}{mvR}$ (*d*) $\dfrac{I}{mvR}$

THE COMBINED TRANSLATIONAL AND ROTATIONAL MOTION OF A RIGID BODY

So far we have considered only pure rotation of a rigid body. However, a rigid body while rotating about an axis can also be moving translationally. We can separate the general motion of a rigid body into translational motion of its center of mass and rotational motion about its center of mass. The motion of rigid body is thus completely characterized by the velocity $\vec{V}_{cm}$ of center of mass and angular velocity $\vec{\omega}$ about the axis passing through the center of mass .In general ,axis of rotation can also change direction but that is dealt with in advanced level course.

Once V_{cm} and ω are specified , the velocity of any point on a rigid body can be determined.

Illustration: A uniform disc of radius R is rotating about the cylindrical axis with angular velocity ω and the cylindrical axis is moving with velocity V with respect to ground.(see figure below). Find the velocity of point A, B, C and D with respect to ground.



Let X and Y axis point in the horizontal and vertical directions respectively. The velocity V_{cm} is equal to V. The velocity of any point can be determined from eqns of relative velocities.

$\vec{V}_A = \vec{V}_{A/cm} + \vec{V}_{cm}$

$= -\omega R\hat{i} + V\hat{i} = (V - \omega R)\hat{i}$

$\vec{V}_B = \vec{V}_{B/cm} + \vec{V}_{cm} = \omega R\hat{i} + V\hat{i} = (V + \omega R)\hat{i}$

$\vec{V}_C = \vec{V}_{C/cm} + \vec{V}_{cm} = -\omega R\hat{j} + V\hat{i} = V\hat{i} - \omega R\hat{j}$

$\vec{V}_D = \omega R\hat{j} + V\hat{i} = V\hat{i} + \omega R\hat{j}$

Kinetic energy: The kinetic energy of a system of particles is given by $KE = KE_{rel/cm} + \frac{1}{2}MV_{cm}^2$ but $KE_{rel/cm} = \frac{1}{2}I_{cm}\omega^2$. Therefore, the total kinetic energy of the body in this case is given by the sum of translational and rotational kinetic energy which is:

$$KE = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

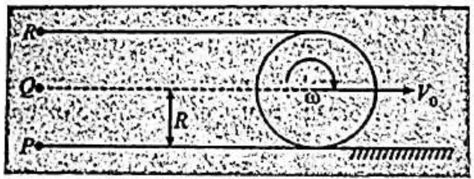
Angular momentum. The total angular momentum in this general case about a point O is given by

$$\vec{L} = \vec{L}_{cm} + \vec{r}_{cm} \times M\vec{V}_{cm}$$

$$= I_{cm}\vec{\omega} + \vec{r}_{cm} \times M\vec{V}_{cm}$$

where $\vec{L}_{cm}$ is the angular momentum relative to center of mass and $\vec{r}_{cm}$ is the position vector of center of mass w.r.t O. This relation can be interpreted as regarding the total angular momentum to be sum of angular momentum about the center of mass and angular momentum of the motion of center of mass C.M. with respect to O as if all the mass was concentrated at the center of mass.

Illustration: A cylinder of mass m and radius R is moving on a horizontal surface with angular velocity ω and its cylindrical axis is moving with velocity V_0 (see figure). Find the angular momentum of the cylinder w.r.t. to P, Q and R shown in the figure.



Let Z-axis be perpendicular to the plane of the paper and point in to the plane of the paper. Since cylinder is rotating in the clockwise direction, $\vec{\omega}$ point in the +ve z-direction.

$$\vec{L}_P = (MV_oR + I_{cm}\omega)\hat{k}$$

$$\vec{L}_Q = I_{cm}\omega\hat{k} + 0 \text{ [momentum arm is zero]}$$

$$\vec{L}_R = (I_{cm}\omega - MV_oR)\hat{k}$$

where $\hat{k}$ is a unit vector in direction of Z-axis.

ROLLING

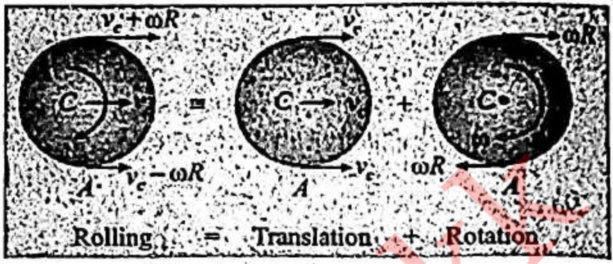
Rolling Motion

A photograph of a rolling bicycle wheel.

The spokes near the top of the wheel are more blurred than those near the bottom of the wheel because they are moving faster.

Pure rolling means no sliding. Now, the motion of any body can be divided into pure translation & pure rotation as discussed. The velocity v_A of point of contact A is $v_c - \omega R$ (see figure).

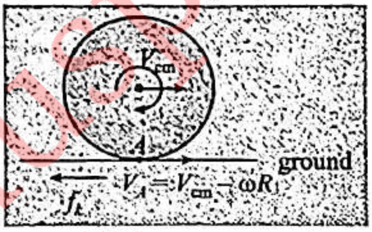
Sliding refers to the condition under which two bodies in contact have relative velocity. And under pure rolling, the relative velocity at point of contact should be zero.



Now for the wheel shown, point A is in contact with the ground. The point A on the ground has zero velocity. Thus, the point A on the wheel should also have zero velocity.

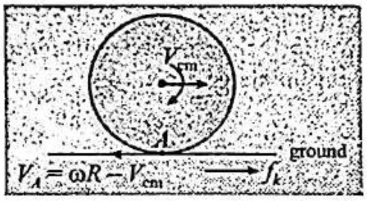
Rolling with Slipping

Case-I: $V_{cm} > \omega R$



Note: The friction will be kinetic in nature & its magnitude can be determined using $f_k = \mu_k N$. Direction will be backwards (because point of contact A is moving forward w.r.t. ground). The effect of f_k is to decrease V_{cm} and increase ω until $V_{cm} = \omega R$ and $V_A = 0$

Case-II: $V_{cm} < \omega R$



Point of contact A moves backwards. So kinetic friction acts in forward direction. The effect of the friction is to increase V_{cm} and decrease ω until $V_{cm} = \omega R$ and $V_A = 0$

Rolling without Slipping

In both the cases considered above, wheel will roll without slipping after some time.

$V_{cm} = \omega R$, $V_A = 0$

This is known as perfect rolling

In Maximum problems we come across this situation

Notes: 1. The friction will be static in nature. Its magnitude is self-adjusting, it can vary from 0 to $\mu_s N$.

2. Static friction may act in forward or backward to prevent slipping.

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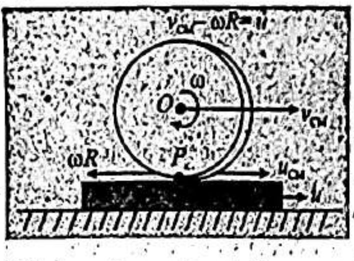
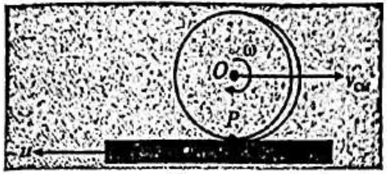
3. The total work done by static friction is zero because point of application of the frictional force does not move.
4. If there is no applied force, mechanical energy will be conserved.

ROLLING WITHOUT SLIPPING ON MOVING PLATFORM

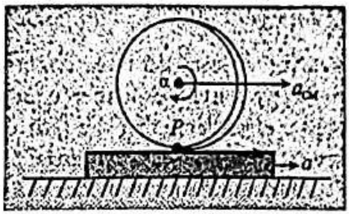
1. If point of contact of the surface is moving with a velocity u with respect to the ground, then



2. For no sliding on the moving platform, $u = \omega R - v_{CM}$



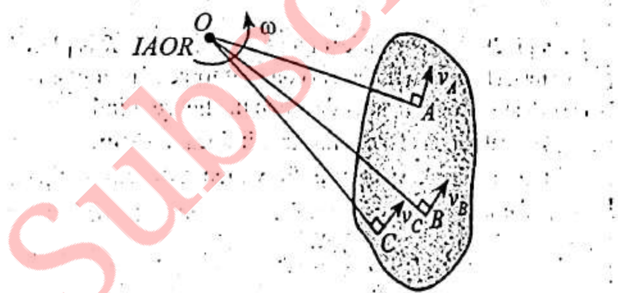
3. For accelerated surface,



$a_{CM} - \alpha R = a$

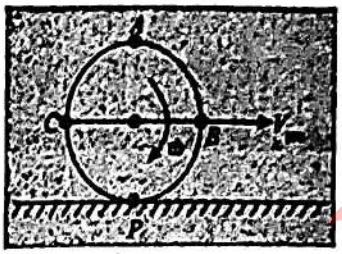
INSTANTANEOUS AXIS OF ROTATION

For a rigid body which has motions both of translation and rotation, it is always possible to find an axis about which body is in pure rotation for an instant. You may imagine it to be a line which is supposed to be rigidly united with the body and which for an instant is at rest.



Such an axis is called the instantaneous axis of rotation (IAOR). This axis is always perpendicular to the plane used to represent the motion and the intersection of the axis with this plane defines the location of instantaneous centre of zero velocity (IC).

For example consider a wheel of mass M and radius R which rolls without slipping. In this case the point of contact with the ground has zero velocity. Hence, this point represents the IC for the wheel. The line perpendicular to the wheel and passing through the point of contact is the IAOR. If it is imagined that the wheel is momentarily pinned at this point, the velocity of any point on the wheel can be found using $v = \omega d$ where d is the distance of the point from IAOR.



To verify that this indeed is pure rotation. Let us find the velocity of point A, B, C in the figure shown above.

$$V_A = V_{cm} + \omega R = \omega(2R)$$

$$V_B = \sqrt{V_{cm}^2 + \omega^2 R^2} = \omega\left(\sqrt{2}R\right)$$

$$V_C = \sqrt{V_{cm}^2 + \omega^2 R^2} = \omega\left(\sqrt{2}R\right)$$

which should be the case for pure rotation because the distance of point A, B and C from axis of rotation is $2R$, $\sqrt{2}R$ and $\sqrt{2}R$ respectively.

The kinetic energy of wheel is

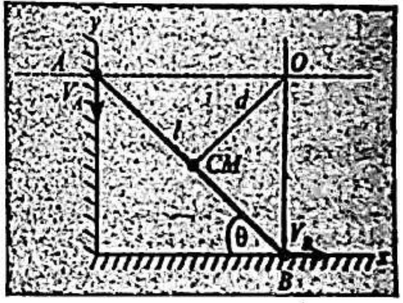
$$KE = \frac{1}{2}\left(I_P\right)\omega^2 = \frac{1}{2}\left(I_{cm} + MR^2\right)\omega^2$$

$$= \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}MR^2\omega^2$$

$$= \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}MV_{cm}^2$$

which is what we will get by treating the motion of the wheel as a combination of translational and rotational motion.

In the figure shown, a rod rests against a wall and makes angle θ with the horizontal If it starts to slip, let us find the instantaneous axis of rotation.



In this case, we know the direction of velocity of point A and point B. Point A moves vertically downwards while point B moves along the horizontal direction. If the instantaneous axis of rotation passes through some point O, then line OA drawn from O to A must be perpendicular to velocity of point A. Therefore, O must lie on line drawn parallel to x-axis and passing through A.

Similarly, OB must be perpendicular to velocity of point B. Therefore O must lie on line drawn parallel to y-axis and passing through point B. Hence O lies on intersection point of OA & OB.

If the velocity of point A is v_0 and length of the rod is L, then The angular velocity ω of rod is

$$\omega = \frac{v_o}{OA} = \frac{v_o}{l\cos\theta}$$

The moment of inertia about an axis passing through point O is

$$I_o = I_{cm} + md^2$$

$$= \frac{ml^2}{12} + m(l/2)^2 \qquad d = \frac{l}{2}$$

$$= \frac{ml^2}{3}$$

Hence $KE = \frac{1}{2} I_o \omega^2$

$$= \frac{1}{2}\left(\frac{ml^2}{3}\right)\left(\frac{v_o}{l\cos\theta}\right)^2 = \frac{1}{6}\,\frac{mv_0^2}{\cos^2\theta}$$

Application of Newton's Second Law in Rolling Motion

1. Write $\vec{F}_{net} = M\vec{a}_{cm}$ for the object as if it were a point-mass, that is, ignoring rotation.
2. Write $\vec{\tau} = I_{cm}\vec{\alpha}$ as if the object were only rotating about the centre of mass, that is ignoring translation.
3. Use of no-slip condition
4. Solve the resulting equations simultaneously for any unknown.

Caution:

- In general, since there is no slipping, $f_s \le \mu_s N$
- Be certain that the sign convention of forces and torques are consistent.

Train Your Brain

Example 27: A spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is-

(a)	2/5	(b)	3/5
(c)	2/7	(d)	3/7

Sol. (c) Total energy,

$$E = (1/2)\, I\omega^2 + (1/2)\, mv^2$$

$$= (1/2)\,(2/5\ mr^2)\,\omega^2 + (1/2)\, mr^2\omega^2$$

$$= (1/5)\, mr^2\omega^2 + (1/2)\, mr^2\omega^2 = (7/10)\, mr^2\omega^2$$

Rotational energy $= (1/5)\, mr^2\omega^2$

$$\therefore \frac{\text{Rotational energy}}{\text{Total energy}} = \frac{\frac{1}{5}mr^2\omega^2}{\frac{7}{10}mr^2\omega^2} = \frac{2}{7}$$

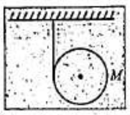
Example 28: A uniform solid cylinder of mass M and radius R starts descending at a moment $t = 0$ due to gravity (see fig. below)

Find

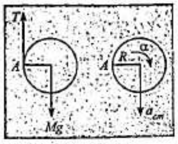
(*a*) tension in the thread

(*b*) acceleration of the centre of mass

(*c*) the angular acceleration



Sol. The F.B.D of the cylinder is as shown below.



From eqns of motion, we have

$$Mg - T = Ma_{cm} \quad \text{... (i)}$$

$$TR = I_{cm}\alpha = \frac{MR^2}{2}\alpha \quad \text{... (ii)}$$

Here, we have two eqns but three unknowns namely a_{cm}, α and T. We must have another equation to solve the problem. Please note that the part of the string which is unwrapped and becomes vertical is stationary. The instantaneous velocity of point A of the cylinder is equal to the velocity of the string which is zero. On the other hand the velocity of point A must be

$$V_A = V_{cm} - \omega R$$

therefore, $V_{cm} - \omega R = 0$

or $a_{cm} = \alpha R$ ... (*iii*)

Solving eqn (*i*), (*ii*) and (*iii*) we obtain

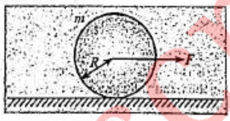
$$a_{cm} = (2/3)g$$

$$\alpha = \frac{2g}{3R} \text{ and } T = \frac{Mg}{3}$$

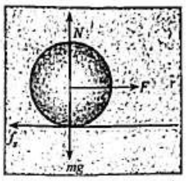
Example 29: A solid cylinder of mass m and radius R is placed on a rough horizontal surface. A constant force F is applied on the cylinder towards right as shown in the figure. Find

(*a*) acceleration of the cylinder if it does not slip.

(*b*) the minimum value of μ_s for which it does not slip on the surface.



Sol. Since we have assumed it is rolling, the force of static friction will act on the cylinder, the value of which can be anything between 0 and $\mu_s N$. Let it be f_s. The F.B.D is shown below.



From eqns of motion:

$$N - mg = 0$$

$$F - f_s = ma_{cm} \quad \text{... (i)}$$

$$f_s R = I_{cm}\alpha = \frac{mR^2}{2}\alpha \quad \text{... (ii)}$$

since cylinder is rolling on the surface

$$V_{cm} - \omega R = 0$$

or $a_{cm} = \alpha R$ ... (*iii*)

Solving (*i*), (*ii*) and (*iii*) we get

$$a_{cm} = \frac{2F}{3m} \text{ and}$$

$$f_s = \frac{F}{3}$$

If it does not slip,

$$f_s \le \mu_s N$$

$$\frac{F}{3} \le \mu_s mg$$

$$\mu_s \ge \frac{F}{3mg}$$

$$\mu_{s,min} = \frac{F}{3mg}$$

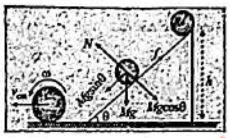
Note: Static friction is a variable force in pure rolling you may take f_s in any direction (forward or backward). If after solving, f_s is negative then it acts in opposite direction to that assumed.

Example 30: A uniform rigid body of mass M and radius R rolls on an rough inclined plane without slipping where K is the radius of gyration of the rigid body about center of mass then find the following parameters.

(*a*) Velocity of center of mass when body reaches the ground

(*b*) Acceleration of center of mass when body reaches the ground

(*c*) Time of descent (*t*)



Sol. (*a*) Applying energy conservation

$$Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}MK^2\omega^2$$

(where K is the radius of gyration)

$$= \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}\cdot\frac{K^2}{R^2}\cdot MR^2\omega^2$$

$$= \frac{1}{2}M\left(1 + \frac{K^2}{R^2}\right)v_{cm}^2, \text{ [where } v_{cm} = R\omega \text{ for pure rolling]}$$

$$\text{or, } v_{cm} = \sqrt{\frac{2gh}{1 + \frac{K^2}{R^2}}}$$

[where $\frac{K^2}{R^2}$ is a pure number between 0 and 1 that depends on the shape of the body]

It is important to note that the velocity of a rolling body is independent of its mass (M) and radius (R). All uniform solid spheres have the same speed at the bottom even if their masses and radii are different because they have same $\frac{K^2}{R^2} = \left(\frac{2}{5}\right)$. All solid cylinders have the same speed $\left(\frac{K^2}{R^2} = \frac{1}{2}\right)$, all hollow cylinders have the same speed $\left(\frac{K^2}{R^2} = 1\right)$ and so on. Smaller the value of $\frac{K^2}{R^2}$, faster the body is moving.

(*b*) **Acceleration:** Let the linear acceleration of the rolling body be a_{cm}. Then for the linear motion, the net force $= Mg\sin\theta - f = Ma_{cm}$ ... (*i*)

For the rotational motion, the net torque

$$fR = I_{cm} \cdot \alpha = MK^2 \cdot \frac{a_{cm}}{R}$$

$$\text{or, } f = \frac{MK^2}{R^2}a_{cm} \quad \text{... (ii)}$$

solving (*i*) and (*ii*) for a_{cm}, we get

$$\boxed{a_{cm} = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}}$$

It is also independent of mass M and radius R.

(c) **Time of descent (t) :** $\frac{1}{2}a_{cm}t^2 = \frac{h}{\sin\theta}$

or, $t = \sqrt{\frac{2h}{a_{cm}\sin\theta}}$

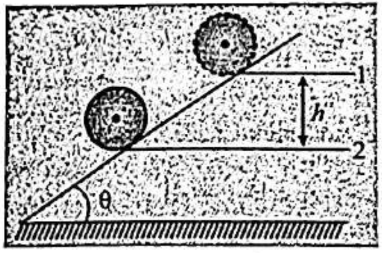
$$\boxed{t = \frac{1}{\sin\theta}\sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{g}}}$$

It is also independent of mass and radius. Clearly, a body of smaller $\frac{K^2}{R^2}$, takes less time.

Example 31: A solid cylinder of mass m and radius r starts rolling down an inclined plane of inclination θ. Friction is enough to prevent slipping. Find the speed of its centre of mass when, its centre of mass has fallen a height h.

Sol. Consider the two shown positions of the cylinder. As it does not slip, total mechanical energy will be conserved.

Energy at position 1 is $E_1 = mgh$



Energy at position 2 is $E_2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

$\therefore \omega = \frac{v_{cm}}{r}$ and $I_{cm} = \frac{mr^2}{2}$

$E_2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{v_{cm}}{r}\right)^2$

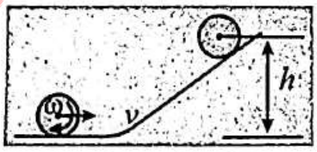
$\Rightarrow E_2 = \frac{3}{4}mv_{c.m.}^2$

From conservation of energy, $E_1 = E_2$

$\Rightarrow v_{cm} = \sqrt{\frac{4}{3}gh}$

Example 32: A ball of radius R and mass m is rolling without slipping on a horizontal surface with velocity of its centre of mass v. It then rolls without slipping up a hill to a height h before momentarily coming to rests. Find h.

Sol. $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$



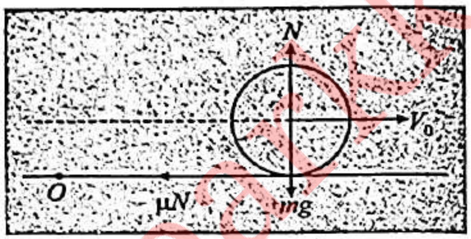
$v = \omega R$, $I = \frac{2}{5}mR^2$

$h = \frac{7v_{CM}^2}{10g}$

Example 33: A bowling ball of mass 'm' is thrown on a horizontal surface in such a way that it slides with speed V_0 initially without rolling (*a*) Find the velocity of the ball when it starts rolling on the surface.

(*b*) How much kinetic energy is lost before it starts rolling. (The co-efficient of kinetic friction is μ)

Sol.



From the free body diagram it is clear that $N = mg$.

From eqns. of motion we have

$m\,a_{cm} = -\mu mg$ ... (*i*)

$I_{cm}\alpha = \mu mg\,R$ ... (*ii*)

R = radius of the ball

$\Rightarrow a_{cm} = -\mu g$

$\Rightarrow \alpha = \frac{\mu mgR}{I_{cm}} = \frac{\mu mgR}{\frac{2}{5}mR^2} = \frac{5\mu g}{2R}$

After time 't', the velocity V_f of center of ball and angular velocity ω_f will be

$V_f = V_o - \mu gt$ ... (*iii*)

$\omega_f = 0 + \frac{5\mu g}{2R}t$ ... (*iv*)

If it starts rolling after time t then

$V_f - \omega_f R = 0$ ... (*v*)

After solving eqns (*iii*), (*iv*) and (*v*) we have

(*a*) $V_f = \frac{5}{7}V_o$ and $\omega_f = \frac{5V_o}{7R}$

(*b*) The change in kinetic energy is

$$\Delta KE = \left(\frac{1}{2}mV_{cm}^2 + \frac{1}{2}\times\frac{2m}{5}\omega^2R^2\right) - \left(\frac{1}{2}mV_0^2\right)$$

$$= \left(\frac{1}{2}m\left(\frac{5}{7}V_o\right)^2 + \frac{m}{5}\left(\frac{5}{7}\frac{V_o}{R}\right)^2R^2\right) - \frac{1}{2}mV_0^2 = -\frac{mV_0^2}{7}$$

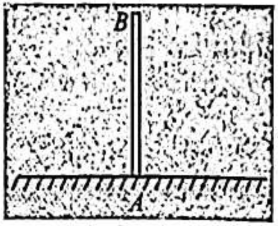
Alternative method: Angular momentum of the cylinder about point O is conserved (see fig.). Since $N = mg$ the torque due to these forces cancels out. The torque due to friction is zero since its moment arm is zero about O. Hence

$$L_i = L_f$$

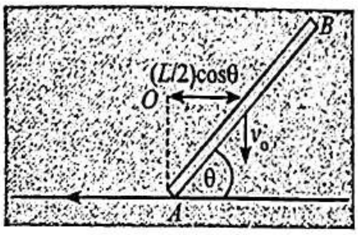
$$mV_oR = mV_fR + I_{cm}\omega_f = \frac{7}{5}mRV_f$$

$$\Rightarrow V_f = \frac{5}{7}V_o$$

Example 34: A uniform rod AB of length L and mass m is held vertically on smooth horizontal surface. At $t = 0$, it is slightly disturbed so that it starts slipping on the smooth surface. Find the angular velocity ω and velocity v_0 of center of mass when rod makes an angle θ with the horizontal.



Sol. Since external force, namely, mg and N act in vertical direction only, the center of mass will move in the vertical direction. End A of the rod can only slip in the horizontal direction (see fig. below). The instantaneous axis rotation will be at point O when rod makes an angle θ with the horizontal.



Conserving mechanical energy, we obtain

$$(mg\ L/2)(1-\sin\theta) = \frac{1}{2}I_o\omega^2$$

in which $I_o = I_{cm} + md^2$

$$= \frac{mL^2}{12} + \frac{mL^2}{4}\cos^2\theta$$

$$\Rightarrow \omega = \sqrt{\left(\frac{1-\sin\theta}{1+3\cos^2\theta}\right)\frac{12g}{L}}$$

The velocity of the center of mass is given by.

$$v_o = \omega(L/2)\cos\theta$$

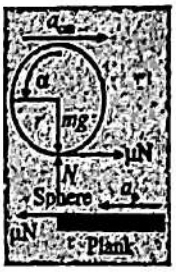
$$= \sqrt{\frac{12(1-\sin\theta)gL}{1+3\cos^2\theta}}\frac{\cos\theta}{2}$$

Example 35: A plank of mass M rests on a smooth horizontal plane. A sphere of mass m and radius r placed on the rough upper surface of the plank and the plank is suddenly given a velocity v in the direction of its length. Find the time after which the sphere begins rolling, if the coefficient of friction between the plank and the sphere is μ and the plank is sufficiently long.

Sol. Let t be the time after which slipping between the sphere and plank ceases.

For the sphere, $N = mg$, $\mu N = ma_{cm}$

$$\Rightarrow a_{cm} = \mu g$$



$$\tau = I\alpha \Rightarrow \mu mgr = (2/5)mr^2\alpha$$

$$\Rightarrow \alpha = \frac{5\mu g}{2r}$$

For the plank,

$$\mu N = Ma_p \Rightarrow a_p = \frac{\mu mg}{M}$$

After time t,

velocity of plank, $v_P = v - \frac{\mu mg}{M}t$

velocity of sphere, $v_{cm} = \mu gt$

angular velocity of sphere, $\omega = \frac{5\mu g}{2r}t$

For no slipping, the point of contact of sphere should have same speed as that of plank,

$$\Rightarrow v_{cm} + \omega r = v_P$$

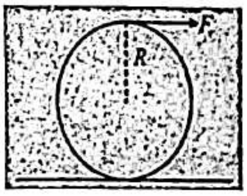
Substituting and solving

$$\mu gt + \left(\frac{5\mu g}{2r}t\right)r = v - \frac{\mu mg}{M}t$$

$$\Rightarrow t = \frac{2v}{g(2\mu + 5\mu + 2\mu m/M)} = \frac{2v}{\mu g[7 + (2m/M)]}$$

Concept Application

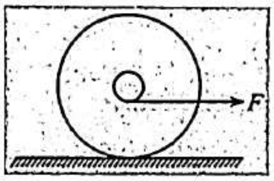
26. A tangential force F acts at the top of a thin spherical shell of mass m and radius R. The acceleration of the shell if it rolls without slipping is:



(a) $\frac{6F}{m}$ (b) $\frac{5F}{6m}$

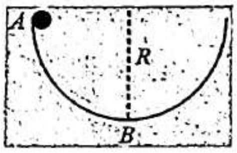
(c) $\frac{F}{m}$ (d) $\frac{6F}{5m}$

27. A yo-yo arranged as shown, rests on a frictionless surface. When a force F is applied to the string, the yo-yo



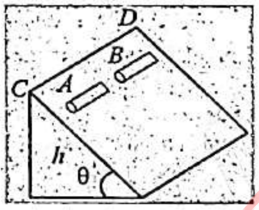
(*a*) moves to the left and rotates counterclockwise
(*b*) moves to the right and rotates counterclockwise
(*c*) moves to the left and rotates clockwise
(*d*) moves to the right and rotates clockwise

28. A small sphere of mass m and radius r rolls without slipping inside a large fixed hemispherical bowl of radius $R(>> r)$ as shown in figure. If the sphere starts from rest at the top point of the hemisphere, then find the normal force exerted by the small sphere on the hemisphere when it is at the bottom B of the hemisphere.



(*a*) $\frac{10}{7}mg$ (*b*) $\frac{17}{7}mg$ (*c*) $\frac{5}{7}mg$ (*d*) $\frac{7}{5}mg$

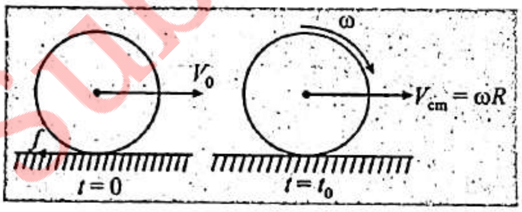
29. Two cylinders A and B are released on a fixed rough inclined plane of angle θ with the horizontal from height 'h'. A is a hollow cylinder and 'B' is a solid cylinder. Both perform pure rolling. Find the distance between the two cylinders when one of them reaches the bottom. Assume that the axis of both cylinders is parallel to the edge CD always.



(*a*) $\frac{h}{2\sin\theta}$ (*b*) $\frac{h}{4\sin\theta}$

(*c*) $\frac{h}{\sin\theta}$ (*d*) $\frac{h}{8\sin\theta}$

30. A uniform disc of mass m and radius R is projected horizontally with velocity V_0 on a rough horizontal floor so that it starts with a purely sliding motion at $t = 0$. After t_0 seconds, it acquires a purely rolling motion as shown in figure. The velocity of the centre of mass of the disc at t_0 is $\frac{xV_0}{3}$ then the value x is.



IMPULSIVE FORCE AND IMPULSIVE TORQUE

When two bodies collide, impulsive force act between them. Associated with an impulsive force is an impulsive torque. Hence if an impulsive force acts on a rigid body it will not only change the velocity of V_{cm} but will also change the angular velocity ω.

From Newton's laws of motion, we have

$$\overline{F}_{ext} = M\,a_{cm}$$

$$\tau_{cm} = r_{\perp}F = I_{cm}\alpha$$

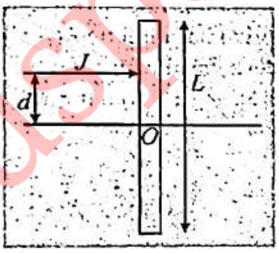
Multiplying both eqn by Δt yield

$$\overline{J} = \left(\overline{F}_{ext}\cdot\Delta t\right) = M\bar{a}_{cm}\,\Delta t = M\left(\overline{V}_f - \overline{V}_i\right)$$

$$Jr_{\perp} = I_{cm}\alpha\Delta t = I_{cm}(\omega_f - \omega_i)$$

where J is the impulse and $Jr_{\perp}$ is the impulsive torque.

Illustration: An impulse J is applied on a rod of mass M and length L lying on a horizontal smooth surface (see fig.). Find the velocity of the centre of mass of the rod and its angular velocity ω after the application of impulse.



Since $V_i = 0$,. $J = MV_{cm}$

$\Rightarrow V_{cm} = J/M$

$$Jd = \frac{ML^2}{12}\omega \quad \Rightarrow \omega = \frac{12Jd}{ML^2}$$

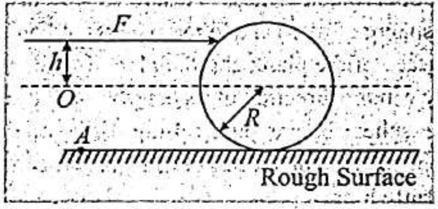
Note: If $d = 0$, then rod will only have translatory motion as external impulsive torque about center of mass vanishes.

Train Your Brain

Example 36: A billiards ball, initially are rest is given a sharp impulse by a cue. The cue is held a distance h above center line (see fig.). The ball leaves the cue with a speed v_0 and because of its forward motion finally acquires a speed of $(9/7)\,v_0$.

Show that $h = (4/5)R$

where R is the radius of ball



Sol. F acts for time Δt. Then

$$V_{cm} = v_o = \frac{F\Delta t}{M} \ \& \ \omega = \frac{F\Delta t h}{(2/5)MR^2}$$

$$\Rightarrow \omega = \frac{5v_o h}{2R^2}$$

After the application of impulse, angular momentum about A will be conserved.

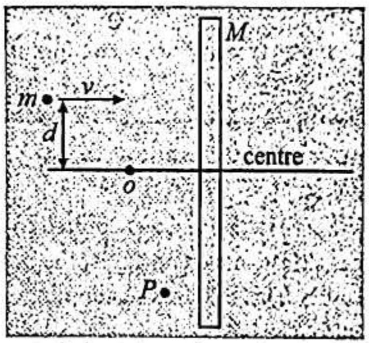
$$\Rightarrow L_i = L_f$$

$$\Rightarrow Mv_o R + I_{cm}\omega = MV_f R + I_{cm}\frac{V_f}{R}$$

$$\Rightarrow Mv_o R + \frac{2}{5}MR^2 \times \frac{5v_o h}{2R^2} = \left(MR + \frac{2}{5}\frac{MR^2}{R}\right)\frac{9}{7}v_o$$

$$\Rightarrow MR + Mh = \frac{9}{5}MR \Rightarrow h = \frac{4}{5}R$$

Example 37: A uniform rod of length L lies on a smooth table. It has a mass M and is free to move in any way on the table. A small ball of mass m moving with speed v as shown collides elastically with rod. (*a*) What quantities are conserved during collision. (*b*) What must be the mass of the ball so that it remains at rest immediately after collision?



Sol. During collision, impulsive force act on the ball and the rod. The ball's velocity will reduce while rod's center of mass acquires certain velocity V_{cm} and it will start rotating with certain angular velocity ω. Let the reduced velocity of the ball be v'.

If we consider the system consisting of ball and the rod, there is no external impulsive force acting on it. Consequently, momentum and angular momentum of the system should be conserved. Since the collision is elastic, mechanical energy will also be conserved.

(*b*) Equating momentum, kinetic energy and angular momentum about O, before and after the collision we have

$$mv = mv' + MV_{cm} \qquad \dots (i)$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}MV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 \qquad \dots (ii)$$

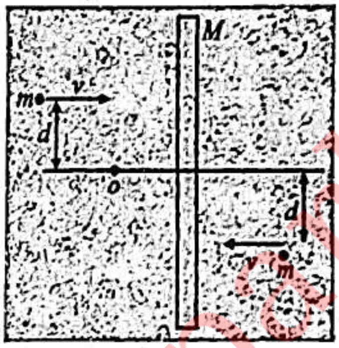
$$mvd = mv'd + I_{cm}\,\omega \qquad \dots (iii)$$

Setting $v' = 0$ in the eqns and solving them, we obtain

$$m = \frac{ML^2}{12d^2 + L^2}$$

Note: Since angular momentum is conserved about any point, you can choose any point about which you write the third eqn. Choose any other point P (see fig.) and write an eqn for angular momentum. Though you shall get a different eqn it has no effect on the final answer.

Example 38: A uniform stick of mass M and length L lie on a smooth horizontal table. It is hit by two bullets fired with velocity v in the opposite direction. The bullets stick to the rod. If the mass of each bullet is m and they simultaneous hit the stick (see fig.), find the angular velocity of the stick after the collision.



Sol. Let the velocity of the center of mass of the stick be V_{cm} and its angular velocity be ω. Since the bullets stick to the rod, mechanical energy will be lost.

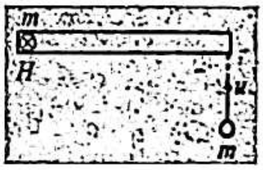
If we consider the system consisting of stick and two bullets, no external force acts. From conservation laws, we obtain

$$mv - mv = (M + 2m)V_{cm} \qquad \dots (i)$$

$$mvd + mvd = \left(\frac{ML^2}{12} + 2md^2\right)\omega \qquad \dots (ii)$$

$$\Rightarrow V_{cm} = 0, \quad \omega = \frac{2mvd}{\frac{ML^2}{12} + 2md^2}$$

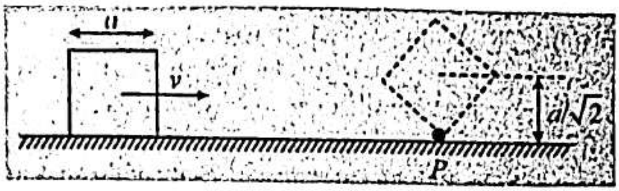
Example 39: A uniform rod of mass m and length ℓ can rotate freely on a smooth horizontal plane about a vertical axis hinged at point H. A point mass having same mass m coming with an initial speed u perpendicular to the rod, strikes the rod perfectly in-elastically at its free end. Find out the angular velocity of the rod just after collision?



Sol. Angular momentum is conserved about H because external impulsive force due to the hinge act at H and it produces no torque about H

$$mu\ell = \left(\frac{m\ell^2}{3} + m\ell^2\right)\omega \Rightarrow \omega = \frac{3u}{4\ell}$$

Example 40: A cube of side length a moves on smooth horizontal surface (see fig.). At point P, it collides inelastically with a small obstacle. As a result it starts rotating about it. Find the minimum velocity v that should be imparted to the block so that it topples over.



Sol. During collision angular momentum about P would be conserved (Why?). Energy will not be conserved as collision is not elastic. Let ω_0 be the angular velocity of cube just after collision.

Conserving angular momentum about P, we obtain

$$mv\frac{a}{2} = I_p\omega_o$$

Where $I_p = I_{cm} + md^2$

$$= \left(m\left(\frac{a^2}{12}+\frac{a^2}{12}\right)\right) + ma^2/2 = \frac{2}{3}ma^2$$

$$\Rightarrow v = \frac{4}{3}a\omega$$

If the cube manages to rotate by $\pi/4$, then it will topple over. Applying energy conservation after the collision, we have

$$\left[0-\frac{1}{2}I_p\omega^2\right] + mg\left[\frac{a}{\sqrt{2}}-\frac{a}{2}\right] = 0$$

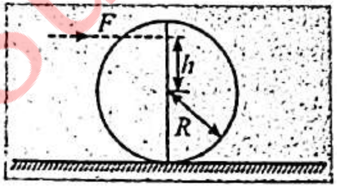
$$\Rightarrow \omega^2 = \frac{2mga}{I_p\sqrt{2}}\left[1-\frac{1}{\sqrt{2}}\right] \Rightarrow \omega^2 = \frac{3}{2}(g/a)\left(\sqrt{2}-1\right)$$

Substituting it in $v = (4/3)a\,\omega$, gives

$$v = \sqrt{\frac{8}{3}ga\left(\sqrt{2}-1\right)}$$

Concept Application

31. An impulsive force F acts horizontally on a uniform solid sphere of radius R placed on a horizontal surface. The line of action of the impulsive force is at a height h above the centre of the sphere. If the rotational and translational kinetic energies of the sphere just after the impulse are equal, then the value of h will be-



(*a*) $\frac{2}{5}R$ (*b*) $\sqrt{\frac{2}{3}}R$

(*c*) $\sqrt{\frac{2}{5}}R$ (*d*) None of these

32. A uniform rod of mass M and length a lies on a smooth horizontal plane. A particle of mass m moving at a speed v perpendicular to the length of the rod strikes it at a distance $a/4$ from the centre and stop after the collision. (*a*) The velocity of the centre of the rod is $\frac{xm}{M}v$ then the value of x is (*b*) The angular velocity of the rod about its centre just after the collision is $\frac{ym}{Ma}v$ then value of y is

33. A thin uniform rod of mass m and length l is free to rotate about its upper end. When it is at rest, it receives an impulse J at its lowest point, normal to its length. Immediately after impact

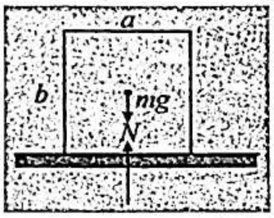
(*a*) The angular velocity of the rod is $\frac{xJ}{ml}$ then the value of x is

(*b*) The linear velocity of the midpoint of the rod is $3J/ym$ then the value of y is

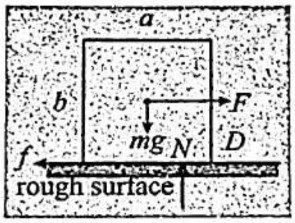
TOPPLING

In many situations an external force is applied to a body to cause it to slide along a surface. In certain cases, the body may tip over before sliding ensues. This is known as toppling.

(1) There is no horizontal force so pressure at bottom is uniform and normal is colinear with mg.

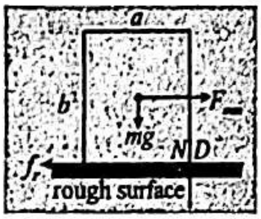


(2) If a force is applied at COM, pressure is not uniform. Normal shifts right so that torque of N can counter balance torque of friction.



(3) If F is continuously increased N keeps shifting towards right until it reaches the right most point D.

Here we have assumed that the surface is sufficiently rough so that there is no sliding as F is increased to F_{max}. If force is increased any further, then torque of N can not counter balance torque of friction f_r and body will topple. The value of force now is the maximum value for which toppling will not occur.



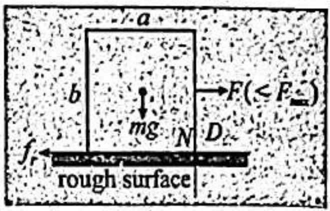
$$F_{max} = f_r$$
$$N = mg$$
$$f_r \cdot b/2 = N \cdot a/2 \Rightarrow f_r = Na/b = mga/b, F_{max} = mga/b$$

(4) If surface is not sufficiently rough and the body slides before F is increased to F_{max} ($= mg\,a/b$) then body will slide before toppling.

This is the case if

$$F_{max} > f_{limit} \Rightarrow mga/b > \mu mg$$

or $\mu < a/b$



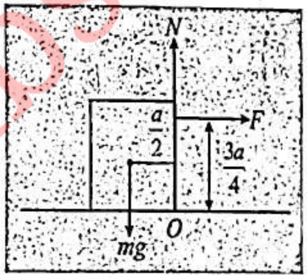
Train Your Brain

Example 41: A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point directly above the centre of the face, at a height $\frac{3a}{4}$ above the base. What is the minimum value of F for which the cube begins to topple about an edge?

Sol. In the limiting case normal reaction will pass through O. The cube will topple about O if torque of F exceeds the torque of mg.

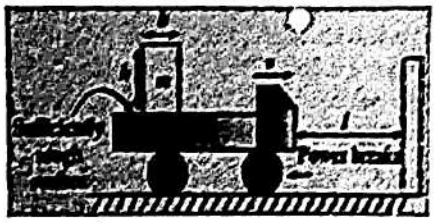
$$\Rightarrow F\left(\frac{3a}{4}\right) > mg\left(\frac{a}{2}\right)$$

$$\Rightarrow F > \frac{2}{3}mg$$



So, the minimum value of F is $\frac{2}{3}mg$

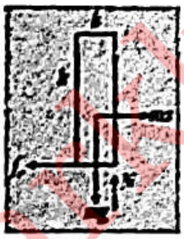
Example 42: Find minimum value of l so that truck can avoid the dead end, without toppling the block kept on it.



Sol. If the truck decelerates at a rate a then in the frame of the truck, pseudo force will act on the block in the forward direction at its center of mass.

$$ma\frac{h}{2} \le mg\frac{b}{2}$$

$$a \le \frac{b}{h}g$$



Final velocity of truck is zero. So that

$$0 = v^2 - 2\left(\frac{b}{h}g\right)l$$

$$l = \frac{h}{2b}\frac{v^2}{g}$$

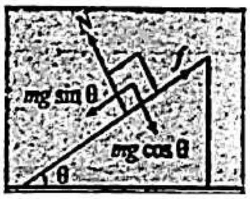
Example 43: A uniform cylinder of height h and radius r is placed with its circular face on a rough inclined plane and the inclination of the plane to the horizontal is gradually increased. If the coefficient of friction is μ, then under what conditions the cylinder will

(*a*) slide before toppling (*b*) topple before sliding

Sol. (*a*) The cylinder will slide if

$$mg\sin\theta > \mu\, mg\cos\theta$$

$$\tan\theta > \mu \qquad (i)$$



The cylinder will topple if

$$(mg\sin\theta)\frac{h}{2} > (mg\cos\theta)r$$

$$\Rightarrow \tan\theta > \frac{2r}{h}$$

Thus, the condition of sliding is $\tan\theta > \mu$ and condition of toppling is $\tan\theta > \frac{2r}{h}$. Hence, the cylinder will slide before toppling if

$$\mu < \frac{2r}{h}$$

(*b*) The cylinder will topple before sliding if

$$\mu > \frac{2r}{h}$$

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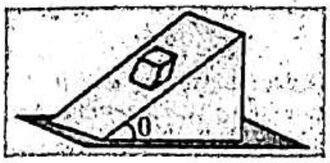
Concept Application

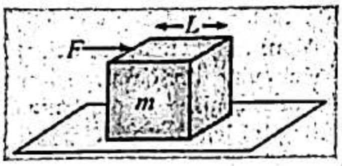
34. A cube is placed on a rough inclined plane of inclination θ as shown in figure. The coefficient of friction between the cube and the plane is μ. If the angle θ is gradually increased, the cube slides before toppling when



(a) $\mu < 1$ (b) $\mu < \frac{1}{2}$

(c) $\mu > 1$ (d) $\mu > \frac{1}{2}$

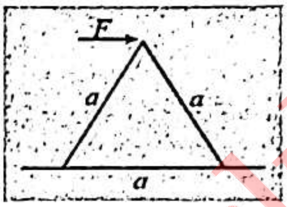
35. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is



(a) Infinitesimal (b) $\frac{mg}{4}$

(c) $\frac{mg}{2}$ (d) $mg(1-\mu)$

36. An equilateral prism of mass 'm' rests on a rough horizontal surface with coefficient of friction μ. A horizontal force 'F' is applied on the prism as shown in Fig. If the coefficient of friction is sufficiently high so that the prism does not slide before toppling, the minimum force required to topple the prism is



(a) $\frac{mg}{\sqrt{3}}$ (b) $\frac{mg}{4}$

(c) $\frac{\mu mg}{\sqrt{3}}$ (d) $\frac{\mu mg}{4}$

37. A solid homogeneous cylinder of height h and base radius r is kept vertically on a conveyor belt moving horizontally with an increasing velocity v, which varies with time 't' as $v = a + bt^2$ (where a and b are constants). If the cylinder is not allowed to slip find the time when the cylinder is about to topple.

(a) $2gr/bh$ (b) $4gr/bh$

(c) gr/bh (d) $gr/2bh$

> *"Obstacles are those frightful things you see when you take your eyes off your goal."*
>
> **– Henry Ford**

Aarambh (Solved Examples)

1. A hollow spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is:
   (*a*) 2/5 (*b*) 2/7 (*c*) 3/5 (*d*) 3/7

Sol. Total energy is, $K = K_R + K_T = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$$= \frac{1}{2}\left(\frac{2}{3}mr^2\right)\omega^2 + \frac{1}{2}mr^2\omega^2$$

$$= \frac{1}{3}mr^2\omega^2 + \frac{1}{2}mr^2\omega^2 = \frac{5}{6}mr^2\omega^2$$

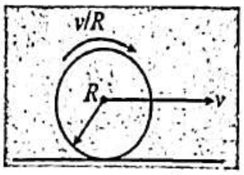
Now, rotational kinetic energy

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{3}mr^2\omega^2$$

$$\therefore \frac{K_R}{K} = \frac{\frac{1}{3}mr^2\omega^2}{\frac{5}{6}mr^2\omega^2} = \frac{2}{5}$$

Therefore, option (*a*) is the correct answer.

2. A disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in figure. At any instant, for the lower most point of the disc.



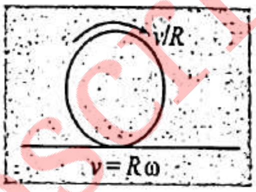
(*a*) Velocity is v, acceleration is zero

(*b*) Velocity is zero, acceleration is zero

(*c*) Velocity is v, acceleration is $\frac{v^2}{R}$

(*d*) Velocity is zero, acceleration is non-zero

Sol. From figure,



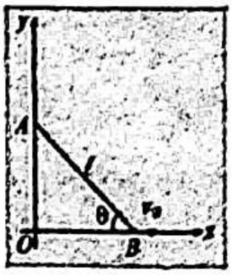
v_{net} (for lowest point) $= v - R\omega = v - v = 0$

and Acceleration $= \frac{v^2}{R} + 0 = \frac{v^2}{R}$

(Since linear speed is constant)

Therefore, option (*d*) is the correct answer.

3. In the figure given below, the end B of the rod AB which makes angle θ with the floor is pulled with a constant velocity v_0 as shown. The length of rod is ℓ. At an instant when $\theta = 37°$



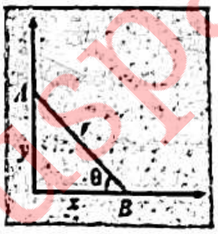
(*a*) Velocity of end A is $\frac{4v_0}{3}$

(*b*) angular velocity of rod is $\frac{5v_0}{6\ell}$

(*c*) angular velocity of rod is constant

(*d*) velocity of end A is constant

Sol.



$$x^2 + y^2 = \ell^2$$

$$\Rightarrow \frac{dy}{dt} = -\left(\frac{x}{y}\right)\frac{dx}{dt} = -\frac{\ell\cos 37°}{\ell\sin 37°}\frac{dx}{dt}$$

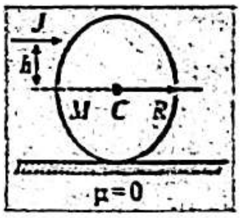
$$\therefore v_A = -\frac{4}{3}v_0$$

Now, $x = \ell\cos\theta$

$$\frac{dx}{dt} = -\ell\sin\theta\frac{d\theta}{dt} \Rightarrow \omega = -\frac{5}{3}\left(\frac{v_0}{\ell}\right)$$

Therefore, option (*a*) is the correct answer.

4. A solid sphere of mass M and radius R is placed on a smooth horizontal surface. It is given a horizontal impulse J at a height h above the centre of mass and sphere starts rolling, then the value of h and speed of centre of mass are:



(*a*) $h = \frac{2}{5}R$ and $v = \frac{J}{M}$

(*b*) $h = \frac{2}{5}R$ and $v = \frac{2}{5}\frac{J}{M}$

(c) $h = \frac{7}{5}R$ and $v = \frac{7}{5}\frac{J}{M}$

(d) $h = \frac{7}{5}R$ and $v = \frac{J}{M}$

Sol. Let the force producing impulse J is F then

$$F \times h = \frac{2}{5}MR^2 \times \alpha$$

$$Fh = \frac{2}{5}MR^2 \times \frac{a}{R}$$

$$Fh = \frac{2}{5}MRa \text{ and } F = Ma \text{ (where } a = R\alpha)$$

$$Mah = \frac{2}{5}MRa$$

$$h = \frac{2R}{5}$$

Also impulse = change in momentum

or $J = Mv$

Therefore, option (*a*) is the correct answer.

5. What must be the relation between length 'L' and radius 'R' of the solid cylinder if its moment of inertia about its axis is equal to that about the equatorial axis?

(*a*) $L = R$ (*b*) $L = 2R$

(*c*) $L = 3R$ (*d*) $L = \sqrt{3}R$

Sol. $$\frac{mR^2}{2} = m\left(\frac{L^2}{12} + \frac{R^2}{4}\right)$$

or $$\frac{R^2}{2} = \frac{L^2}{12} + \frac{R^2}{4}$$

or $L = \sqrt{3}R$

Therefore, option (*d*) is the correct answer.

6. Two loops P and Q are made from a uniform wire. The radii of P and Q are r_1 and r_2 respectively, and their moments of inertia are I_1 and I_2 respectively. If $I_2 = 4I_1$, then $\frac{r_2}{r_1}$ equals:

(*a*) $4^{2/3}$ (*b*) $4^{1/3}$

(*c*) $4^{-2/3}$ (*d*) $4^{-1/3}$

Sol. $I = MR^2 = (2\pi R\lambda)R^2$

or $I \alpha R^3$ or $R \alpha I^{1/3}$

or $$\frac{R_2}{R_1} = \left(\frac{I_2}{I_1}\right)^{1/3} = \left(\frac{4}{1}\right)^{1/3}$$

Therefore, option (*b*) is the correct answer.

7. A loop of radius 3 meter weighs 150 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 15 cm/sec. How much work has to be done to stop it:

(*a*) 3.375 J (*b*) 7.375 J (*c*) 5.375 J (*d*) 9.375 J

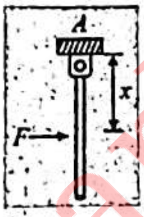
Sol. Required work = Total K.E.

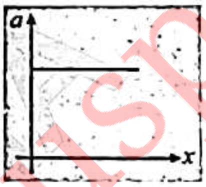
$$= \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)$$

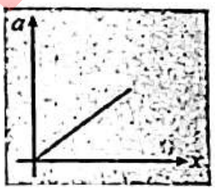
$$= \frac{1}{2} \times 150 \times (0.15)^2 (1+1) = 3.375 \text{ J}$$

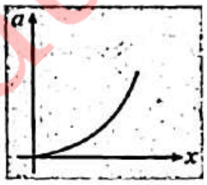
Therefore, option (*a*) is the correct answer.

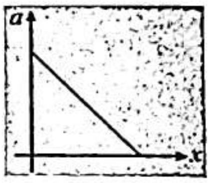
8. A rod of mass m and length l is hinged at one of its end A as shown in figure. A force F is applied at a distance x from A. The acceleration of centre of mass (a) varies with x as:



(*a*)


(*b*)


(*c*)


(*d*)


Sol. The rod will rotate about point A with angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{Fx}{\frac{ml^2}{3}} = \frac{3Fx}{ml^2}$$

$$\therefore a = \frac{l}{2}\alpha = \frac{3}{2}\frac{Fx}{ml} \text{ or } a \propto x$$

i.e., a-x graph is a straight line passing through origin.

Therefore, option (*b*) is the correct answer.

9. Two points of a rod move with velocities $3v$ and v perpendicular to the rod and in the same direction, separated by a distance 'r'. Then the angular velocity of the rod is -

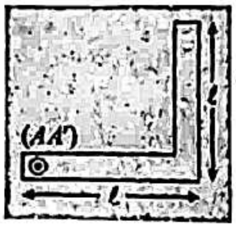
(*a*) $\frac{3v}{r}$ (*b*) $\frac{4v}{r}$ (*c*) $\frac{5v}{r}$ (*d*) $\frac{2v}{r}$

Sol. $\omega_{rod} = \left[\frac{v_{rel.}}{r}\right]$; v_{rel} being the velocity of one point w.r.t other

$= \frac{3v - v}{r}$ and 'r' being the distance between them $= \frac{2v}{r}$

Therefore, option (*d*) is the correct answer.

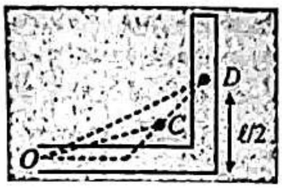
10. A L shaped rod of mass M is free to rotate in a vertical plane about axis AA' as shown in figure. Maximum angular acceleration of rod is-



(a) $\frac{3g}{\sqrt{10\ell}}$ (b) $\frac{9g\ell}{10}$

(c) $\frac{9g\ell}{5}$ (d) $\frac{6g}{5\sqrt{2\ell}}$

Sol. Moment of inertia about axis AA', is



$$\therefore OD = \frac{\sqrt{5}\ell}{2}$$

$$OC = \frac{\sqrt{10}\ell}{4}$$

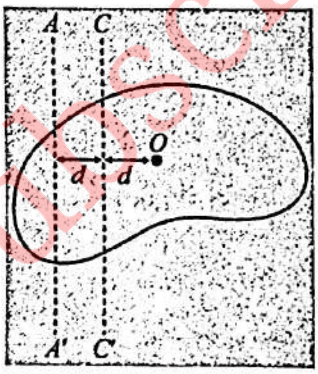
$$I_{AA'} = \left\{\left(\frac{M}{2}\right)\frac{\ell^2}{12} + \left(\frac{M}{2}\right).\frac{5\ell^2}{4} + \left(\frac{M}{2}\right)\frac{\ell^2}{3}\right\}$$

Torque of Mg will be maximum when 'OC' is horizontal

$$\therefore \alpha = \frac{Mg.\sqrt{10}\ell/4}{\frac{5M\ell^2}{6}} = \frac{3g}{\sqrt{10\ell}}$$

Therefore, option (a) is the correct answer.

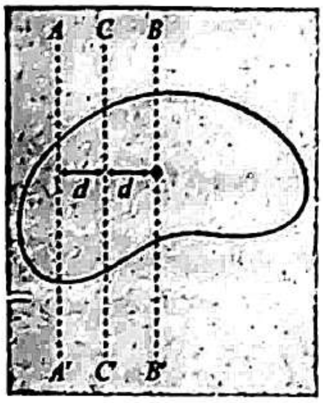
11. Figure shows a body of arbitrary shape 'O' is the centre of mass of the body and mass of the body is M. If $I_{CC'} = I_o$ then $I_{AA'}$ will be equal to –



(a) $I_{CC'} + Md^2$ (b) $I_{CC'} - Md^2$

(c) $I_{CC'} + 3Md^2$ (d) $I_{CC'} + 4Md^2$

Sol. Let BB' be an axis passing through centre of mass and parallel to CC'.

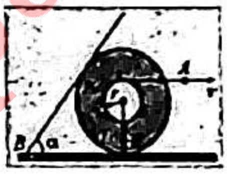


$$\therefore I_{AA}' = I_{BB}' + 4\,Md^2 = I_{CC}' - Md^2 + 4\,Md^2$$

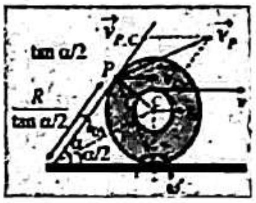
$$I_{AA}' = I_{CC}' + 3\,Md^2$$

Therefore, option (c) is the correct answer.

12. A bobbin rolls without slipping over a horizontal surface so that the velocity v of the end of the thread (point A) is directed along the horizontal. A board hinged at B leans against the bobbin as shown in figure. The inner and outer radii of the bobbin are r and R respectively. Determine the angular velocity ω of the board as a function of angle α.



Sol. Let P be the point of contact of the board and the bobbin at a certain instant. As the bobbin is in pure rolling, the velocity of point P, $\vec{v}_P = \vec{v}_{\text{rotation}} + \vec{v}_{\text{translation}} = \vec{v}_{P.C} + \vec{v}_C$



If the angular velocity of the board at this instant is ω, the linear velocity of the point of the board in contact with the bobbin is $\frac{\omega R}{\tan\frac{\alpha}{2}}$ normal to the board.

If the board is to remain in contact with the bobbin, their normal velocities should be same, i.e.,

$$\frac{\omega R}{\tan\frac{\alpha}{2}} = v_0 \sin\alpha \qquad \text{... }(i)$$

Comparing angular speed about instantaneous centre of rotation,

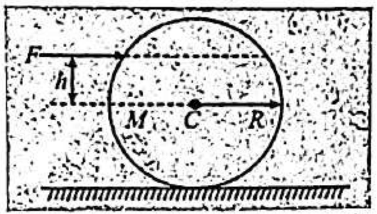
$$\frac{v_0}{R} = \frac{v}{R+r} = \text{angular velocity of bobbin, } \omega$$

$$\therefore v_0 = \frac{vR}{R+r} \qquad \text{... }(ii)$$

Now Eq.(i) becomes $\omega R = \frac{vR}{R+r}\sin\alpha\tan\frac{\alpha}{2}$

$$\text{or } \omega = \frac{2v\sin^2\frac{\alpha}{2}\cos\frac{\alpha}{2}}{(R+r)}$$

13. A solid sphere of radius R and M is placed on a smooth horizontal floor. If it given a horizontal impulse F at a height h above centre of mass and the sphere starts rolling, then its angular speed ω is –



(a) $\frac{2Fh}{5R^2M}$ (b) $\frac{5}{2}\times\frac{Fh}{MR^2}$

(c) $\frac{F}{hM}$ (d) $\frac{7Fh}{5MR^2}$

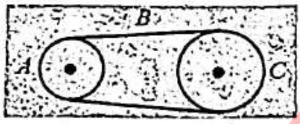
Sol. Angular impulse = Change in angular momentum

$\therefore F\times h = \frac{2}{5}MR^2\times\omega$

$\therefore \omega = \frac{5}{2}\times\frac{Fh}{MR^2}$

Therefore, option (b) is the correct answer.

14. As shown in figure, wheel A of radius $r_A = 10$ cm is coupled by belt B to wheel C of radius $r_C = 25$ cm. The angular speed of wheel A is increased from rest at a constant rate of 1.6 rad/s^2. Time after which wheel C reaches a rotational speed of 100 rpm, assuming the belt does not slip, is nearly -



(a) 4 sec (b) 8 sec (c) 12 sec (d) 16 sec

Sol. $a_t = \alpha_A r_A = \alpha_C r_C$

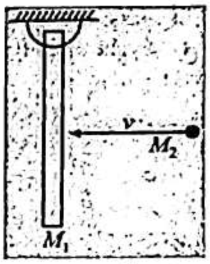
$\alpha_C = \alpha_A\left(\frac{r_A}{r_C}\right)$

$= 1.6\times\frac{10}{25} = 0.64$ rad/s^2

$t = \frac{\omega_C}{\alpha_C} = \frac{\frac{100\times 2\pi}{60}}{0.64} = 16.35$ sec.

Therefore, option (d) is the correct answer.

15. A uniform rod of mass M_1 is hinged at its upper end. A particle of mass M_2 moving horizontally strikes the rod at its mid point elastically. If the particle comes to rest after collision, the value of $\frac{M_1}{M_2}$ is –



(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

Sol. $L_H = M_2 v\,\frac{L}{2} = \frac{M_1L^2}{3}\,\omega$

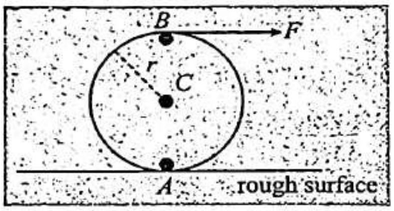
$\omega = \frac{3M_2v}{2M_1L}$ ...(i

for $e = 1$, $v_{CM} = v = \frac{\omega L}{2}$ ...(ii

$\Rightarrow \frac{2v}{L} = \frac{3M_2v}{2M_1L} \Rightarrow \frac{M_1}{M_2} = \frac{3}{4}$

Therefore, option (a) is the correct answer.

16. A force F acts tangentially at the highest point of a sphere of mass m kept on a rough horizontal plane. If the sphere rolls without slipping, find the acceleration of the centre (C) and tangential acceleration of points A and B of the sphere.



Sol. The situation is shown in figure. As the force F rotates the sphere, the point of contact has a tendency to slip towards left so that the static friction on the sphere will act towards right. Let r be the radius of the sphere and a be the linear acceleration of the centre of the sphere. The angular acceleration about the centre of the sphere is $\alpha = a/r$, as there is no slipping.

For the linear motion of the centre, $F + f = ma$ ...(i)

and for the rotational motion about the centre,

$Fr - fr = I\alpha = \left(\frac{2}{5}mr^2\right)\left(\frac{a}{r}\right)$ or $F - f = \frac{2}{5}ma$, ...(ii)

From (i) and (ii), $2F = \frac{7}{5}ma$ or $a = \frac{10F}{7m}$.

Tangential acceleration of point A is zero.

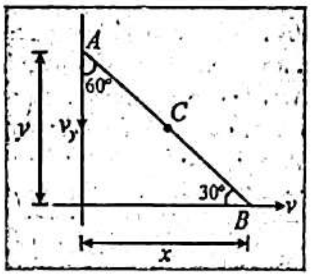
Tangential acceleration of point B is $2a = \left(\frac{20F}{7m}\right)$

17. A ladder of length L is slipping with its ends against a vertical wall and a horizontal floor at a certain moment, the speed of the end in contact with the horizontal floor is v and the angle $\alpha = 30°$ with the horizontal. Then the speed of the ladder's center of mass must be

(*a*) $\frac{2v}{\sqrt{3}}$ (*b*) $\frac{v}{2}$ (*c*) v (*d*) None

Sol. Let the length of the ladder be L.

Since the ladder is constrained to move along the x and y-axis, let the position co-ordinate of the ends of the ladder be $(x, 0), (0, y)$.



$AB = \sqrt{x^2 + y^2} = L$; where L is constant.

Differentiating both sides.

$$\left(\frac{1}{2}(x^2+y^2)^{\frac{-1}{2}}\right)2\left(x\frac{dx}{dt}+y\frac{dy}{dt}\right)=0$$

$$\Rightarrow\left(x\frac{dx}{dt}+y\frac{dy}{dt}\right)=0 \qquad \text{... (i)}$$

Let the velocity of the upper end along the y-axis is v_y.

The velocity of the lower end along the x-axis is $v_x = v$ as given.

Component of v_y along AB = Component of v along AB.

$v_y \cos 60° = v \sin 60°$

$$\Rightarrow \quad v_y \times \frac{1}{2} = v \times \frac{\sqrt{3}}{2} \Rightarrow v_y = \sqrt{3}v$$

The end points of the ladder have co-ordinates $(x, 0), (0, y)$. Position co-ordinates of the center of mass of the ladder $= \frac{1}{2}(x\hat{i} + y\hat{j})$

Since v_y is along the –ve y-axis, we take it to be negative.

$$\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{1}{2}\left(\hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt}\right)$$

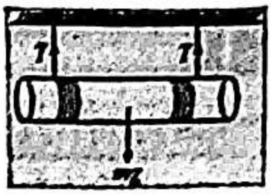
$$\Rightarrow \quad \vec{v}_{cm} = \frac{1}{2}(\hat{i}v_x + \hat{j}v_y) \Rightarrow \vec{v}_{cm} = \frac{1}{2}(\hat{i}v - \hat{j}\sqrt{3}v)$$

∴ Magnitude of the velocity of center of mass: $|v_{cm}|$

$$= \frac{1}{2}\sqrt{((v^2 + (\sqrt{3}v)^2)} = v$$

Therefore, option (*c*) is the correct answer.

18. A cylinder of mass m is suspended through two strings wrapped around it as shown in figure. Find (*a*) the tension T in the string and (*b*) the speed of the cylinder as it falls through a distance h.



Sol. The portion of the strings between the ceiling and the cylinder is at rest. Hence the points of the cylinder where the strings leave it are at rest. The cylinder is thus rolling without slipping on the strings. Suppose the centre of the cylinder falls with an acceleration a. The angular acceleration of the cylinder about its axis is $\alpha = a/R$, as the cylinder does not slip over the strings.

The equation of motion for the centre of mass of the cylinder is

$$mg - 2T = ma \qquad \text{– (i)}$$

and for the motion about the centre of mass, it is

$$2Tr = I\alpha$$

$$2Tr = \frac{1}{2}mr^2\frac{a}{r} \text{ or } 2T = \frac{ma}{2} \qquad \text{– (ii)}$$

From (*i*) and (*ii*), $a = \frac{2}{3}g$ and $T = \frac{mg}{6}$.

As the centre of the cylinder starts moving from rest, the velocity after it has fallen through a distance h is given by

$$v^2 = 2\left(\frac{2}{3}g\right)h \text{ or } v = \sqrt{\frac{4gh}{3}}.$$

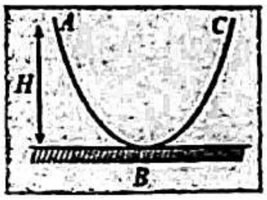
19. A sphere of mass m and radius r is released from rest at point A on a track in vertical plane. The track is rough enough to support rolling between A and B and from B onwards it is smooth. The maximum height attained by sphere from ground on its journey from B onwards is

(*a*) H

(*b*) $\frac{5}{7}H$

(*c*) $\frac{2}{5}H$

(*d*) $\frac{2}{7}H$



Sol. At B

$$mgH = \frac{1}{2}mV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}m\omega^2r^2 + \frac{1}{2}\times\frac{2}{5}mr^2\omega^2 = \frac{7}{10}mr^2\omega^2$$

$$\Rightarrow \omega^2 = \frac{10gH}{7r^2}$$

At highest point along curve BC, $mgH = mgh + \frac{1}{2}I_{cm}\omega^2$

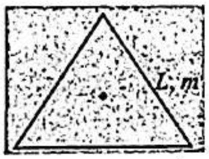
$$\Rightarrow mgH = mgh + \frac{1}{2}\times\frac{2}{5}mr^2\times\frac{10gH}{7r^2} \Rightarrow h = H\left(1-\frac{2}{7}\right) = \frac{5H}{7}.$$

Therefore, option (*b*) is the correct answer.

School Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. Three rods each of mass m and length L are joined to form an equilateral triangle as shown in the figure. What is the moment of inertia about an axis passing through the centre of mass of the system and perpendicular to the plane?



(a) $2\,mL^2$ (b) $\frac{mL^2}{2}$

(c) $\frac{mL^2}{3}$ (d) $\frac{mL^2}{6}$

2. A circular disc is to be made by using iron and aluminium so that it possesses maximum moment of inertia about geometrical axis. It is possible with

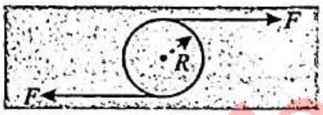
(a) Aluminium at interior and iron surrounding it

(b) Iron at interior surrounded by aluminium

(c) Using iron and aluminium layers in alternate order

(d) Sheet of iron is used at both external surfaces and aluminium as interior layer

3. Two equal and opposite forces are applied tangentially to a uniform disc of mass M and radius R as shown in the figure. If the disc is pivoted at its centre and free to rotate in its plane, the angular acceleration of the disc is



(a) $\frac{F}{MR}$ (b) $\frac{2F}{3MR}$

(c) $\frac{4F}{MR}$ (d) Zero

4. A force $\vec{F} = \left(2\hat{i} + 3\hat{j} - 5\hat{k}\right)$N acts at a point $\vec{r}_1 = \left(2\hat{i} + 4\hat{j} + 7\hat{k}\right)$m. The torque of the force about the point $\vec{r}_2 = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$m is

(a) $\left(17\hat{j} + 5\hat{k} - 3\hat{i}\right)$Nm

(b) $\left(2\hat{i} + 4\hat{j} - 6\hat{k}\right)$Nm

(c) $\left(12\hat{i} - 5\hat{j} + 7\hat{k}\right)$Nm

(d) $\left(-22\hat{i} + 13\hat{j} - \hat{k}\right)$Nm

5. A horizontal disc rotating freely about a vertical axis through its centre makes 90 revolutions per minute. A small piece of wax of mass m falls vertically on the disc and sticks to it at a distance r from the axis. If the number of revolutions per minute reduce to 60, then the moment of inertia of the disc is

(a) mr^2 (b) $\frac{3}{2}mr^2$ (c) $2\,mr^2$ (d) $3\,mr^2$

6. If two discs of moment of inertia I_1 and I_2 rotating about collinear axis passing through their centres of mass and perpendicular to their plane with angular speeds ω_1 and ω_2 respectively in opposite directions are made to rotate combinedly along same axis, then the magnitude of angular velocity of the system is

(a) $\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$ (b) $\frac{I_1\omega_1 - I_2\omega_2}{I_1 + I_2}$

(c) $\frac{I_1\omega_1 + I_2\omega_2}{\omega_1 + \omega_2}$ (d) $\frac{I_1\omega_1 - I_2\omega_2}{\omega_1 - \omega_2}$

7. A hollow sphere of mass 1 kg and radius 10 cm is free to rotate about its diameter. If a force of 30 N is applied tangentially to it, its angular acceleration is (in rad/s^2)

(a) 5000 (b) 450 (c) 50 (d) 5

8. A swimmer while jumping into river from a height easily form a loop in air if

(a) He pulls his arms and legs in

(b) He spreads his arms and legs

(c) He keep himself straight

(d) None of these.

9. The moment of inertia of a body depends on

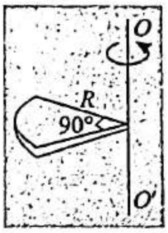
(a) The mass of the body.

(b) The distribution of the mass in the body.

(c) The axis of rotation of the body

(d) All of these

10. A quarter disc of radius R and mass m is rotating about the axis OO' (perpendicular to the plane of the disc) as shown. Rotational kinetic energy of the quarter disc is



(a) $\frac{1}{2}mR^2\omega^2$ (b) $\frac{1}{4}mR^2\omega^2$

(c) $\frac{1}{8}mR^2\omega^2$ (d) $\frac{1}{16}mR^2\omega^2$

Directions: These questions consist of two statements each, printed as Assertion and Reason. While answering these questions, you are required to choose any one of the following four responses.

(*a*) Both Assertion and Reason are True and the Reason is a correct explanation of the Assertion.

(*b*) Both Assertion and Reason are True but Reason is not a correct explanation of the Assertion.

(*c*) Assertion is True but the Reason is False.

(*d*) Assertion and Reason are False

11. **Assertion:** When a rigid body rotates about any fixed axis, then all the particles of it move in circles of different radii but with same angular velocity.

Reason: In rigid body relative position of particles are fixed.

12. **Assertion:** If angular momentum of an object is constant about a point, then net torque on it about that point is zero

Reason: Torque is equal to the rate of change of angular momentum.

VERY SHORT ANSWER TYPE QUESTIONS

13. The cross product of two vectors is zero. What is the angle between them?

14. Name the physical quantity, which is conserved when no external torque acts on a body.

15. Define moment of inertia of a rigid body.

16. Give the expression for the total kinetic energy of a rigid body rolling down an inclined plane without slipping.

17. What is the relation between moment of inertia (I), angular momentum (L) and angular velocity (ω)?

18. Which rule is used to determine the direction of torque?

19. Which physical quantity is expressed by the rate of change of angular velocity?

SHORT ANSWER TYPE QUESTIONS

20. Do the comparison between translational motion and rotational motion by making a list of variables of translational motion and their analogue rotational variable.

21. Obtain an expression for the instantaneous power of a particle in terms of torque (τ) and angular velocity (ω).

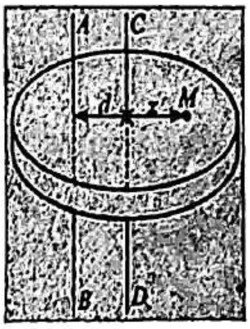
LONG ANSWER TYPE QUESTIONS

22. State the law of conservation of angular momentum and prove that torque is given by the product of moment of inertia and angular acceleration and it is the rate of change of angular momentum.

23. Define the moment of inertia. A thin uniform rod of mass 6 *m* is bent to form a regular hexagon of side *a*. What is the moment of inertia of the hexagon about an axis passing through its centre and perpendicular to its plane.

CASE STUDY BASED QUESTIONS

24. The parallel axis theorem states that, the moment of inertia of a body about any axis is equal to the moment of inertia about parallel axis through its center of mass plus the product of the mass of the body and the square of the perpendicular distance between the two parallel axes.



Mathematically,

$I = I_{CM} + Md^2$

where,

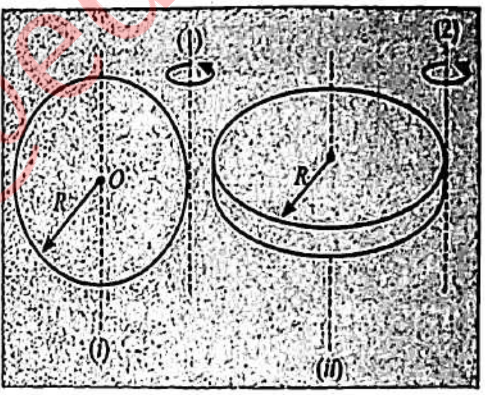
I = moment of inertia of the body

I_{CM} = moment of inertia about of the center mass

M = mass of the body

d^2 = square of the distance between two axes

Find out the moment of inertia of a disc having uniform mass distribution (mass M and radius R) about an axis which is tangent to the disc if



(i) The tangent is in the plane of the disc as in (Fig. (i))

(*a*) $1/2MR^2$ (*b*) $3/2MR^2$

(*c*) $5/4MR^2$ (*d*) MR^2

(ii) The tangent is perpendicular to the plane of the disc as in (Fig. (ii)).

(*a*) $1/2MR^2$ (*b*) $3/2MR^2$

(*c*) $5/4MR^2$ (*d*) MR^2

25. The angular momentum (L) of a rigid body rotating about a fixed axis can be calculated using the following formula:

$L = I\omega$

where:

(1) L represent the angular momentum of the rigid body.

(2) I is the moment of inertia of the rigid body,

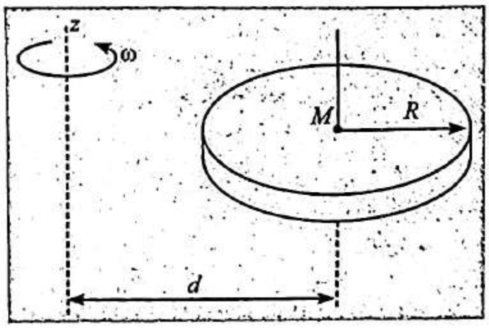
(3) ω represents the angular velocity of the rigid body,

Also, According to Law of conservation angular momentum, "The total angular momentum of a system is constant in both magnitude and direction if the net external torque acting on the system is zero, that is, if the system is isolated."

i.e, $L_i = L_f$ or $I_i\omega_i = I_f\omega_f$

Answer the following question with the help of above passage.

(i) Find the angular momentum of a disc about the axis shown in fig.



(*a*) $\left(\frac{3MR^2}{2}+Md^2\right)\omega(-\hat{k})$

(*b*) $\left(\frac{MR^2}{2}+Md^2\right)\omega(\hat{k})$

(*c*) $(Md^2)\omega(\hat{k})$

(*d*) $\left(\frac{MR^2}{2}\right)\omega(\hat{k})$

(ii) A flywheel having moment of inertia 20 kg m^2 rotates about its axis with an angular velocity of 100 $rads^{-1}$. If the moment of inertia of the flywheel is reduced to 10 kg m^2 without applying external torque, calculate the new angular velocity of the Flywheel.

(*a*) 50 $rads^{-1}$

(*b*) 100 $rads^{-1}$

(*c*) 150 $rads^{-1}$

(*d*) 200 $rads^{-1}$

Prarambh (Topicwise)

ANGULAR DISPLACEMENT, VELOCITY AND ACCELERATION

1. Two bodies of mass 10 kg and 5 kg moving in concentric orbits of radius r_1 and r_2 such that their periods are same. The ratio of centripetal accelerations is

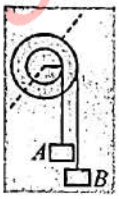
(*a*) r_1/r_2 (*b*) r_2/r_1

(*c*) $(r_1/r_2)^3$ (*d*) $(r_2/r_1)^2$

2. A wheel starts rotating from rest and attains an angular velocity of 60 rad/sec in 5 seconds. The total angular displacement in radians will be-

(*a*) 60 (*b*) 80 (*c*) 100 (*d*) 150

3. Figure shows a small wheel fixed coaxially on a bigger one of double the radius. The system rotates uniformly about the common axis. The strings supporting *A* and *B* do not slip on the wheels. If *x* and *y* be the distances travelled by *A* and *B* in the same time interval, then-



(*a*) $x = 2y$ (*b*) $x = y$

(*c*) $y = 2x$ (*d*) None of these

4. The linear and angular acceleration of a particle are 10 m/sec^2 and 5 rad/sec^2 respectively It will be at a distance from the axis of rotation.

(*a*) 50 m (*b*) $\frac{1}{2}$ m

(*c*) 1 m (*d*) 2 m

5. A particle is moving with a constant angular velocity about an exterior axis. Its linear velocity will depend upon -

(*a*) perpendicular distance of the particle form the axis

(*b*) the mass of particle

(*c*) angular acceleration of the particle

(*d*) the linear acceleration of particle

6. A body is in pure rotation. The linear speed v of a particle, the distance r of the particle from the axis and the angular velocity ω of the body are related as $\omega = \frac{v}{r}$. Thus:

(*a*) $\omega \propto \frac{1}{r}$

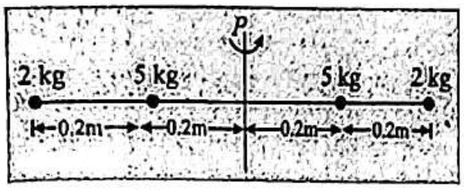
(*b*) $\omega \propto r$

(*c*) $\omega = 0$

(*d*) ω is independent of r.

MOMENT OF INERTIA

7. Four masses are fixed on a massless rod as shown in fig. The moment of inertia about the axis P is nearly



(*a*) 2 kg m^2 (*b*) 1 kg m^2

(*c*) 0.5 kg m^2 (*d*) 0.3 kg m^2

8. By the theorem of perpendicular axes, if a body be in X–Z-plane then:

(*a*) $I_x - I_y = I_z$ (*b*) $I_x + I_z = I_y$

(*c*) $I_x + I_y = I_z$ (*d*) $I_y + I_z = I_x$

9. The axis X and Z in the plane of a disc are mutually perpendicular and Y-axis is perpendicular to the plane of the disc. If the moment of inertia of the body about X and Y axes is respectively 30 kg m^2 and 40 kg m^2 then *M.I.* about Z-axis in kg m^2 will be:

(*a*) 70 (*b*) 50 (*c*) 10 (*d*) Zero

10. Two rods each of mass m and length ℓ are joined at the centre to form a cross. The moment of inertia of this cross about an axis passing through the common centre of the rods and perpendicular to the plane formed by them, is:

(*a*) $\frac{m\ell^2}{12}$ (*b*) $\frac{m\ell^2}{6}$ (*c*) $\frac{m\ell^2}{3}$ (*d*) $\frac{m\ell^2}{2}$

11. For the same total mass which of the following will have the largest moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of the body

(*a*) A disc of radius a

(*b*) A ring of radius a

(*c*) A square lamina of side $2a$

(*d*) Four rods forming a square of side $2a$

12. The moment of inertia of a thin uniform circular disc about one of the diameters is I. Its moment of inertia about an axis perpendicular to the plane of disc and passing through its centre is:

(*a*) $(\sqrt{2})I$ (*b*) $2I$ (*c*) $I/2$ (*d*) $I/\sqrt{2}$

13. The moment of inertia of a uniform semicircular wire of mass M and radius r about a line perpendicular to the plane of the wire through the centre is:

(*a*) Mr^2 (*b*) $\frac{1}{2}Mr^2$

(*c*) $\frac{1}{4}Mr^2$ (*d*) $\frac{2}{5}Mr^2$

14. The density of a rod AB increases linearly from A to B. Its midpoint is O and its centre of mass is at C. Four axes pass through A, B, O and C, all perpendicular to the length of the rod. The moments of inertia of the rod about these axes are I_A, I_B, I_O and I_C respectively.

(*a*) $I_A > I_B$ (*b*) $I_A < I_B$

(*c*) $I_O = I_C$ (*d*) $I_O < I_C$

15. A stone of mass 4 kg is whirled in a horizontal circle of radius 1m and makes 2 rev/sec. The moment of inertia of the stone about the axis of rotation is-

(*a*) 64 kg × m^2 (*b*) 4 kg × m^2

(*c*) 16 kg × m^2 (*d*) 1 kg × m^2

16. In an arrangement four particles, each of mass 2 units are situated at the coordinate points (3, 2, 0), (1, –1, 0), (0, 0, 0) and (–1, 1, 0). The moment of inertia of this arrangement about the Z-axis will be-

(*a*) 8 units (*b*) 16 units

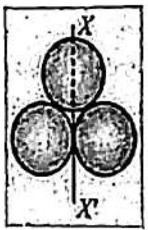
(*c*) 43 units (*d*) 34 units

17. A solid sphere and a hollow sphere of the same mass have the same *M.I.* about their respective diameters. The ratio of their radii will be:

(*a*) 1 : 2 (*b*) $\sqrt{3}:\sqrt{5}$

(*c*) $\sqrt{5}:\sqrt{3}$ (*d*) 5 : 4

18. Three solid spheres of mass M and radius R are shown in the figure. The moment of inertia of the system about XX' axis wil be:



(*a*) $\frac{7}{2}MR^2$ (*b*) $\frac{14}{5}MR^2$

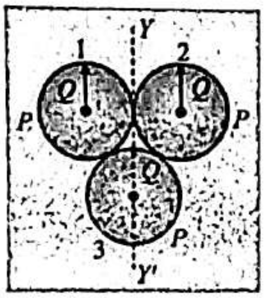
(*c*) $\frac{16}{5}MR^2$ (*d*) $\frac{21}{5}MR^2$

19. The moment of inertia of a square lamina about the perpendicular axis through its centre of mass is 20 kg-m^2. Then the moment of inertia about an axis touching its side and in the plane of the lamina will be:

(*a*) 10 kg-m^2 (*b*) 30 kg-m^2

(*c*) 40 kg-m^2 (*d*) 25 kg-m^2

20. Three rings, each of mass P and radius Q are arranged as shown in the figure. The moment of inertia of the arrangement about YY' axis will be:



(a) $\frac{7}{2}PQ^2$ (b) $\frac{2}{7}PQ^2$ (c) $\frac{2}{5}PQ^2$ (d) $\frac{5}{2}PQ^2$

21. The moment of inertia of a rod of mass M and length L about an axis passing through one edge and perpendicular to its length will be:

(a) $\frac{ML^2}{12}$ (b) $\frac{ML^2}{6}$ (c) $\frac{ML^2}{3}$ (d) ML^2

22. Three thin uniform rods each of mass M and length L and placed along the three axis of a Cartesian coordinate system with one end of each rod at the origin. The $M.I.$ of the system about z-axis is:

(a) $\frac{ML^2}{3}$ (b) $\frac{2ML^2}{3}$ (c) $\frac{ML^2}{6}$ (d) ML^2

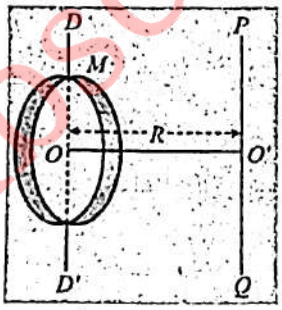
23. Four particles each of mass m are placed at the corners of a square of side length ℓ. The radius of gyration of the system about an axis perpendicular to the square and passing through centre is:

(a) $\frac{\ell}{\sqrt{2}}$ (b) $\frac{\ell}{2}$ (c) ℓ (d) $\ell\sqrt{2}$

24. The $M.I.$ of a thin rod of length ℓ about the perpendicular axis through its centre is I. The $M.I.$ of the square structure made by four such rods about a axis perpendicular to the plane and through the centre will be:

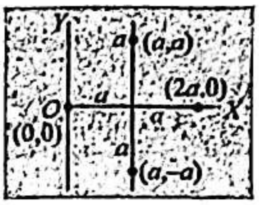
(a) $4I$ (b) $8I$ (c) $12I$ (d) $16I$

25. The moment of inertia of a ring of mass M and radius R about PQ axis will be:



(a) MR^2 (b) $\frac{MR^2}{2}$ (c) $3/2MR^2$ (d) $2MR^2$

26. Four point masses (each of mass m) are arranged in the $X-Y$ plane the moment of inertia of this array of masses about Y-axis is



(a) ma^2 (b) $2ma^2$ (c) $4ma^2$ (d) $6ma^2$

FIXED AXIS OF ROTATION, TOPPLING

27. For a system to be in equilibrium, the torques acting on it must balance. This is true only if the torques are taken about

(a) the centre of the system

(b) the centre of mass of the system

(c) any point on the system

(d) any point on the system or outside it

28. A rectangular block has a square base measuring $a \times a$, and its height is h. It moves on a horizontal surface in a direction perpendicular to one of the edges h being vertical. The coefficient of friction is μ. It will topple if

(a) $\mu > \frac{h}{a}$ (b) $\mu > \frac{a}{h}$

(c) $\mu > \frac{2a}{h}$ (d) $\mu > \frac{a}{2h}$

29. A force of $\left(2\hat{i}-4\hat{j}+2\hat{k}\right)$ Newton acts at a point $\left(3\hat{i}+2\hat{j}-4\hat{k}\right)$ metre from the origin. The magnitude of torque is:

(a) zero (b) 24.4 N-m

(c) 0.244 N-m (d) 2.444 N-m

30. Rate of change of angular momentum with respect to time is proportional to:

(a) angular velocity (b) angular acceleration

(c) moment of inertia (d) torque

31. When constant torque is acting on a body then:

(a) body maintain its state or moves in straight line with same velocity

(b) acquire linear acceleration

(c) acquire angular acceleration

(d) rotates with a constant angular velocity

32. If $I = 50$ kg-m^2, then how much torque will be applied to stop it in 10 sec. Its initial angular speed is 20 rad/sec:

(a) 100 N-m (b) 150 N-m

(c) 200 N-m (d) 250 N-m

33. A particle is at a distance r from the axis of rotation. A given torque τ produces some angular acceleration in it. If the mass of the particle is doubled and its distance from the axis is halved, the value of torque to produce the same angular acceleration is:

(a) $\tau/2$ (b) τ (c) 2τ (d) 4τ

ENERGY ANALYSIS

34. A ring of radius r and mass m rotates about an axis passing through its centre and perpendicular to its plane with angular velocity ω. Its kinetic energy is

(*a*) $mr\omega$ (*b*) $\frac{1}{2}mr\omega^2$

(*c*) $mr^2\omega^2$ (*d*) $\frac{1}{2}mr^2\omega^2$

35. A rod of length L is hinged at one end. It is brought to a horizontal position and released. The angular velocity of the rod when it is in vertical position is

(*a*) $\sqrt{2g/L}$ (*b*) $\sqrt{3g/L}$

(*c*) $\sqrt{g/2L}$ (*d*) $\sqrt{g/L}$

36. Two bodies A and B having same angular momentum and $I_A > I_B$, then the relation between $(K.E.)_A$ and $(K.E.)_B$ will be:

(*a*) $(K.E.)_A > (K.E.)_B$ (*b*) $(K.E.)_A = (K.E.)_B$

(*c*) $(K.E.)_A < (K.E.)_B$ (*d*) $(K.E.)_A \le (K.E.)_B$

ANGULAR MOMENTUM

37. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Two objects, each of mass m are attached gently to the opposite ends of a diameter of the ring. The wheel now rotates with an angular velocity

(*a*) $\frac{\omega M}{(M+m)}$ (*b*) $\frac{\omega(M-2m)}{(M+2m)}$

(*c*) $\frac{\omega M}{(M+2m)}$ (*d*) $\frac{\omega(M+2m)}{M}$

38. A rotating table completes one rotation in 10 sec and its moment of inertia is 100 kg-m². A person of 50 kg mass stands at the centre of the rotating table. If the person moves 2m. from the centre, the angular velocity of the rotating table in rad/sec. will be:

(*a*) $\frac{2\pi}{30}$ (*b*) $\frac{20\pi}{30}$

(*c*) $\frac{2\pi}{3}$ (*d*) 2π

39. A uniform heavy disc is rotating at constant angular velocity (ω) about a vertical axis through its centre O. Some wax is dropped gently on the disc. The angular velocity of the disc:

(*a*) does not change (*b*) increases

(*c*) decreases (*d*) becomes zero

40. A girl sits near the edge of a rotating circular platform. If the girl moves from circumference towards the centre of the platform then the angular velocity of the platform will:

(*a*) decrease (*b*) increase

(*c*) remain same (*d*) becomes zero

41. Two wheels P and Q are mounted on the same axle. The moment of inertia of P is 6 kg-m² and it is rotating at 600 rotations per minute and Q is at rest. If the two are joined by means of a clutch then they combined and rotate at 400 rotations per minute. The moment of inertia of Q will be -

(*a*) 3 kg-m² (*b*) 4 kg-m²

(*c*) 5 kg-m² (*d*) 8 kg-m²

COMBINED ROTATION AND TRANSLATION

42. If a spherical ball rolls on a table without slipping, the fraction of its total kinetic energy associated with rotation is

(*a*) 3/5 (*b*) 2/7 (*c*) 2/5 (*d*) 3/7

43. The speed of a homogeneous solid sphere after rolling down an inclined plane of vertical height h, from rest without sliding is

(*a*) $\sqrt{gh}$ (*b*) $\sqrt{(g/5)gh}$

(*c*) $\sqrt{(4/3)gh}$ (*d*) $\sqrt{(10/7)gh}$

44. The rotational kinetic energy of a body is E. In the absence of external torque, if mass of the body is halved and radius of gyration doubled, then its rotational kinetic energy will be:

(*a*) 0.5 E (*b*) 0.25E

(*c*) E (*d*) 2E

45. A ring is rolling without slipping. Its energy of translation is E. Its total kinetic energy will be:

(*a*) E (*b*) 2E (*c*) 3E (*d*) 4E

46. One hollow and one solid cylinder of the same outer radius rolls down on a rough inclined plane. The foot of the inclined plane is reached by

(*a*) solid cylinder earlier

(*b*) hollow cylinder earlier

(*c*) simultaneously

(*d*) the heavier earlier irrespective of being solid or hollow

47. If a solid sphere, disc and cylinder are allowed to roll down an inclined plane from the same height

(*a*) cylinder will reach the bottom first

(*b*) disc will reach the bottom first

(*c*) sphere will reach the bottom first

(*d*) all will reach the bottom at the same time

48. A solid homogeneous sphere is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of the sphere

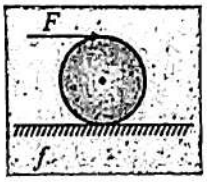
(*a*) total kinetic energy is conserved

(*b*) angular momentum of the sphere about the point of contact with the plane is conserved

(*c*) only the rotational kinetic energy about the centre of mass is conserved

(*d*) angular momentum about centre of mass is conserved

49. A ring of mass M is kept on a horizontal rough surface. A force F is applied tangentially at its rim as shown. The coefficient of friction between the ring and the surface is μ. Then



(*a*) friction will act in the forward direction
(*b*) friction will act in the backward direction
(*c*) frictional force will not act
(*d*) frictional force will be μ Mg.

50. A ring takes time t_1 and t_2 for sliding down and rolling down an inclined plane of length L respectively for reaching the bottom. The ratio of t_1 and t_2 is:

(*a*) $\sqrt{2}:1$ (*b*) $1:\sqrt{2}$
(*c*) $1:2$ (*d*) $2:1$

51. A ladder rests against a frictionless vertical wall, with its upper end 6 m above the ground and the lower end 4 m away from the wall. The weight of the ladder is 500 N and its C.G. at 1/3rd distance from the lower end. Wall's reaction will be, (in Newton)

(*a*) 111 (*b*) 333
(*c*) 222 (*d*) 129

Prabal (JEE Main Level)

1. A fan is running at 3000 rpm. It is switched off. It comes to rest by uniformly decreasing its angular speed in 10 seconds. The total number of revolutions in this period.

(*a*) 150 (*b*) 250
(*c*) 350 (*d*) 300

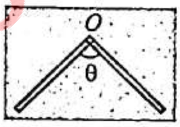
2. The moment of inertia of a non-uniform semicircular wire having mass m and radius r about a line perpendicular to the plane of the wire through the centre is

(*a*) mr^2 (*b*) $\frac{1}{2}mr^2$ (*c*) $\frac{1}{4}mr^2$ (*d*) $\frac{2}{5}mr^2$

3. Let I_A and I_B be the moments of inertia of two solid cylinders of identical geometrical shape and size about their axes, the first made of aluminium and the second of iron.

(*a*) $I_A < I_B$
(*b*) $I_A = I_B$
(*c*) $I_A > I_B$
(*d*) relation between I_A and I_B depends on the actual shapes of the bodies.

4. A uniform thin rod of length L and mass M is bent at the middle point O as shown in figure. Consider an axis passing through its middle point O and perpendicular to the plane of the bent rod. Then moment of inertia about this axis is:



(*a*) $\frac{2}{3}mL^2$ (*b*) $\frac{1}{3}mL^2$
(*c*) $\frac{1}{12}mL^2$ (*d*) dependent on θ

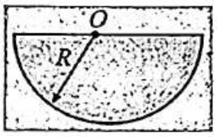
5. Two spheres of same mass and radius are in contact with each other. If the moment of inertia of a sphere about its diameter is I, then the moment of inertia of both the spheres about the tangent at their common point would be -

(*a*) $3I$ (*b*) $7I$ (*c*) $4I$ (*d*) $5I$

6. The *M.I.* of a disc about its diameter is 2 units. Its *M.I.* about axis through a point on its rim and in the plane of the disc is

(*a*) 4 units. (*b*) 6 units
(*c*) 8 units (*d*) 10 units.

7. Moment of inertia of a thin semicircular disc (mass = M & radius = R) about an axis through point O and perpendicular to plane of disc, is given by:



(*a*) $\frac{1}{4}MR^2$ (*b*) $\frac{1}{2}MR^2$
(*c*) $\frac{1}{8}MR^2$ (*d*) MR^2

8. A rigid body can be hinged about any point on the x-axis. When it is hinged such that the hinge is at x, the moment of inertia is given by $I = 2x^2 - 12x + 27$ The x-coordinate of centre of mass is

(*a*) $x = 2$ (*b*) $x = 0$
(*c*) $x = 1$ (*d*) $x = 3$

9. On applying a constant torque on a body about centre of mass-

(*a*) linear velocity may increase
(*b*) angular velocity may increase
(*c*) it will rotate with constant angular velocity
(*d*) it will move with constant velocity

10. A wheel starting with angular velocity of 10 radian/sec acquires angular velocity of 100 radian/sec in 15 seconds. If moment of inertia is 10kg-m^2, then applied torque (in N-m) is

(*a*) 900 (*b*) 100

(*c*) 90 (*d*) 60

11. A torque of 2 N-m produces an angular acceleration of 2 rad/sec^2 in a body. If its radius of gyration is 2m, its mass will be:

(*a*) 2kg (*b*) 4 kg

(*c*) 1/2 kg (*d*) 1/4 kg

12. A uniform circular disc A of radius r is made from a metal plate of thickness t and another uniform circular disc B of radius $4r$ is made from the same metal plate of thickness $t/4$. If equal torques act on the discs A and B, initially both being at rest. At a later instant, the angular speeds of a point on the rim of A and another point on the rim of B are ω_A and ω_B respectively. We have

(*a*) $\omega_A > \omega_B$

(*b*) $\omega_A = \omega_B$

(*c*) $\omega_A < \omega_B$

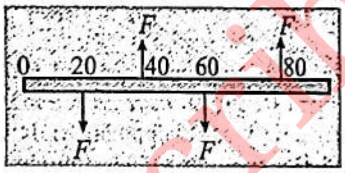
(*d*) the relation depends on the actual magnitude of the torques.

13. A force $\vec{F} = 4\hat{i} - 10\hat{j}$ acts on a body at a point having position vector $-5\hat{i} - 3\hat{j}$ relative to origin of co-ordinates on the axis of rotation. The torque acting on the body about the origin is:

(*a*) $38\,\hat{k}$ (*b*) $-25\,\hat{k}$

(*c*) $62\,\hat{k}$ (*d*) None of these

14. Four equal and parallel forces are acting on a rod (as shown in figure) at distances of 20 cm, 40 cm, 60 cm and 80 cm respectively from one end of the rod. Under the influence of these forces the rod:



(*a*) is at rest

(*b*) experiences a torque

(*c*) experiences a linear acceleration

(*d*) experiences a torque and also a linear acceleration

15. Two men support a uniform horizontal rod at its two ends. If one of them suddenly lets go, the force exerted by the rod on the other man will:

(*a*) remain unaffected

(*b*) increase

(*c*) decrease

(*d*) become unequal to the force exerted by him on the beam.

16. A cubical block of mass M and edge a slides down a rough inclined plane of inclination θ with a uniform velocity. The torque of the normal force on the block about its centre has a magnitude.

(*a*) zero (*b*) Mga

(*c*) $Mga \sin\theta$ (*d*) $\frac{1}{2} Mga \sin\theta$

17. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force 'F' is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $\frac{3a}{4}$ above the base. The minimum value of 'F' for which the cube begins to tilt about the edge is (assume that the cube does not slide).

(*a*) $\frac{2}{3}mg$ (*b*) $\frac{4}{3}mg$ (*c*) $\frac{5}{4}mg$ (*d*) $\frac{1}{2}mg$

18. The moment of inertia and rotational kinetic energy of a fly wheel are 20kg-m^2 and 1000 joule respectively. Its rotational frequency per minute would be:

(*a*) $\frac{600}{\pi}$ (*b*) $\frac{25}{\pi^2}$

(*c*) $\frac{5}{\pi}$ (*d*) $\frac{300}{\pi}$

19. A circular ring of wire of mass M and radius R is making n revolutions/sec about an axis passing through a point on its rim and perpendicular to its plane. The kinetic energy of rotation of the ring is given by-

(*a*) $4\pi^2MR^2n^2$ (*b*) $2\pi^2MR^2n^2$

(*c*) $\pi^2MR^2n^2$ (*d*) $8\pi^2MR^2n^2$

20. Rotational kinetic energy of a disc of constant moment of inertia is:

(*a*) directly proportional to angular velocity

(*b*) inversely proportional to angular velocity

(*c*) inversely proportional to square of angular velocity

(*d*) directly proportional to square of angular velocity

21. A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ in 4 sec. the magnitude of this torque is:

(*a*) $4A_0$ (*b*) A_0

(*c*) $3A_0/4$ (*d*) $12A_0$

22. A particle moves with a constant velocity parallel to the Y-axis. Its angular momentum about the origin.

(*a*) is zero (*b*) remains constant

(*c*) goes on increasing (*d*) goes on decreasing

23. A boy sitting firmly over a rotating stool has his arms folded. If he stretches his arms, his angular momentum about the axis of rotation

(*a*) increases (*b*) decreases

(*c*) remains unchanged (*d*) doubles

24. The rotational kinetic energy of a rigid body of moment of inertia 5 kg-m^2 is 10 joules. The angular momentum about the axis of rotation would be:

(*a*) 100 joule-sec (*b*) 50 joule-sec

(*c*) 10 joule-sec (*d*) 2 joule -sec

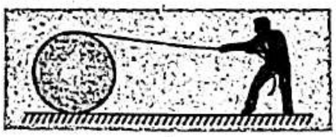
25. The angular velocity of a body changes from one revolution per 9 second to 1 revolution per second without applying any torque. The ratio of its radius of gyration in the two cases is

(*a*) 1 : 9 (*b*) 3 : 1 (*c*) 9 : 1 (*d*) 1 : 3

26. A disc rolls on a table. The ratio of its K.E. of translation to the total K.E. is:

(*a*) 2/5 (*b*) 1/3 (*c*) 5/6 (*d*) 2/3

27. A thin string is wrapped several times around a cylinder kept on a rough horizontal surface. A boy standing at a distance ℓ from the cylinder draws the string towards him as shown in figure. The cylinder rolls without slipping. The length of the string passed through the hand of the boy while the cylinder reaches his hand is



(*a*) ℓ (*b*) 2ℓ (*c*) 3ℓ (*d*) 4ℓ

28. A solid sphere, a hollow sphere and a ring, all having equal mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are equal and but not sufficient to allow pure rolling. The greatest kinetic energy at the bottom of the incline will be achieved by

(*a*) the solid sphere

(*b*) the hollow sphere

(*c*) the ring

(*d*) all will achieve same kinetic energy.

29. A body is given translational velocity and kept on a surface that has sufficient friction. Then:

(*a*) body will move forward before pure rolling

(*b*) body will move backward before pure rolling

(*c*) body will start pure rolling immediately

(*d*) none of these

30. A disc and a ring of the same mass are rolling to have the same kinetic energy. What is ratio of their velocities of centre of mass

(*a*) $(4:3)^{1/2}$ (*b*) $(3:4)^{1/2}$ (*c*) $(2:3)^{1/2}$ (*d*) $(3:2)^{1/2}$

31. A body kept on a smooth horizontal surface is pulled by a constant horizontal force applied at the top point of the body. If the body rolls purely on the surface, its shape can be:

(*a*) thin pipe (*b*) uniform cylinder

(*c*) uniform sphere (*d*) thin spherical shell

32. A sphere is released on a smooth inclined plane from the top. When it moves down its angular momentum is:

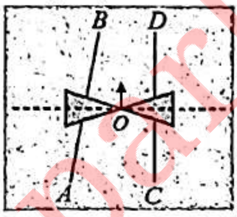
(*a*) Conserved about every point

(*b*) Conserved about the point of contact only

(*c*) Conserved about the centre of the sphere only

(*d*) Conserved about any point on a fixed line parallel to the inclined plane and passing through the centre of the ball.

33. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to:



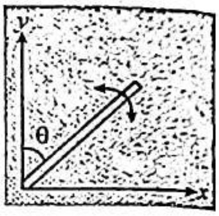
(*a*) turn left and right alternately

(*b*) turn left.

(*c*) turn right.

(*d*) go straight.

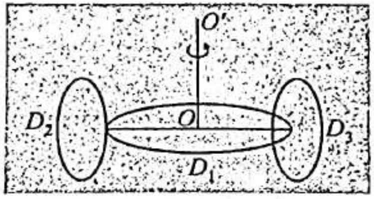
34. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is:



(*a*) $\dfrac{3g}{2\ell}\cos\theta$ (*b*) $\dfrac{2g}{3\ell}\cos\theta$

(*c*) $\dfrac{3g}{2\ell}\sin\theta$ (*d*) $\dfrac{2g}{3\ell}\sin\theta$

35. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly as its opposite ends with the centre of D_2 and D_3 (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1 as shown in the figure, will: (D_2 and D_3 are perpendicular to the plane of D_1)



(*a*) MR^2 (*b*) $3MR^2$

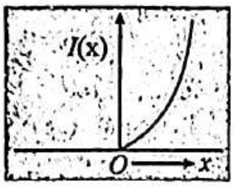
(*c*) $\dfrac{4}{5}MR^2$ (*d*) $\dfrac{2}{3}MR^2$

36. The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians):

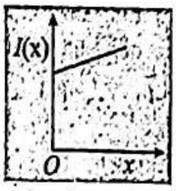
(*a*) $\frac{\pi}{6}$ (*b*) $\frac{\pi}{3}$ (*c*) $\frac{\pi}{8}$ (*d*) $\frac{\pi}{4}$

37. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is '$I(x)$'. Which one of the graphs represents the variation of $I(x)$ with x correctly?

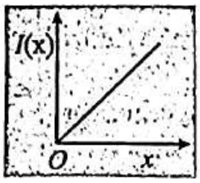
(*a*)



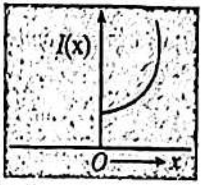
(*b*)



(*c*)



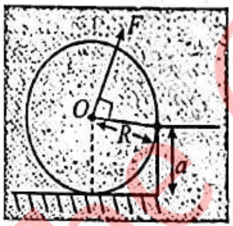
(*d*)



38. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is:

(*a*) $\frac{k}{2I}\theta$ (*b*) $\frac{k}{I}\theta$ (*c*) $\frac{k}{4I}\theta$ (*d*) $\frac{2k}{I}\theta$

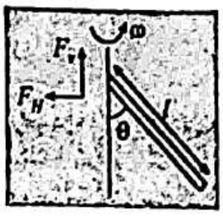
39. A uniform cylinder of mass M and radius R is to be pulled over a step of height a ($a < R$) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is



(*a*) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$ (*b*) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

(*c*) $Mg\sqrt{1-\frac{a^2}{R^2}}$ (*d*) $Mg\frac{a}{R}$

40. A uniform rod of length 'l' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω, the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that



(*a*) $\cos\theta = \frac{g}{l\omega^2}$ (*b*) $\cos\theta = \frac{2g}{3l\omega^2}$

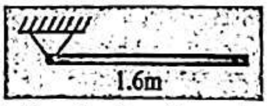
(*c*) $\cos\theta = \frac{g}{2l\omega^2}$ (*d*) $\cos\theta = \frac{3g}{2l\omega^2}$

INTEGER TYPE QUESTIONS

41. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 revolutions per minute (*rpm*). The person now starts moving towards the centre of the platform. What will be the rotational speed (in *rpm*) of the platform when the person reaches its centre _________.

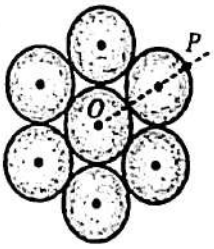
42. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is p% of the initial energy. Value of p is _____.

43. The uniform rod of mass 20 kg and length 1.6 m is pivoted at its end and swings freely in the vertical plane. Angular acceleration of rod just after the rod is released from rest in the horizontal position as shown in figure is $\frac{xg}{16}$. Find the value of x.



44. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. For minimum moment of inertia, the ratio of $\frac{\ell}{R}$ is given as $\sqrt{\frac{x}{2}}$. Find the value of x.

45. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is $\frac{xMR^2}{2}$. Find the value of x.

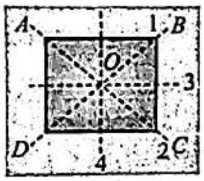


46. The radius of gyration of a uniform rod of length ℓ, about an axis passing through a point $\ell/4$ away from the centre of the rod, and perpendicular to it, is $\sqrt{\frac{7}{x}}\ell$. Find the value of x.

Parikshit (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. The moment of inertia of a thin uniform square plate $ABCD$ of uniform thickness about an axis passing through the centre O and perpendicular to the plate is



where I_1, I_2, I_3, and I_4 are respectively the moments of inertia about axes 1, 2, 3, and 4 which are in the plane of the plate.

(a) $I_1 + I_2$ (b) $I_3 + I_4$

(c) $I_1 + I_3$ (d) $I_1 + I_2 + I_3 + I_4$

2. A rod of weight w is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at a distance x from A.

(a) The normal reaction at A is $\frac{wx}{d}$

(b) The normal reaction at A is $\frac{w(d-x)}{d}$

(c) The normal reaction at B is $\frac{wx}{d}$

(d) The normal reaction at B is $\frac{w(d-x)}{d}$

3. A body is in equilibrium under the influence of a number of forces. Each force has a different line of action. The minimum number of forces required is

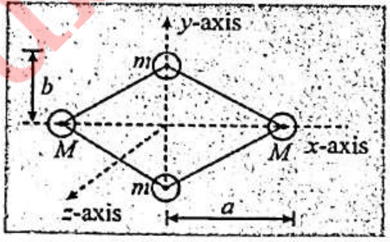
(a) 2, if their lines of action pass through the centre of mass of the body

(b) 3, if their lines of action are not parallel

(c) 3, if their lines of action are parallel

(d) 4, if their lines of action are parallel and all the forces have the same magnitude

4. Four point masses are fastened to the corners of a frame of negligible mass lying in the xy plane. Let ω be the angular speed of rotation. Then



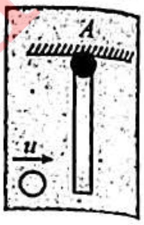
(a) Rotational kinetic energy associated with a given angular speed depends on the axis of rotation.

(b) Rotational kinetic energy about y-axis is independent of m and its value is $Ma^2\omega^2$

(c) Rotational kinetic energy about z-axis depends on m and its value is $(Ma^2 + mb^2)\omega^2$

(d) Rotational kinetic energy about z-axis is independent of m and its value is $Mb^2\omega^2$

5. In the given figure a ball strikes a rod elastically and rod is hinged at point A. Then which of the statement(s) is/are correct for the collision?



(a) Linear momentum of system (ball + rod) is conserved

(b) Angular momentum of system about hinged point A is conserved

(c) Initial KE of the system is equal to final KE of the system

(d) Linear momentum of ball is conserved.

6. A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground.

(a) Angular momentum of the particle about O is increasing

(b) Torque of the gravitational force on the particle about O is decreasing

(c) The moment of inertia of the particle about O is decreasing

(d) The angular velocity of the particle about O is increasing

7. A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground is v.

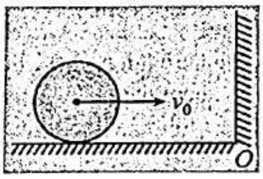
(a) $0 \le v \le 2u$

(b) $v = u$, if CP is horizontal

(c) $v = u$, if CP makes an angle of 30° with the horizontal and P is below the horizontal level of C

(d) $v = \sqrt{2}u$, if CP is horizontal

8. Consider a sphere of mass 'm' radius 'R' doing pure rolling motion on a rough surface having velocity $\vec{v}_0$ as shown in the figure. It makes an elastic impact with the smooth wall and moves back and starts pure rolling after some time again.



(*a*) Change in angular momentum about 'O' in the entire motion equals $2mv_0R$ in magnitude.

(*b*) Moment of impulse provided by wall during impact about O equals $2mv_0R$ in magnitude

(*c*) Final velocity of ball will be $\frac{3}{7}\vec{v}_0$

(*d*) Final velocity of ball will be $-\frac{3}{7}\vec{v}_0$

9. A uniform rod of length l and mass $2m$ rests on a smooth horizontal table. A point mass m moving horizontally at right angle to the rod with velocity v collides with one end of the rod and sticks to it, then

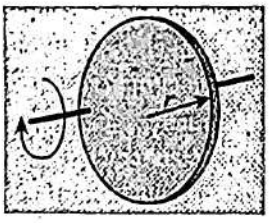
(*a*) angular velocity of the system after collision is v/l.

(*b*) angular velocity of the system after collision is $v/2l$.

(*c*) the loss in kinetic energy of the system as a whole as a result of the collision is $mv^2/6$.

(*d*) the loss in kinetic energy of the system as a whole as a result of the collision is $7mv^2/24$.

10. A uniform disk of radius r and mass M is pivoted about a horizontal axis parallel to its symmetry axis and passing through its edge such that it can swing freely in a vertical plane (Figure). It is released from rest with its center of mass at the same height as the pivot.



(*a*) The maximum angular velocity of disc is $2\sqrt{\frac{g}{R}}$.

(*b*) The initial angular acceleration is $\frac{2g}{3R}$.

(*c*) The speed of centre of mass is maximum when the disc has rotated by 90°.

(*d*) The force exerted by the hinge on disc is equal to weight at initial moment.

11. Three solid cylinders A, B and C each of mass m and radius R are allowed to roll down on three inclined planes A', B' and C' respectively (each of inclination θ) such that for A' and A coefficient of friction is zero, for B' and B coefficient of friction is greater than zero but less than $\frac{\tan\theta}{3}$ and for C and C coefficient of friction is more than $\frac{\tan\theta}{3}$. On reaching bottom of the inclined plane

(*a*) C has maximum angular speed

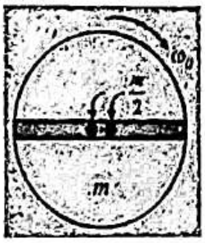
(*b*) B has minimum total kinetic energy

(*c*) A takes minimum time to reach the bottom

(*d*) B has minimum translational kinetic energy

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 12 to 14): A uniform disc of mass 'm' and radius R is free to rotate in horizontal plane about a vertical smooth fixed axis passing through its centre. There is a smooth groove along the diameter of the disc and two small balls of mass $\frac{m}{2}$ each are placed in it on either side of the centre of the disc as shown in figure. The disc is given initial angular velocity ω_0 and released.



12. The angular speed of the disc when the balls reach the end of the disc is:

(*a*) $\frac{\omega_0}{2}$ (*b*) $\frac{\omega_0}{3}$ (*c*) $\frac{2\omega_0}{3}$ (*d*) $\frac{\omega_0}{4}$

13. The speed of each ball relative to ground just after they leave the disc is:

(*a*) $\frac{R\omega_0}{\sqrt{3}}$ (*b*) $\frac{R\omega_0}{\sqrt{2}}$ (*c*) $\frac{2R\omega_0}{3}$ (*d*) $\frac{R\omega_0}{3}$

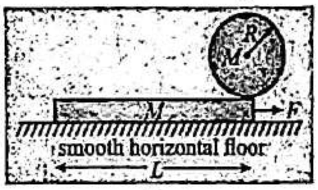
14. The net work done by forces exerted by disc on one of the ball for the duration ball remains on the disc is

(*a*) $\frac{2mR^2\omega_0^2}{9}$ (*b*) $\frac{mR^2\omega_0^2}{18}$

(*c*) $\frac{mR^2\omega_0^2}{6}$ (*d*) $\frac{mR^2\omega_0^2}{9}$

Comprehension (Q. 15 to 17): A uniform disc of mass M and radius R initially stands vertically on the right end of a horizontal plank of mass M and length L, as shown in the figure.

The plank rests on smooth horizontal floor and friction between disc and plank is sufficiently high such that disc rolls on plank without slipping. The plank is pulled to right with a constant horizontal force of magnitude F.



15. Th magnitude of acceleration of plank is-

(*a*) $\frac{F}{8M}$ (*b*) $\frac{F}{4M}$ (*c*) $\frac{3F}{2M}$ (*d*) $\frac{3F}{4M}$

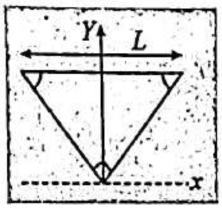
16. The magnitude of angular acceleration of the disc is:

(*a*) $\frac{F}{4MR}$ (*b*) $\frac{F}{8MR}$ (*c*) $\frac{F}{2MR}$ (*d*) $\frac{3F}{2MR}$

17. The distance travelled by centre of disc from its initial position till the left end of plank comes vertically below the centre of disc is

(*a*) $\frac{L}{2}$ (*b*) $\frac{L}{4}$ (*c*) $\frac{L}{8}$ (*d*) L

[illegible] figure shows an isosceles [illegible] plate of mass M and base L. The angle at the apex is 90°. The apex lies at the origin and the base is parallel to x-axis.



18. The moment of inertia of the plate about the z-axis is

(a) $\frac{ML^2}{12}$ (b) $\frac{ML^2}{24}$

(c) $\frac{ML^2}{6}$ (d) none of these

19. The moment of inertia of the plate about the x-axis is

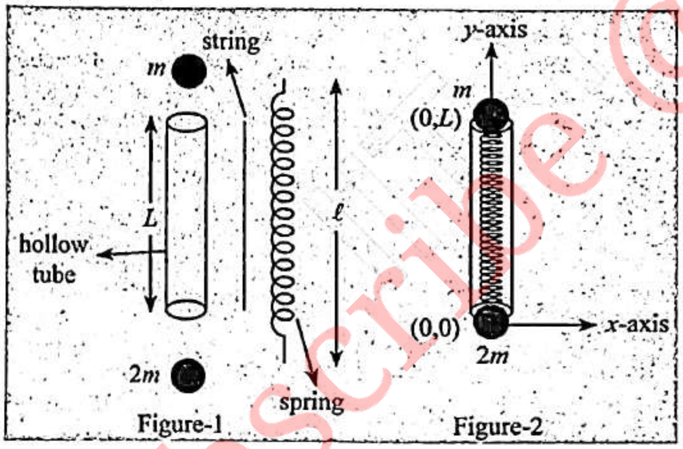
(a) $\frac{ML^2}{8}$ (b) $\frac{ML^2}{32}$ (c) $\frac{ML^2}{24}$ (d) $\frac{ML^2}{6}$

20. The moment of inertia of the plate about its base parallel to the x-axis is

(a) $\frac{ML^2}{18}$ (b) $\frac{ML^2}{36}$

(c) $\frac{ML^2}{24}$ (d) none of these

Comprehension (Q. 21 to 23): The following set of figures shows components of a mechanical system in figure-1 and assembled system in figure-2. There are two small balls of mass m and $2m$, a massless tube of length L, a spring of natural length ℓ $(> L)$ and an inextensible, massless rope of length L. In the assembly, the string holds the two balls in position, keeping the spring compressed.



The arrangement lies on a smooth horizontal table (x-y plane) as shown. At this moment, the velocity of centre of mass of system is $v_0\,\hat{i}$ and angular velocity is $\vec{\omega} = \omega_0\hat{k}$. Take this moment as $t = 0$.

21. The kinetic energy of the system in centre of mass frame is

(a) $\frac{3}{2}m\omega_0^2L^2$ (b) $\frac{1}{3}m\omega_0^2L^2$

(c) $\frac{2}{3}m\omega_0^2L^2$ (d) $\frac{4}{3}m\omega_0^2L^2$

22. Angular momentum of the system about origin of the table frame (the point where $2m$ is located at $t = 0$) is

(a) $3\,mL\left[\frac{\omega_0 L}{2} - 2v_0\right](-\hat{k})$

(b) $-mL\left[v_0 + \frac{2}{3}\omega_0 L\right]\hat{k}$

(c) $-mL\left(v_0 - \frac{2}{3}\omega_0 L\right)\hat{k}$

(d) $-mL\left(v_0 - \frac{2}{9}\omega_0 L\right)\hat{k}$

23. At a time $t = t_0$, the string is suddenly broken and the balls are released to move. Mark the incorrect statement. Consider the momentum and angular momentum after and before string breaks.

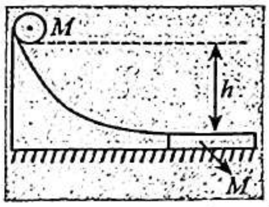
(a) Total angular momentum of system is conserved in table reference frame as well as centre of mass reference frame.

(b) In the centre of mass frame, angular momentum of each of the masses is separately conserved.

(c) Total linear momentum of system is conserved in table reference frame.

(d) In the table frame momentum of individual balls is conserved.

Comprehension (Q. 24 to 27): A solid cylinder of radius r and mass M slides down a smooth hill by a height h from rest and gets onto the long plank of mass M lying on the smooth horizontal plane at the base of the hill as shown in the figure. Due to friction between the cylinder and plank, the cylinder slows down and starts rolling without friction over the plank. While the cylinder is rolling (without sliding) over the plank, the velocity of the plank is v_P, the velocity of centre of mass of the cylinder is v_C and angular velocity of the cylinder is ω.



24. The net work performed by friction in ground frame and in plank frame

(a) are equal

(b) are unequal but both negative

(c) are positive and negative, respectively

(d) are negative and positive, respectively

25. The velocity of centre of mass (v_{cm}) of the plank-cylinder system when the cylinder is rolling over the plank is

(a) constant and equals to $\frac{1}{2}\sqrt{2gh}$

(b) variable with $(v_{cm})_{max} = \frac{1}{2}\sqrt{2gh}$

(c) constant and equal to $\sqrt{2gh}$

(d) variable with $(v_{cm})_{max} = \sqrt{2gh}$

26. Which of the following is correct

(*a*) $v_C = \omega r$ (*b*) $v_C < \omega r$

(*c*) $v_C > \omega r$ (*d*) none

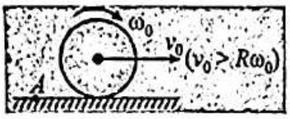
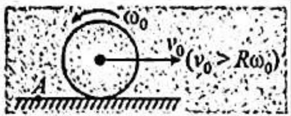
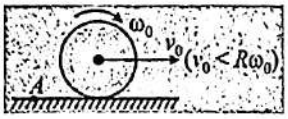
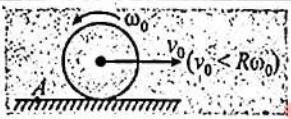
27. Which of the following is incorrect

(*a*) $2v_p + \omega r = \sqrt{2gh}$ (*b*) $2v_C - \omega r = \sqrt{2gh}$

(*c*) $v_p = v_C - \omega r$ (*d*) none

MATCH THE COLUMN TYPE QUESTIONS

28. In each situation of column-I, a uniform disc of mass m and radius R rolls on a rough fixed horizontal surface as shown in the figure. At $t = 0$ (initially) the angular velocity of disc is ω_o and velocity of centre of mass of disc is v_o (in horizontal direction). The relation between v_o and ω_o for each situation and also initial sense of rotation is given for each situation in column-I. Then match the statements in column-I with the corresponding results in column-II.

	Column-I		Column-II
A.		p.	The angular momentum of disc about point A (as shown in figure) remains conserved.
B.		q.	The kinetic energy of disc after it starts rolling without slipping is less than its initial kinetic energy.
C.		r.	In the duration disc rolls with slipping, the friction acts on disc towards left.
D.		s.	In the duration disc rolls with slipping, the friction acts on disc for some time towards right and for some time towards left.

(*a*) A → (p,r); B → (p,r); C → (p); D → (q,r)

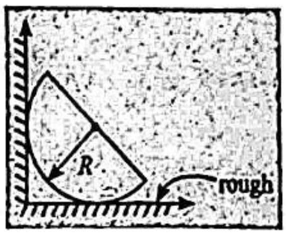
(*b*) A → (p,q,r); B → (p,q,r); C → (p,q); D → (p,q,r)

(*c*) A → (q,r); B → (q,r); C → (q); D → (q,r)

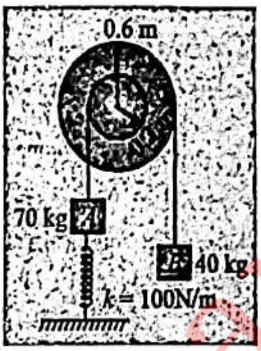
(*d*) A → (p,q); B → (p,q); C → (q); D → (p,q,r)

NUMERICAL TYPE QUESTIONS

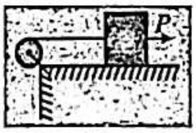
29. A solid hemisphere rests in equilibrium on a rough ground and against a smooth wall. The curved surface touches the wall and the ground. The angle of inclination of the circular base to the horizontal is 30°, the minimum coefficient of friction required between ground and hemisphere is μ. Find the value of 32μ.



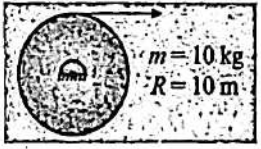
30. The figure shown is in equilibrium. Find out the extension in the spring (in cm).



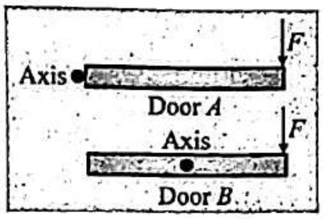
31. A wheel of radius 0.20 m is mounted on a frictionless horizontal axis. The rotational inertia of the wheel about the axis is 0.07 kg m^2. A massless cord wrapped around the wheel is attached to a 2.0 kg block that slides on a horizontal frictionless surface. If a horizontal force of magnitude $P = 3.0$ N is applied to the block as shown in figure, what is the magnitude of the angular acceleration (in rad/s^2) of the wheel? Assume the string does not slip on the wheel.



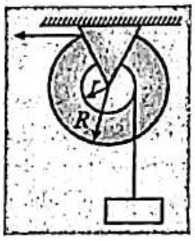
32. A force of 100 N is applied on a disc along the tangent for 10 sec. During this time, the disc attains an angular velocity of 10 rad/s. There is constant frictional torque at the axis which opposes the rotation of the disc. Now the force is removed. After what time (in sec) from the removal of force will the disc come to rest?



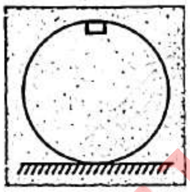
33. The drawing shows the top view of two doors. The doors are uniform and identical. Door A rotates about an axis through its left edge, while door B rotates about an axis through the center. The same force F is applied perpendicular to each door at its right edge, and the force remains perpendicular as the door turns. Starting from rest, door A rotates through a certain angle in 3 s. How long (in sec.) does it take door B to rotate through the same angle?



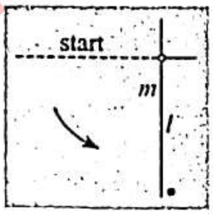
34. A yo-yo-shaped device mounted on a horizontal frictionless axis is used to lift a 30 kg box as shown in figure. The outer radius R of the device is 0.50 m, and the radius r of the hub is 0.20 m. When a constant horizontal force of magnitude 152 N is applied in the left direction to a rope wrapped around the outside of the device, the box, which is suspended from a rope wrapped around the hub, has an upward acceleration of magnitude 0.80 m/s^2. What is the rotational inertia I (in kg-m^2) of the device about its axis of rotation?



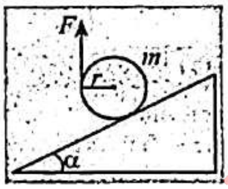
35. A ring of mass $m = 1$ kg and radius $R = 1.25$ m is kept on a rough horizontal ground. A small body of mass m is struck to the top of the ring. When it was given a slight push forward, the ring started rolling purely on the ground. What is the maximum speed of the centre of the ring (in m/s)?



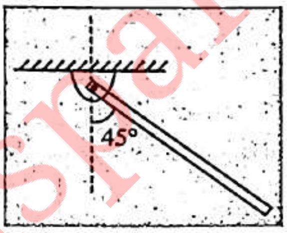
36. A stick of mass $m = 300$ gm and length $\ell = 30$ cm is pivoted at an end. It is held horizontally and then released. It swings down, and when it is vertical, the free end elastically collides with a ball, as shown. (Assume that the ball is initially held, and then released a split second before the stick strikes it.) If the stick loses half of its angular velocity during the collision maintaining its direction of motion, what is the mass of the ball (in gm)?



37. A solid cylinder of mass $m = 6$ kg and radius $r = 0.1$ m is kept in balance on a slope of inclination $\alpha = 37°$ with the help of a thread fastened to its jacket. The cylinder does not slip on the slope. What is the minimum required coefficient of friction to keep the cylinder in balance when the thread is held vertically?

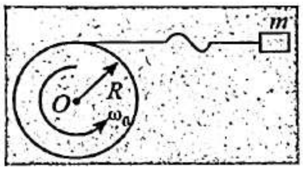


38. A uniform rigid rod of mass m hinged at one end is released from rest in the position shown in the vertical plane. Find the magnitude of reaction force (in N) at hinge just after its release. (Use: $m = 0.8$kg., $\theta = 45°$, $g = 10$ m/s^2)

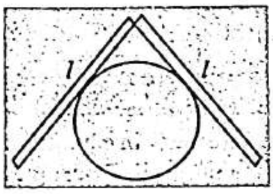


INTEGER TYPE QUESTIONS

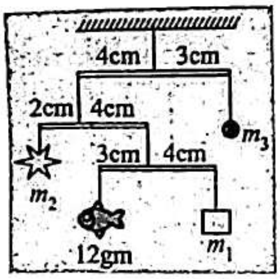
39. Disc of mass $M = 10$ kg and radius $R = 50$ cm is revolving about fixed vertical axis on smooth horizontal surface with angular velocity $\omega_0 = 10$ rad/sec. A block of mass $m = 5$kg is connected to disc through slack string as shown in the diagram. Find the angular velocity (in rad/s) when string become taut.



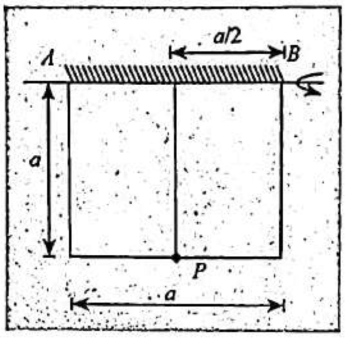
40. On a smooth horizontal beam of radius 1 m, we have placed two identical boards, connected by a hinge, as shown in Figure. Boards are in equilibrium, and the angle between the boards is 90°. Find the length of each board l (in m).



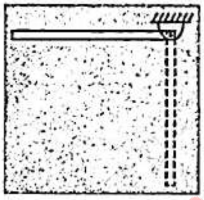
41. A souvenir is constructed by 3 light rods, light strings and four objects as shown. If the system is in equilibrium, what is mass m_3 (in gm)



42. A thin plate of mass 3 kg lies in the x-y plane and is free to rotate about an axis (AB) passing through its upper edge AB. A particle of mass 1 kg moving at 7.5 m/s makes a perfectly elastic collision at point P horizontally & perpendicular to its surface. Find angular velocity (in rad/s) of the plate just after collision. Take $a = 0.5$ m.

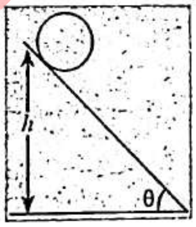


43. A uniform rod of mass 4 kg and length 1m is hinged at its end and held horizontal. The rod is released. At the moment the rod becomes vertical, the hinge breaks and the rod rotates about its centre of mass while falling freely. What is the rotational kinetic energy (in J) of the rod thereafter? [Take $g = 10$ m/s^2]



44. An L-shaped body having two parts of mass $\frac{1}{3}$ kg and $\frac{2}{3}$ kg and length 6m each rotates about an axis passing through the centre of mass and perpendicular to its plane. Find the moment of inertia of the body (in kgm^2) about this axis.

45. When a solid sphere is released on a fixed rough inclined plane so that it roll purely. When it reaches the ground it has translational kinetic energy of 100J. What would have been the translational kinetic energy at bottom if the plane was smooth? (In J)



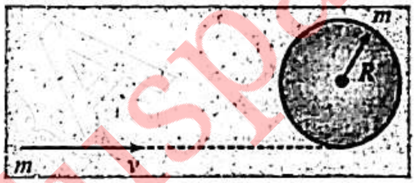
46. A solid sphere rolling on a rough horizontal surface with a linear speed 7 m/s collides elastically with a fixed smooth vertical wall. Then the speed of the sphere when it has started pure rolling after collision in m/s is.

SINGLE CORRECT TYPE QUESTIONS

47. An equilateral triangular wire frame is made from 3 rods of equal mass and length ℓ each. The frame is rotated about an axis perpendicular to the plane of the frame and passing through its end. What is the radius of gyration of the frame?

(*a*) $\frac{\ell}{2}$ (*b*) ℓ (*c*) $\frac{\ell}{\sqrt{2}}$ (*d*) $\frac{\ell}{2\sqrt{3}}$

48. A circular hoop of mass m, and radius R rests flat on a horizontal frictionless surface. A bullet, also of mass m, and moving with a velocity v, strikes the hoop and gets embedded in it. The thickness of the hoop is much smaller than R. The angular velocity with which the system rotates after the bullet strikes the hoop is



(*a*) $\frac{v}{4R}$ (*b*) $\frac{v}{3R}$ (*c*) $\frac{2v}{3R}$ (*d*) $\frac{3v}{4R}$

49. A solid sphere rolls down without slipping two different inclined planes of the same height but of different inclinations

(*a*) in both cases the speeds and time of descend will be same

(*b*) the speeds will be same but time of descend will be different

(*c*) the speeds will be different but time of descend will be same

(*d*) speeds and time of descend both will be different

50. A solid sphere with a velocity (of centre of mass) v and angular velocity ω is gently placed on a rough horizontal surface. The frictional force on the sphere:

(*a*) must be forward (in direction of v)

(*b*) must be backward (opposite to v)

(*c*) cannot be zero

(*d*) none of the above

51. A cylinder is pure rolling up an incline plane. It stops momentarily and then rolls back. The force of friction.

(*a*) on the cylinder is zero throughout the journey

(*b*) is directed opposite to the velocity of the centre of mass throughout the journey

(*c*) is directed up the plane throughout the journey

(*d*) is directed down the plane throughout the journey

52. When a bicycle is in motion (accelerating) on a rough horizontal plane, the force of friction exerted by the plane on the two wheels is such that it acts: (chain is attached to back wheel)

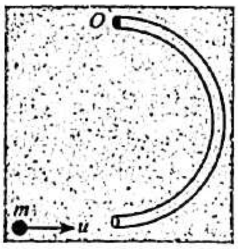
(*a*) In the backward direction on the front wheel and in the forward direction on the rear wheel

(*b*) In the forward direction on the front wheel and in the backward direction on the rear wheel

(*c*) In the backward direction on both front and the rear wheels

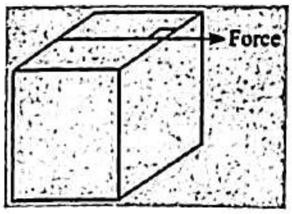
(*d*) In the forward direction on both the front and the rear wheels

53. A uniform semi circular ring of mass m is placed on smooth horizontal surface and is free to rotate about an axis passing through point 'O' and perpendicular to the plane of the ring. A particle of same mass is striking the semi circular ring at other end, elastically as shown in the figure. Find out angular velocity of the ring just after impact.



(*a*) $\frac{2u}{3R}$ (*b*) $\frac{3u}{2R}$ (*c*) $\frac{u}{2R}$ (*d*) $\frac{u}{R}$

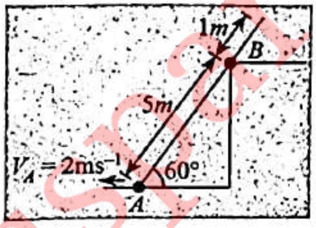
54. In the given figure, a solid uniform cube is placed on a horizontal surface. The co-efficient of friction between them is μ where $\mu < \frac{1}{2}$. A variable horizontal force perpendicular to one edge and passing through the midpoint of that edge is applied on the cube's upper face. The maximum acceleration with which it can move without toppling is



(*a*) μg (*b*) $\frac{g(1-2\mu)}{2}$

(*c*) $g(1-2\mu)$ (*d*) $\left(\mu+\frac{1}{2}\right)g$

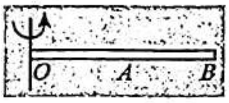
55. Velocity of point A on the rod is 2 m/s at the instant shown in the figure. The velocity of the point B on the rod at this instant is



(*a*) $\frac{2}{\sqrt{3}}$ m/s (*b*) 1m/s

(*c*) $\frac{1}{2\sqrt{3}}$ m/s (*d*) $\frac{\sqrt{3}}{2}$ m/s

56. A rod OB of length l, hinged about its end O is rotating with an angular velocity ω. The rod breaks at its mid point A and as a result the part OA starts rotating with angular velocity 2ω. Then the angular velocity of AB about its centre of mass is



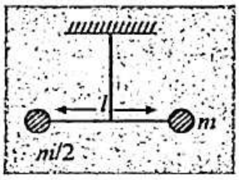
(*a*) ω (*b*) 3ω (*c*) 4ω (*d*) $\frac{3}{2}\omega$

PYQ's (Past Year Questions)

ANGULAR DISPLACEMENT VELOCITY AND ACCELERATION

1. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be

[09 Jan, 2019 (Shift-I)]

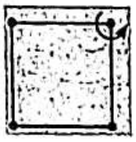


(*a*) $\frac{3k\theta_0^2}{l}$ (*b*) $\frac{2k\theta_0^2}{l}$

(*c*) $\frac{k\theta_0^2}{l}$ (*d*) $\frac{k\theta_0^2}{2l}$

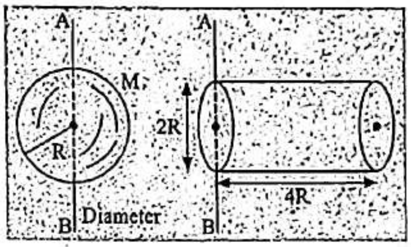
TORQUE, MOMENT OF INERTIA AND ITS THEOREMS

2. Four particles each of mass 1 kg are placed at four corners of a square of side 2 m. Moment of inertia of system about an axis perpendicular to its plane and passing through one of its vertex is _____ kgm^2. **[27 Jan, 2024 (Shift-I)]**



3. Two identical spheres each of mass 2 kg and radius 50 cm are fixed at the ends of a light rod so that the separation between the centers is 150 cm. Then, moment of inertia of the system about an axis perpendicular to the rod and passing through its middle point is $\frac{x}{20}$ kg m^2, where the value of x is **[31 Jan, 2024 (Shift-II)]**

4. Ratio of radius of gyration of a hollow sphere to that of a solid cylinder of equal mass, for moment of Inertia about their diameter axis AB as shown in figure is $\sqrt{\frac{8}{x}}$. The value of x is: **[05 April, 2024 (Shift-I)]**



(*a*) 34 (*b*) 17 (*c*) 67 (*d*) 51

5. Three balls of masses 2 kg, 4 kg and 6 kg respectively are arranged at centre of the edges of an equilateral triangle of side 2 m. The moment of inertia of the system about an axis through the centroid and perpendicular to the plane of triangle, will be _______ kg m^2. **[06 April, 2024 (Shift-II)]**

6. A uniform rod AB of mass 2 kg and Length 30 cm at rest on a smooth horizontal surface. An impulse of 0.2 Ns is applied to end B. The time taken by the rod to turn through right angles will be $\frac{\pi}{x}$s, where x = _______. **[01 Feb, 2024 (Shift-II)]**

7. A string is wrapped around the rim of a wheel of moment of inertia 0.40 kgm^2 and radius 10 cm. The wheel is free to rotate about its axis. Initially the wheel is at rest. The string is now pulled by a force of 40 N. The angular velocity of the wheel after 10 s is x rad/s, where x is _______. **[09 April, 2024 (Shift-I)]**

8. A heavy iron bar of weight 12 kg is having its one end on the ground and the other on the shoulder of a man. The rod makes an angle 60° with the horizontal, the weight experienced by the man is: **[27 Jan, 2024 (Shift-II)]**

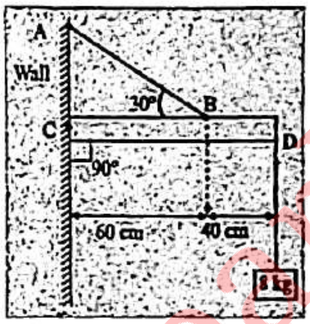
(*a*) 6 kg (*b*) 12 kg (*c*) 3 kg (*d*) $6\sqrt{3}$ kg

9. A heavy iron bar, of weight W is having its one end on the ground and the other on the shoulder of a person. The bar makes an angle θ with the horizontal. The weight experienced by the person is: **[09 April, 2024 (Shift-I)]**

(*a*) $W/2$ (*b*) W (*c*) $W\cos\theta$ (*d*) $W\sin\theta$

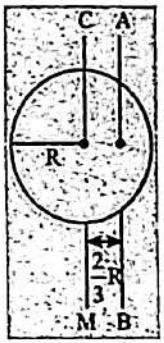
10. An object of mass 8 kg is hanging from one end of a uniform rod CD of mass 2 kg and length 1 m pivoted at its end C on a vertical wall as shown in figure below. It is supported by a cable AB such that the system is in equilibrium. The tension in the cable is:

(Take g = 10m/s^2) **[25 Jan, 2023 (Shift-I)]**

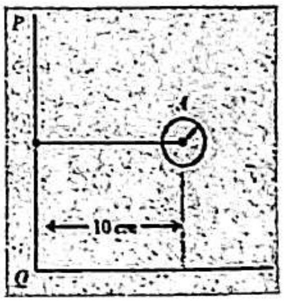


(*a*) 240 N (*b*) 90 N (*c*) 300 N (*d*) 30 N

11. I_{CM} is the moment of inertia of a circular disc about an axis (CM) passing through its center and perpendicular to the plane of the disc. I_{AB} is it's moment of inertia about an axis AB perpendicular to plane and parallel to the axis CM at a distance $\frac{2}{3}R$ from the center (where R is the radius of the disc). The ratio of I_{AB} and I_{CM} is x : 9. The value of x is _________ **[25 Jan, 2023 (Shift-I)]**



12. Solid sphere A is rotating about an axis PQ. If the radius of the sphere is 5 cm then its radius of gyration about PQ will be $\sqrt{x}\,cm$. The value of x is _________. **[24 Jan, 2023 (Shift-I)]**



13. A bar of mas $M = 1.00$ kg and length $L = 0.20$ m is lying on a horizontal frictionless surface. One end of the bar is pivoted at a point about which it is free to rotate. A small mass $m = 0.10$ kg is moving on the same horizontal surface with $5.00\ \text{ms}^{-1}$ speed on a path perpendicular to the bar. It hits the bar at a distance $L/2$ from the pivoted end and returns back on the same path with speed v. After this elastic collision, the bar rotates with an angular velocity ω. Which of the following statement is correct? **[JEE Adv, 2023]**

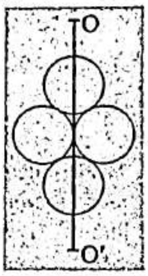
(*a*) $\omega = 6.98\ \text{rad s}^{-1}$ and $v = 4.30\ \text{m s}^{-1}$

(*b*) $\omega = 3.75\ \text{rad s}^{-1}$ and $v = 4.30\ \text{m s}^{-1}$

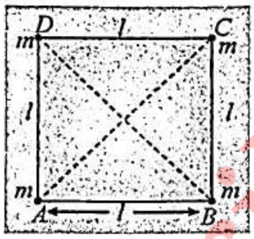
(*c*) $\omega = 3.75\ \text{rad s}^{-1}$ and $v = 10.0\ \text{m s}^{-1}$

(*d*) $\omega = 6.80\ \text{rad s}^{-1}$ and $v = 4.10\ \text{m s}^{-1}$

14. Four identical discs each of mass 'M' and diameter 'a' are arranged in a small plane as shown in figure. If the moment of inertia of the system about 'OO'' is $\frac{x}{4}Ma^2$. Then, the value of x will be **[28 July, 2022 (Shift-I)]**

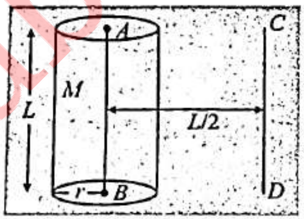


15. Four equal masses, m each are placed at the corners of a square of length (l) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be: **[16 March, 2021 (Shift-I)]**



(*a*) $2ml^2$ (*b*) ml^2 (*c*) $\sqrt{3}ml^2$ (*d*) $3ml^2$

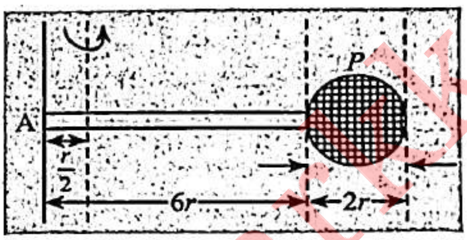
16. The solid cylinder of length 80 cm and mass M has a radius of 20 cm. Calculate the density of the material used if the moment of inertia of the cylinder about an axis CD parallel to AB as shown in figure is $2.7\ \text{kg m}^2$. **[26 Aug, 2021 (Shift-II)]**



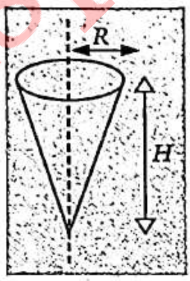
(*a*) $7.5 \times 10^1\ \text{kg/m}^3$ (*b*) $1.49 \times 10^2\ \text{kg/m}^3$

(*c*) $14.9\ \text{kg/m}^3$ (*d*) $7.5 \times 10^2\ \text{kg/m}^3$

17. Consider a badminton racket with length scales as shown in the figure. If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at, $\frac{r}{2}$ distance from the end A of the handle will ____________ Mr^2. **[26 Aug, 2021 (Shift-I)]**



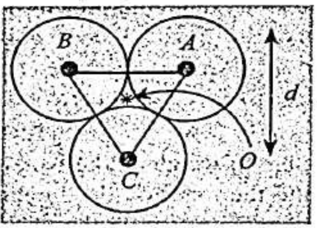
18. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is: **[06 Sep, 2020 (Shift-I)]**



(*a*) $\frac{MR^2}{2}$ (*b*) $\frac{M(R^2+H^2)}{4}$

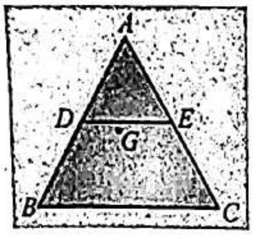
(*c*) $\frac{MH^2}{3}$ (*d*) $\frac{MR^2}{3}$

19. Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_O/I_A of moment of inertia I_O of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is: **[09 Jan, 2020 (Shift-I)]**

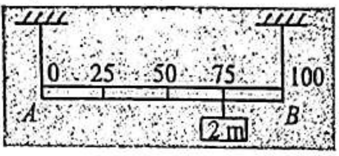


(*a*) $\frac{15}{13}$ (*b*) $\frac{13}{15}$ (*c*) $\frac{23}{13}$ (*d*) $\frac{13}{23}$

20. ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is ______. **[04 Sep, 2020 (Shift-I)]**



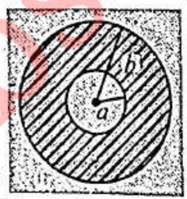
21. Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2m hung at a distance of 75 cm from A. The tension in the string at A is: **[02 Sep, 2020 (Shift-I)]**



(*a*) 0.75 mg (*b*) 1 mg (*c*) 2 mg (*d*) 0.5 mg

22. Put a uniform meter scale horizontally on your extended index fingers with the left one at 0.00 cm and the right one at 90.00 cm. When you attempt to move both the fingers slowly towards the center, initially only the left finger slips with respect to the scale and the right finger does not. After some distance the left finger stops and the right one starts slipping. Then the right finger stops at a distance x_R from the center (50.00 cm) of the scale and the let one starts slipping again. This happens because of the difference in the frictional forces on he two fingers. If the coefficients of static and dynamic friction between the fingers and the scale are 0.40 and 0.32, respectively, the value of x_R (in cm) is ________ **[JEE Adv, 2020]**

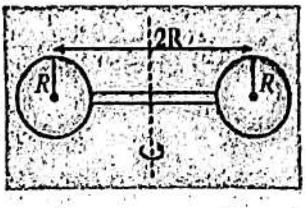
23. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\rho = \rho_0/r$ then the radius of gyration of the disc about its axis passing through the centre is: **[12 April, 2019 (Shift-I)]**



(*a*) $\frac{a+b}{2}$ (*b*) $\frac{a+b}{3}$

(*c*) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (*d*) $\sqrt{\frac{a^2+b^2+ab}{3}}$

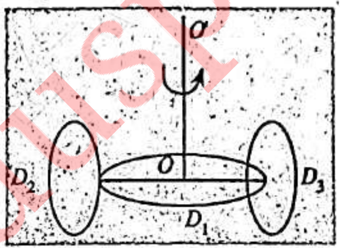
24. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is: **[10 Jan, 2019 (Shift-II)]**



(*a*) $\frac{137}{15}MR^2$ (*b*) $\frac{17}{15}MR^2$

(*c*) $\frac{209}{15}MR^2$ (*d*) $\frac{152}{15}MR^2$

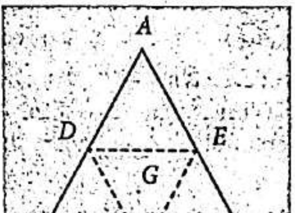
25. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly as its opposite ends (see figure). The moment of inertia of the system about the axis 'OO'', passing through the centre of D_1 as shown in the figure, will: **[11 Jan, 2019 (Shift-II)]**



(*a*) MR^2 (*b*) $3MR^2$

(*c*) $\frac{4}{5}MR^2$ (*d*) $\frac{2}{3}MR^2$

26. The equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then: **[11 Jan, 2019 (Shift-I)]**



(*a*) $I = \frac{15}{16}I_0$ (*b*) $I = \frac{3}{4}I_0$

(*c*) $I = \frac{9}{16}I_0$ (*d*) $I = \frac{I_0}{4}$

ENERGY ANALYSIS OF ROLLING MOTION

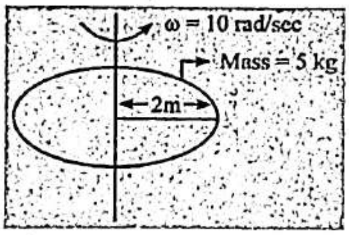
27. A body of mass 5 kg moving with a uniform speed $3\sqrt{2}\ \text{ms}^{-1}$ in X – Y plane along the line $y = x + 4$. The angular momentum of the particle about the origin will be ______ kg m^{2}s^{-1}. **[29 Jan, 2024 (Shift-II)]**

28. A particle of mass m projected with a velocity 'u' making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is: **[30 Jan, 2024 (Shift-I)]**

(*a*) $\dfrac{\sqrt{3}}{16}\dfrac{mu^3}{g}$ (*b*) $\dfrac{\sqrt{3}}{2}\dfrac{mu^2}{g}$

(*c*) $\dfrac{mu^3}{\sqrt{2}g}$ (*d*) zero

29. Consider a Disc of mass 5 kg, radius 2m, rotating with angular velocity of 10 rad/s about an axis perpendicular to the plane of rotation. An identical disc is kept gently over the rotating disc along the same axis. The energy dissipated so that both the discs continue to rotate together without slipping is _____ J. **[30 Jan, 2024 (Shift-I)]**



30. Two discs of moment of inertia $I_1 = 4\ \text{kg m}^2$ and $I_2 = 2\text{kg m}^2$ about their central axes & normal to their planes, rotating with angular speeds 10 rad/s & 4 rad/s respectively are brought into contact face to face with their axe of rotation coincident. The loss in kinetic energy of the system in the process is___________J. **[30 Jan, 2024 (Shift-II)]**

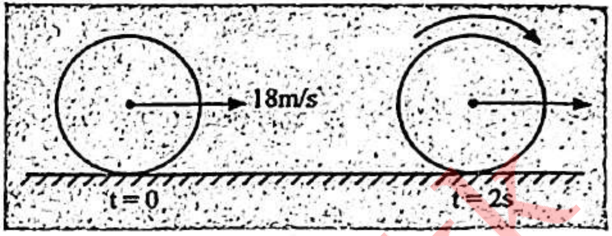
31. A body of mass 'm' is projected with a speed 'u' making an angle of 45° with the ground. The angular momentum of the body about the point of projection, at the highest point is expressed as $\dfrac{\sqrt{2}mu^3}{Xg}$ The value of 'X' is_______. **[31 Jan, 2024 (Shift-II)]**

32. If the radius of earth is reduced to three-fourth of its present value without change in its mass then value of duration of the day of earth will be _____ hours 30 minutes. **[06 April, 2024 (Shift-I)]**

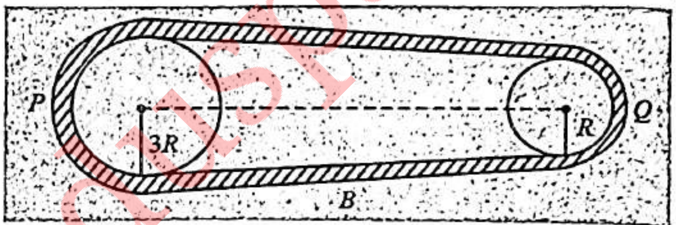
33. A thin circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with angular velocity ω. If another disc of same dimensions but of mass $M/2$ is placed gently on the first disc co-axially, then the new angular velocity of the system is: **[08 April, 2024 (Shift-II)]**

(*a*) $\frac{4}{5}\omega$ (*b*) $\frac{5}{4}\omega$ (*c*) $\frac{2}{3}\omega$ (*d*) $\frac{3}{2}\omega$

34. A uniform disc of mass 0.5 kg and radius r is projected with velocity 18 m/s at $t = 0$ s on a rough horizontal surface. It starts off with a purely sliding motion at $t = 0$ s. After 2 s it acquires a purely rolling motion (see figure). The total kinetic energy of the disc after 2 s will be ___________ J (given, coefficient of friction is 0.3 and $g = 10\ \text{m/s}^2$) **[30 Jan, 2023 (Shift-II)]**

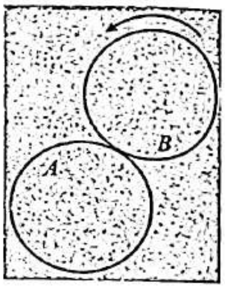


35. In the given figure, two wheels P and Q are connected by a belt B. The radius of P is three times as that of Q. In case of same rotational kinetic energy, the ratio of rotational inertias $\left(\dfrac{I_1}{I_2}\right)$ will be x : 1. The value of x will be __________. **[27 July, 2021 (Shift-II)]**



36. A particle of mass 1 kg is subjected to a force which depends on the position as with $\vec{F} = -k\left(x\hat{i} + y\hat{j}\right)\text{kg ms}^{-2}$. At time $t = 0$, the particle's position $\vec{r} = \left(\dfrac{1}{\sqrt{2}}\hat{i} + \sqrt{2}\hat{j}\right)\text{m}$ and its velocity $\vec{v} = \left(-\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \dfrac{2}{\pi}\hat{k}\right)\ \text{ms}^{-1}$. Let v_x and v_y denote the x and the y components of the particle's velocity, respectively. Ignore gravity. When $z = 0.5$ m, the value of $(xv_y - yv_x)$ is ________ m^2 s^{-1}. **[JEE Adv, 2022]**

37. A flat surface of a thin uniform disk A of radius R is glued to a horizontal table. Another thin uniform disk B of mass M and with the same radius R rolls without slipping on the circumference of A, as shown in the figure. A flat surface of B also lies on the plane of the table. The center of mass of B has fixed angular speed ω about the vertical axis passing through the center of A. The angular momentum of B is $nM\omega R^2$ with respect to the center of A. Which of the following is the value of n? **[JEE Adv, 2022]**



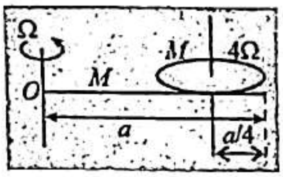
(*a*) 2 (*b*) 5

(c) 7/2 (*d*) 9/2

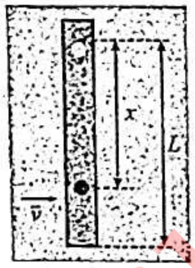
of a [illegible] square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is **[06 Sep, 2020 (Shift-I)]**

(*a*) $3\,m\ell^2\omega$ (*b*) $5\,m\ell^2\omega$ (*c*) $m\ell^2\omega$ (*d*) $2\,m\ell^2\omega$

39. A thin rod of mass M and length a is free to rotate in horizontal plane about a fixed vertical axis passing through point O. A thin circular disc of mass M and of radius $a/4$ is pivoted on this rod with its center at a distance $a/4$ from the free end so that it can rotate freely about its vertical axis, as shown in the figure. Assume that both the rod and the disc have uniform density and they remain horizontal during the motion. An outside stationary observer finds the rod rotating with an angular velocity Ω and the disc rotating about its vertical axis with angular velocity $4\,\Omega$. The total angular momentum of the system about the point O is $\left(\frac{Ma^2\Omega}{48}\right)n$. The value of n is. **[JEE Adv, 2020]**



40. A rod of mass m and length L, pivoted at one of its ends, is hanging vertically. A bullet of the same mass moving at speed v strikes the rod horizontally at a distance x from its pivoted end and gets embedded in it. The combined system now rotates with angular speed ω about the pivot. The maximum angular speed ω_m is achieved for $x = x_M$. Then **[JEE Adv, 2020]**



(*a*) $\omega = \frac{3vx}{L^2+3x^2}$ (*b*) $\omega = \frac{12vx}{L^2+12x^2}$

(*c*) $x_M = \frac{L}{\sqrt{3}}$ (*d*) $\omega_M = \frac{v}{2L}\sqrt{3}$

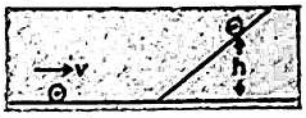
COMBINED ROTATION AND TRANSLATION

41. A ring and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of both bodies are identical and the ratio of their kinetic energies is $\frac{7}{x}$ where x is ______. **[27 Jan, 2024 (Shift-II)]**

42. A cylinder is rolling down on an inclined plane of inclination 60°. It's acceleration during rolling down will be $\frac{x}{\sqrt{3}}$ m/s^2, where x = ______. (use $g = 10$ m/s^2). **[29 Jan, 2024 (Shift-I)]**

floor so that its center of mass has a speed of 0.4 m/s. The absolute value of work done on the disc to stop it is ____ J. **[31 Jan, 2024 (Shift-I)]**

44. A disc of radius R and mass M is rolling horizontally without slipping with speed v. It then moves up an inclined smooth surface as shown in figure. The maximum height that the disc can go up the incline is: **[01 Feb, 2024 (Shift-II)]**



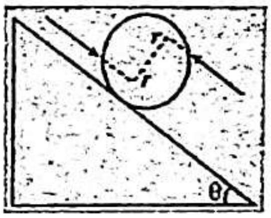
(*a*) $\frac{v^2}{g}$ (*b*) $\frac{3}{4}\frac{v^2}{g}$ (*c*) $\frac{1}{2}\frac{v^2}{g}$ (*d*) $\frac{2}{3}\frac{v^2}{g}$

45. A solid sphere and a hollow cylinder roll up without slipping on same inclined plane with same initial speed v. The sphere and the cylinder reaches upto maximum heights h_1 and h_2, respectively, above the initial level. The ratio $h_1 : h_2$ is $\frac{n}{10}$. The value of n is_____. **[04 April, 2024 (Shift-I)]**

46. A hollow sphere is rolling on a plane surface about its axis of symmetry. The ratio of rotational kinetic energy to its total kinetic energy is $x/5$. The value of x is ________. **[05 April, 2024 (Shift-II)]**

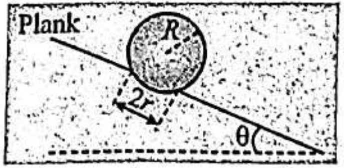
47. A circular disc reaches from top to bottom of an inclined plane of length l. When it slips down the plane, it takes t s. When it rolls down the plane then it takes $\left(\frac{\alpha}{2}\right)^{1/2} t$ s, where α is .... **[09 April, 2024 (Shift-II)]**

48. A solid sphere of mass 1 kg and radius 1 m rolls without slipping on a fixed inclined plane with an angle of inclination $\theta = 30°$ from the horizontal. Two forces of magnitude 1 N each, parallel to the incline, act on the sphere, both at distance $r = 0.5$ m from the center of the sphere, as shown in the figure. The acceleration of the sphere down the plane is____ ms^{-2}. (Take $g = 10$ ms^{-2}.) **[JEE Adv, 2022]**



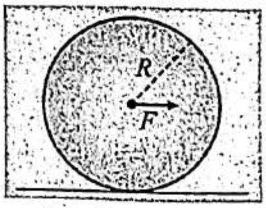
49. At time $t = 0$, a disk of radius 1 m starts to roll without slipping on a horizontal plane with an angular acceleration of $\alpha = \frac{2}{3}$ rad s^{-2}. A small stone is stuck to the disk. At $t = 0$, it is at the contact point of the disk and the plane. Later, at time $t = \sqrt{\pi}$ s, the stone detaches itself and flies off tangentially from the disk. The maximum height (in m) reached by the stone measured from the plane is $\frac{1}{2}+\frac{x}{10}$. The value of x is [Take $g = 10$ ms^{-2}]. **[JEE Adv, 2020]**

50. A football of radius R is kept on a hole of radius r $(r < R)$ made on a plank kept horizontally. One end of the plank is now lifted so that it gets tilted making an angle θ from the horizontal as shown in the figure below. The maximum value of θ so that the football does not start rolling down the plank satisfies (figure is schematic and not drawn to scale)
[JEE Adv, 2020]



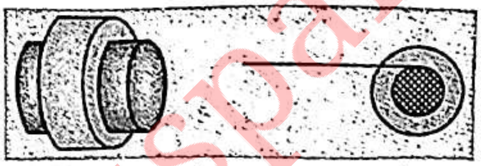
(*a*) $\sin\theta = \dfrac{r}{R}$
(*b*) $\tan\theta = \dfrac{r}{R}$
(*c*) $\sin\theta = \dfrac{r}{2R}$
(*d*) $\cos\theta = \dfrac{r}{2R}$

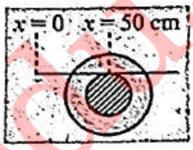
51. A horizontal force F is applied at the centre of mass of a cylindrical object of mass m and radius, perpendicular to its axis as shown in the figure. The coefficient of friction between the object and the ground is μ. The centre of mass of the object has an acceleration a. The acceleration due to gravity is g. Given that the object rolls without slipping, which of the following statement(s) is(are) correct?
[JEE Adv, 2020]

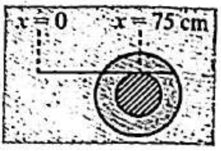


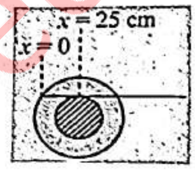
(*a*) For the same F, the value of a does not depend on whether the cylinder is solid or hollow
(*b*) For a solid cylinder, the maximum possible value of a is $2\mu g$
(*c*) The magnitude of the frictional force on the object due to the ground is always μmg
(*d*) For a thin-walled hollow cylinder, $a = \dfrac{F}{2m}$

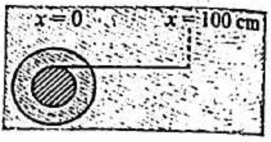
52. A small roller of diameter 20 cm has an axle of diameter 10 cm (see figure below on the left). It is on a horizontal floor and a meter scale is positioned horizontally on its axle with one edge of the scale on top of the axle (see figure on the right). The scale is now pushed slowly on the axle so that it moves without slipping on the axle, and the roller starts rolling without slipping. After the roller has moved 50 cm, the position of the scale will look like (figures are schematic and not drawn to scale)
[JEE Adv, 2020]



(*a*)


(*b*)


(*c*)


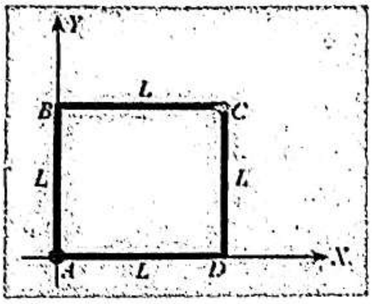
(*d*)


VISUAL TRACKING

- Follow a slow-moving object, such as a pencil or finger, with your eyes in a circular or figure-8 motion.
- This activity will help you improve focus through enhanced eye coordination.

PW Challengers

1. A very light square frame *ABCD* of side length *L* is placed on a smooth horizontal surface. A particle of mass *m* is attached to vertex *A* and insect of mass *m* sits on vertex *C* as shown in the figure. Initially insect and frame are at rest. Now insect moves on the frame on side *BC* and reaches point *B*. At this moment coordinates of point *A* will be



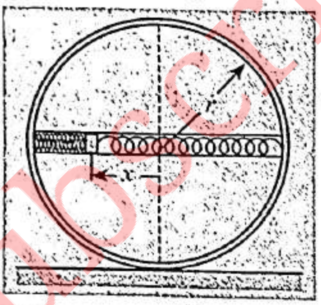
(*a*) $\left(\frac{L}{2},0\right)$

(*b*) $\left(\frac{L}{2\sqrt{2}},\frac{L}{2\sqrt{2}}\right)$

(*c*) $\left(\frac{L}{2}\left[1-\frac{1}{\sqrt{2}}\right],\frac{L}{2}\left[1-\frac{1}{\sqrt{2}}\right]\right)$

(*d*) $\left(\frac{L}{\sqrt{2}},\frac{L}{2}\right)$

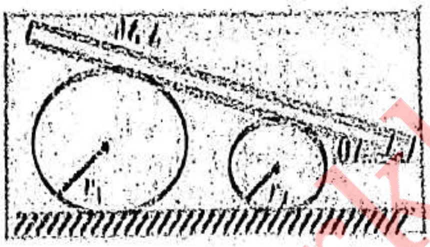
2. A light rigid tube is welded along a diameter of a uniform cylindrical pipe of mass *M* and radius *r*. A bead of mass *m* connected by two identical light springs each of force constant *k* can slide inside the tube. Other ends of the springs are connected to the pipe as shown in the figure. The pipe is held motionless on a horizontal floor, the bead is pulled aside a distance *x* away from the centre of the pipe and then the pipe and the bead both are released simultaneously. If the cylinder rolls without slipping on the ground, find acceleration of its centre immediately after the system is released.



$\frac{(mg+2kr)xr}{mx^2+2Mr^2}$ (*b*) $\frac{(mg+2kr)xr}{2Mr^2}$

$\frac{(mg+2kr)xr}{Mr^2}$ (*d*) $\frac{(mg+2kr)xr}{mx^2+Mr^2}$

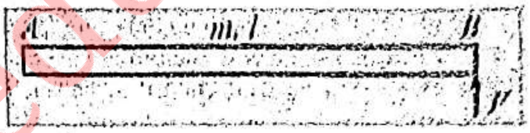
3. A slab of mass *M* is placed on two very light cylinders of unequal radii which are lying on a rough horizontal surface. The slab makes an angle θ with the horizontal (see figure). If the slab is released from rest and there is no slipping anywhere, the acceleration of the slab with respect to ground is



(*a*) $g\sin(2\theta)$ (*b*) $g\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

(*c*) $\frac{g}{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ (*d*) $g\sin\frac{\theta}{2}$

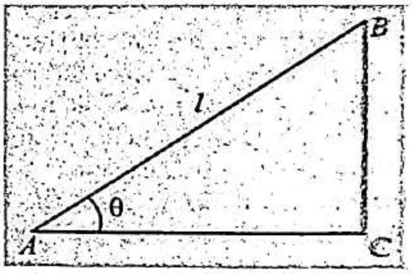
4. A rod of mass *m* and length *l* is is lying on a rough horizontal surface. The coefficient of friction between the rod and horizontal surface is μ. A force *F* is applied on the rod at end *B* perpendicular to the length of the rod. Find the minimum force *F* at which rod begins to slide on the surface.



(*a*) $\frac{\mu mg}{3}$ (*b*) $\frac{\mu mg}{\sqrt{2}}$

(*c*) $\frac{2\mu mg}{3}$ (*d*) $\mu mg(\sqrt{2}-1)$

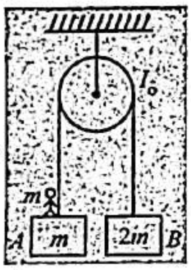
5. A uniform right angled triangular lamina is placed on a rough horizontal floor. One of the acute angle of the lamina is θ as shown in the figure by the top view of the situation. If τ_A and τ_B are the minimum torques required to rotate the lamina about stationary vertical axis passing through *A* and *B* respectively then the minimum torque required to rotate the lamina about a stationary vertical axis passing through vertex *C* is



(*a*) $\tau_A\cos^2\theta+\tau_B\sin^2\theta$ (*b*) $\tau_A\sin^2\theta+\tau_B\cos^2\theta$

(*c*) $\tau_A\cos^3\theta+\tau_B\sin^3\theta$ (*d*) $\tau_A\sin^3\theta+\tau_B\cos^3\theta$

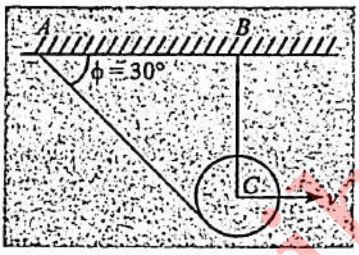
6. In figure shown below, a man of mass m stands on box A. The two boxes A and B of mass m and $2m$ respectively are connected by a light cord and are hanging in air. The cord passes over a pulley of moment inertia $I_o = \frac{mR^2}{2}$. The friction between the cord and pulley is sufficient to prevent slipping. If the man starts climbing the rope with an acceleration $\frac{g}{7}$ with respect to the cord, then the force applied by the man on the cord while he is climbing is



(a) $\frac{8}{7}mg$ (b) $\frac{9}{8}mg$

(c) $\frac{11}{10}mg$ (d) $\frac{10}{9}mg$

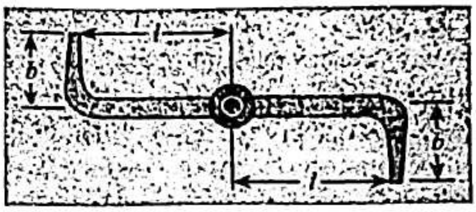
7. Free end of a cord wrapped on disc of mass m and radius R is affixed to a ceiling at point A. Another cord is affixed to center of the disc and point B on the ceiling as shown in figure below. If the velocity of the center of the disc at an instant is v in the horizontal direction, then the kinetic energy of the disc is



(a) $\frac{11}{16}mv^2$ (b) $\frac{9}{16}mv^2$

(c) $\frac{7}{16}mv^2$ (d) $\frac{5}{16}mv^2$

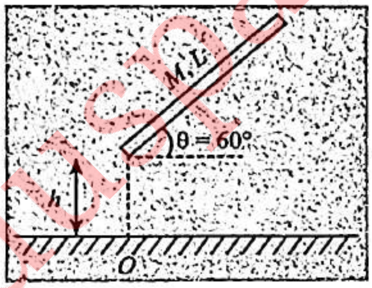
8. A water sprinkler system consists of a tube bent into S shape. This tube rotates in a horizontal plane about a vertical axis through its centre when water flows out from its nozzles. Top view of the tube is shown in the figure. Moment of inertia of the rotating tube including water about the axis is I, cross-section of outlet of a nozzle is S, density of water is ρ, water flow velocity relative to the nozzles is u and length b of the nozzles is much smaller than the length l of the tube arms. Neglect all types of frictional and viscous effects. An expression for the angular velocity ω of the tube as a function of time t.



(a) $\frac{2u^2\rho Sl}{I}t$ (b) $\frac{u^2\rho Sl}{2I}t$

(c) $\frac{u}{l}\left[1-e^{-\frac{2\rho Sl^2u}{I}t}\right]$ (d) $\frac{u}{l}\left[1-e^{-\frac{\rho Sl^2u}{2I}t}\right]$

9. A rod of mass M and length L is released from rest as shown in the figure. It strikes a smooth floor at point O. If the collision is perfectly inelastic, then

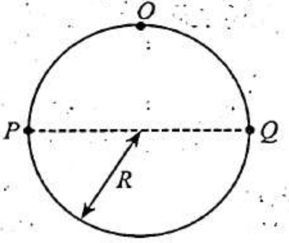


the impulse on the rod during collision is

(a) $\frac{3M}{7}\sqrt{2gh}$ (b) $\frac{2M}{7}\sqrt{2gh}$

(c) $\frac{6M}{7}\sqrt{2gh}$ (d) $\frac{4M}{7}\sqrt{2gh}$

10. A ring of mass m and radius R wears two identical beads P and Q each of mass m. Ring is hinged at its top point O so that it can rotate in its plane about the horizontal axis passing through point O. Bead P is affixed on the ring and bead Q is held so that they occupy position of the ends of the horizontal diameter of the ring and the system is at rest. Frictional forces are negligible. The initial acceleration of bead P when bead Q is released is $\frac{\sqrt{2}g}{nR}$. Find n.



ANSWER KEY

CONCEPT APPLICATION

1. (a) [2], (b) [1]   2. [16,4]   3. [800]   4. (a) [8], (b) [4]   5. (d)   6. (c)   7. (a)   8. (c)
9. (b)   10. (b)   11. (d)   12. (d)   13. (d)   14. (a)   15. (d)   16. (c)   17. (c)   18. (b)
19. (b)   20. [6,10]   21. [2]   22. (a)   23. [$-\hat{k}, 2\sqrt{2}$]   24. (a)   25. (a)   26. (d)   27. (b)
28. (b)   29. (b)   30. [2]   31. (c)   32. [1,3]   33. [3,2]   34. (a)   35. (c)   36. (a)   37. (c)

SCHOOL LEVEL PROBLEMS

1. (b)   2. (a)   3. (c)   4. (d)   5. (c)   6. (b)   7. (b)   8. (a)   9. (d)   10. (b)
11. (a)   12. (a)   24. (i)-(c), (ii)-(b)   25. (i)-(b), (ii)-(d)

PRARAMBH (TOPICWISE)

1. (a)   2. (d)   3. (c)   4. (d)   5. (a)   6. (d)   7. (b)   8. (b)   9. (c)   10. (b)
11. (d)   12. (b)   13. (a)   14. (a)   15. (b)   16. (d)   17. (c)   18. (c)   19. (c)   20. (a)
21. (c)   22. (b)   23. (a)   24. (d)   25. (c)   26. (d)   27. (d)   28. (b)   29. (b)   30. (d)
31. (c)   32. (a)   33. (a)   34. (d)   35. (b)   36. (c)   37. (c)   38. (a)   39. (c)   40. (b)
41. (a)   42. (b)   43. (d)   44. (a)   45. (b)   46. (a)   47. (c)   48. (b)   49. (c)   50. (b)
51. (a)

PRABAL (JEE MAIN LEVEL)

1. (b)   2. (a)   3. (a)   4. (c)   5. (b)   6. (d)   7. (b)   8. (d)   9. (b)   10. (d)
11. (d)   12. (a)   13. (c)   14. (b)   15. (c)   16. (d)   17. (a)   18. (d)   19. (a)   20. (d)
21. (c)   22. (b)   23. (c)   24. (c)   25. (b)   26. (d)   27. (b)   28. (a)   29. (a)   30. (a)
31. (a)   32. (d)   33. (b)   34. (c)   35. (b)   36. (a)   37. (d)   38. (d)   39. (a)   40. (d)
41. [9]   42. [20]   43. [15]   44. [3]   45. [181]   46. [48]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (a, b, c)   2. (b, c)   3. (b, c, d)   4. (a, b, c)   5. (b, c)   6. (a, c, d)   7. (a, c, d)   8. (a, b, d)   9. (a, c)   10. (b, c)
11. (a, b, c)   12. (b)   13. (c)   14. (d)   15. (d)   16. (c)   17. (a)   18. (c)   19. (a)   20. (c)
21. (b)   22. (c)   23. (d)   24. (a)   25. (a)   26. (c)   27. (d)   28. (b)   29. [6.00]   30. [100.00]
31. [4.00]   32. [10.00]   33. [2.00]   34. [2.80]   35. [5.00]   36. [33.33]   37. [0.75]   38. [5.83]   39. [5]   40. [4]
41. [84]   42. [15]   43. [5]   44. [7]   45. [140]   46. [3]   47. (c)   48. (b)   49. (b)   50. (d)
51. (c)   52. (a)   53. (a)   54. (c)   55. (b)   56. (b)

PYQ's (PAST YEAR QUESTIONS)

1. (c)   2. [16]   3. [53]   4. (c)   5. (4)   6. [4]   7. [100]   8. (c)   9. (a)   10. (c)
11. [17]   12. [110]   13. (a)   14. [3]   15. (d)   16. (b)   17. [52]   18. (a)   19. (d)   20. [11]
21. (b)   22. [25.60]   23. (d)   24. (a)   25. (b)   26. (a)   27. [60]   28. (a)   29. [250]   30. (24)
31. [8]   32. [13]   33. (c)   34. [54]   35. [9]   36. [3]   37. (b)   38. (a)   39. [49]   40. (a, c, d)
41. [7]   42. [10]   43. [6]   44. (c)   45. [7]   46. [2]   47. (3)   48. [2.86]   49. [0.52]   50. (a)
51. (b, d)   52. (b)

PW CHALLENGERS

1. (c)   2. (a)   3. (d)   4. (d)   5. (d)   6. (d)   7. (a)   8. (c)   9. (d)   10. [5]